# Fuzzy Resource Allocation Mechanisms in Workflow Nets

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Keywords: Petri Net, Workflow Net, Resource Allocation, Fuzzy Time, Possibility Theory.

Abstract: The purpose of Workflow Management Systems is to execute Workflow processes. Workflow processes represent the sequence of activities that have to be executed within an organization to treat specific cases and to reach a well-defined goal. Therefore, it is to manage in the best possible way time and resources. The proposal of this work is to express in a more realistic way the resource allocation mechanisms when human behavior is considered in Workflow activities. In order to accomplish this, fuzzy sets delimited by possibility distributions will be associated with the Petri net models that represent human type resource allocation mechanisms. Additionally, the duration of activities that appear on the routes (control structure) of the Workflow process, will be represented by fuzzy time intervals produced through a kind of constraint propagation mechanism. New firing rules based on a joint possibility distribution will then be defined.

## 1 INTRODUCTION

The purpose of Workflow Management Systems is to execute Workflow processes. Workflow processes represent the sequence of activities that have to be executed within an organization to treat specific cases and to reach a well-defined goal. Of all notations used for the modeling of Workflow processes, Petri nets are very suitable (van der Aalst and van Hee, 2004), as they represent basic routings. Moreover, Petri nets can be used for specifying the real time characteristics of Workflow Management Systems (in the time Petri net case) as well as complex resource allocation mechanisms. As a matter of fact, late deliveries in an organization are generally due to resources overload.

Many papers have already considered the Petri net theory as an efficient tool for the modeling and analysis of Workflow Management Systems. In (van der Aalst and van Hee, 2004), Workflow nets, which are acyclic Petri net models used to represent Workflow process, are defined.

Workflow nets have been identified and widely used as a solid model of Workflow processes, for example in (Aalst, 1997), (van Hee et al., 2006), (Martos-Salgado and Rosa-Velardo, 2011), (Wang and Li, 2013). In (Ling and Schmidt, 2000), an extension of Workflow nets is presented. This model is called time Workflow net and associates time intervals with the transitions of the corresponding Petri net model. In (Kotb and Badreddin, 2005), an extended Workflow Petri net model is defined. Such a model allows for the treatment of critical resources which have to be used for specific activities in real time. In (Wang et al., 2009), a resource-oriented Workflow net (ROWN) based on a two-transition task model was introduced for resource-constrained Workflow modeling and analysis. Considering the possibility of task failure during execution, in (Wang and Li, 2013), a three-transition task model to specify a task start, end and failure was proposed. Additional research can be found in (Adogla and Collins, 2014), (He et al., 2014), (Deng et al., 2014) and (Guo et al., 2014).

The majority of existing models put their focus on the process aspect and do not consider important characteristics of the Workflow Management System. In (Aalst, 1997) and (van Hee et al., 2006) for example, the resource allocation mechanisms are represented only in an informal way. In (Ling and Schmidt, 2000), (Kotb and Badreddin, 2005) and (Wang and Li, 2013) resource allocation mechanisms are represented by simple tokens in places as it is generally the case in production systems (Lee and DiCesare, 1994). But a simple token in a place will not represent in a realistic way human employees who can treat simultaneously different cases in a single day, as it is usually the case in most Business processes.

The proposal of this work is to express in a more realistic way resource allocation mechanisms when

Cristina Jeske de Freitas J., Julia S. and Pereira de Rezende L..
Fuzzy Resource Allocation Mechanisms in Workflow Nets.
DOI: 10.5220/0005365304710478
In *Proceedings of the 17th International Conference on Enterprise Information Systems* (ICEIS-2015), pages 471-478
ISBN: 978-989-758-096-3

human behavior is considered. For that, fuzzy sets delimited by possibility distributions (Dubois and Prade, 1988) will be associated with the Petri net models that represent human type resource allocation mechanisms and fuzzy time intervals will be associated to activity durations. A firing mechanism using a joint possibility distribution will then be defined in order to associate through a single formalism explicit time constraints as well as resource availability information.

The remainder of this paper is as follows. Section 2 introduces the concepts of fuzzy sets and possibility measures. Section 3 shows Workflow modeling. Section 4 presents resource allocation mechanisms. Section 5 presents a fuzzy time constraint propagation mechanism. Section 6 defines new firing rules that consider fuzzy time constraints as well as fuzzy resource allocation mechanisms. Finally, section 7 concludes the paper and provides references for additional works.

## 2 FUZZY SETS AND POSSIBILITY MEASURES

The notion of fuzzy set was introduced by (Zadeh, 1965) in order to represent the gradual nature of human knowledge. For example, the size of a man could be considered by the majority of a population as small, normal, tall, etc. A certain degree of belief can be attached to each possible interpretation of symbolic information and can simply be formalized by a fuzzy set F of a reference set X that can be defined by a membership function $\mu_F(x) \in [0,1]$. In particular, for a given element $x \in X$, $\mu_F(x) = 0$ denote that x is not a member of the set F, $\mu_F(x) = 1$ denotes that $x$ is definitely a member of the set F, and intermediate values denote the fact that x is more or less an element of $F$. Normally, a fuzzy set is represented by a trapezoid $A = [a1, a2, a3, a4]$ as the one represented in figure 1 where the smallest subset corresponding to the membership value equal to 1 is called the core, and the largest subset corresponding to the membership value greater than 0 is called the support.

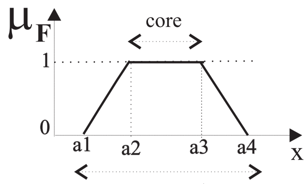


Figure 1: Representation of a fuzzy set.

There exist three particular cases of fuzzy sets that are generally considered:

- the triangular form where a2=a3,
- the imprecise case where a1=a2 and a3=a4,
- the precise case where a1=a2=a3=a4.

When considering two distincts fuzzy sets A and B, the basic operations are as follows (Klir and Yuan, 1995):

- the fuzzy sum $A \oplus B$ defined as: $[a1, a2, a3, a4] \oplus [b1, b2, b3, b4] = [a1 + b1, a2 + b2, a3 + b3, a4 + b4]$,
- the fuzzy subtraction $A \ominus B$ defined as: $[a1, a2, a3, a4] \ominus [b1, b2, b3, b4] = [a1 - b4, a2 - b3, a3 - b2, a4 - b1]$,
- the fuzzy product $A \otimes B$ defined as: $[a1, a2, a3, a4] \otimes [b1, b2, b3, b4] = [a1.b1, a2.b2, a3.b3, a4.b4]$.

A fuzzy set $F$ can be delimited by a possibility distribution $\Pi_f$, such as: $\forall x \in X$, $\Pi_f(x) = \mu_F(x)$ (Dubois and Prade, 1988), (Cardoso et al., 1999). Given a possibility distribution $\Pi_a(x)$, the measure of possibility $\Pi(S)$ and necessity $N(S)$ that a data $a$ belongs to a crisp set $S$ of $X$ is defined by $\Pi(S) = sup_{x \in S} \Pi_a(x)$ and $N(S) = inf_{x \notin S}(1 - \Pi_a(x)) = 1 - \Pi(\overline{S})$. If $\Pi(S) = 0$, it is impossible that $a$ belongs to $S$. If $\Pi(S) = 1$, it is possible that $a$ belongs to $S$, but it also depends on the value of $N(S)$. If $N(S) = 1$, it is certain that (the larger the value of $N(S)$, the more the proposition is believed in). In particular, there exists a duality relationship between the modalities of the possible and the necessary which postulates that an event is necessary when its contrary is impossible. Some practical examples of possibility and necessity measures are presented in (Dubois and Prade, 1988).

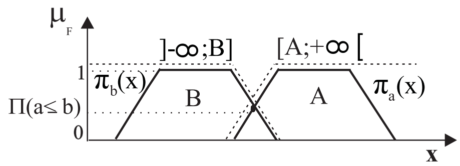


Figure 2: Possibility Measure.

Given two data $a$ and $b$ characterized by two fuzzy sets $A$ and $B$ as shown in figure 2, the measure of possibility and necessity of having $a \leq b$ are defined as:

$$\Pi(a \leq b) = sup_{x \leq y}(min(\Pi_a(x), min(\Pi_b(y))) = max([A, +\infty[\cap] - \infty, B]) \quad (1)$$

and

$$N(a \leq b) = 1 - sup_{x \leq y}(min(\Pi_a(x), min(\Pi_b(y))). \quad (2)$$

Given a normalized possibility distribution $\pi_a$, (Dubois and Prade, 1989) defines the following fuzzy sets of the time point that are:

- possibly after a: $\mu_{[A,+\infty[}(x) = sup_{x \in X} \pi_a(s)$ (see figure 3);
- necessarily after a: $\mu_{]A,+\infty[}(x) = inf_{x \in X}(1 - \pi_a(s))$ (see figure 3);
- possibly before a: $\mu_{]-\infty,A]}(x) = sup_{x \in X} \pi_a(s)$ (see figure 4);
- necessarily before a: $\mu_{]-\infty,A[}(x) = inf_{x \in X}(1 - \pi_a(s))$ (see figure 4).

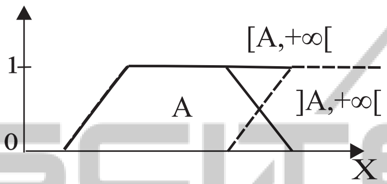


Figure 3: Possibly/necessarily after *a*.

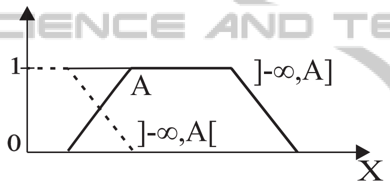


Figure 4: Possibly/necessarily before *a*.

A visibility time interval $[a,b]$ is a period of time between two dates $a$ and $b$. In the case where $a$ and $b$ are fuzzy dates $A$ and $B$ (delimited by $\pi_a$ and $\pi_b$) respectively, the interval $[a,b]$ is represented by the the following pair of fuzzy sets:

- $[A,B]$, the conjunctive set of time instants that represents the set of dates possibly after $A$ and possibly before $B$;
- $]A,B[$, the conjunctive set of time instants that represents the set of dates necessarily after $A$ and necessarily before $B$.

The joint possibility admits as upper bound in (Dubois and Prade, 1988):

$$\forall x \in X \quad \forall y \in Y \quad \pi(x,y) = min(\pi_X(x), \pi_Y(y)) \quad (3)$$

when the reference sets are non-interactive (the value of $x$ in $X$ has no influence on the value of $y$ in $Y$, and vice versa).

## 3 WORKFLOW MODELING

Modeling a Workflow process in terms of a Workflow net is rather straightforward: transitions are active components and models the tasks, places are passive components and model conditions (pre and post), and tokens model the cases to be treated (van der Aalst and van Hee, 2004).

To illustrate the mapping of a process into a Workflow net, the process for handling complaints, shown in (van der Aalst and van Hee, 2004) is considered: an incoming complaint is first recorded. Then the client who has complained along with the department affected by the complaint are contacted. The client is approached for more information. The department is informed of the complaint and may be asked for its initial reaction. These two tasks may be performed in parallel, i.e. simultaneously or in any order. After this, data is gathered and a decision is made. Depending upon the decision, either a compensation payment is made or a letter is sent. Finally, the complaint is filed. In Figure 5, a Workflow net that correctly models this process is shown.

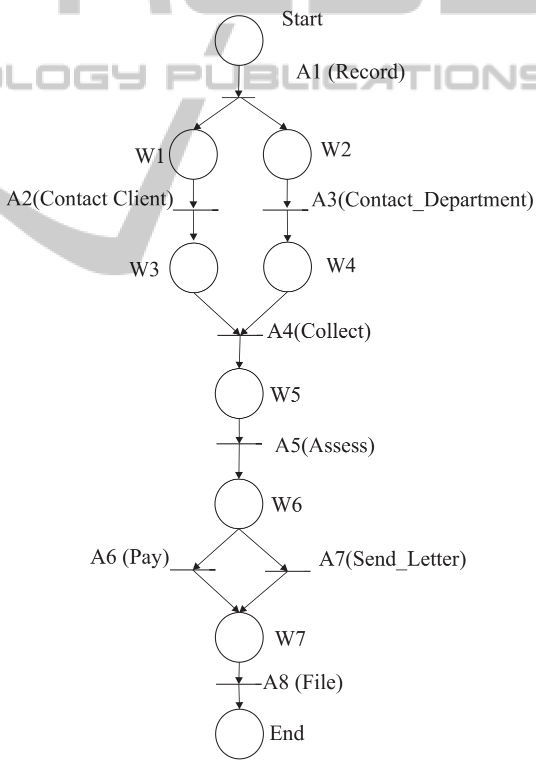


Figure 5: “Handle Complaint Process”.

As mentioned previously, a task can be associated to a transition in a Workflow net. However, in order to catch resources in use when a task is in execution and released them when the task is done, we use two sequential transitions plus a place to model a task. The first transition represents the beginning of the task, the place the task, and the second transition the end of the task (Wang et al., 2009).

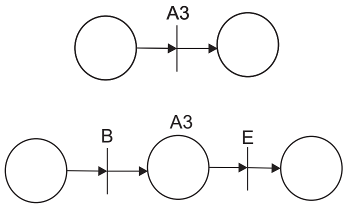


Figure 6: Petri net model of a task.

As shown in figure 6, transition $B$ represents the beginning of a task execution; $E$ represents the end of the task execution. Place $A3$ represents the task in execution. From reachability analysis perspective, figure 6 can be reduced to a single transition which represents the entire task execution as a single logic unit.

## 4 RESOURCE ALLOCATION MECHANISM



Resources in Workflow Management Systems are non-preemptive (van der Aalst and van Hee, 2004): once a resource has been allocated to a specific activity, it cannot be free before ending the corresponding activity. As already mentioned, there exists different kinds of resources in Workflow processes. Some of which are of the discrete type and can be represented by a simple token. For example, a printer used to treat a specific class of documents will be represented as a non-preemptive resource and could be allocated to a single document at a same time. On the contrary, some other resources cannot be represented by simple tokens. This is generally the case with human type resources. As a matter of fact, it is not unusual for an employee who works in an administration to treat several cases simultaneously. For example, in an insurance company, a single employee can normally treat several documents during a working day and not necessarily in a pure sequential order. In this case, a simple token could not model human behavior in a proper manner.

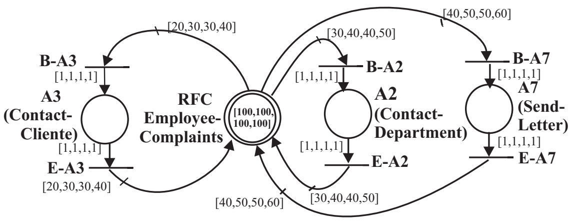


Figure 7: Fuzzy Continuous Resource.

Fuzzy allocation mechanisms were presented in (Jeske et al., 2009). An example of fuzzy continuous resource is given in figure 7. For example, this figure shows that $30\% \pm 10\%$ of the resource availability $R2$ is necessary to realize the activity $A3$ (Contact-Client).

The behavior of a fuzzy continuous resource allocation model can be defined through the concepts of "enabled transition" and "fundamental equation".

In an ordinary Petri net, a transition $t$ is enabled if and only if for all the input places $p$ of the transition, $M(p) \geq Pre(p,t)$, which means that the number of tokens in each input place is greater or equal to the weight associated to the arcs which connect the input places to the transition $t$. With a fuzzy continuous resource allocation mechanism, considering a transition $t$, the marking of an input place $p$ and the weights associated to the arc which connects this place to the transition $t$ are defined through different fuzzy sets. In this case, a transition $t$ is enabled if and only if (for all the input places of the transition $t$):

$$\Pi_t = \Pi(Pre_{FCR}(p,t) \leq M_{FCR}(p)) > 0. \quad (4)$$

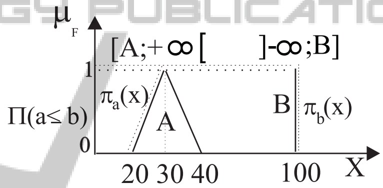


Figure 8: Possibility Measure of $B$-$A3$.

For example, the transition $B$-$A3$ in figure 7 is enabled because $\Pi_{B-A3} = \Pi(Pre_{FCR}(R2,\text{B}-\text{A3}) \leq M_{FCR}(R2)) = 1 > 0$ as shown in figure 8 ($a = Pre_{FCR}(R2,\text{B}-\text{A3})$ and $b = M_{FCR}(R2)$).

For an ordinary Petri net, once a transition is enabled by a marking $M$, it can be fired and a new marking $M'$ is obtained according to the fundamental equation:

$$M'(p) = M(p) - Pre(p,t) + Pos(p,t). \quad (5)$$

With a fuzzy continuous resource allocation model, the marking evolution is defined through the following fundamental equation:

$$M'_{FCR}(p) = M_{FCR}(p) \ominus Pre_{FCR}(p,t) \boxplus Pos_{FCR}(p,t) \quad (6)$$

The operation "$\ominus$" corresponds to the fuzzy subtraction. The operation "$\boxplus$", when considering the sum of two fuzzy sets, is different from the one given in fuzzy logic and is defined as:
$[a1,a2,a3,a4] \boxplus [b1,b2,b3,b4] =$
$[a1+b1,a2+b2,a3+b3,a4+b4]$.
This difference is due to the fact that the fuzzy operation "$\oplus$" does not maintain the marking of the

fuzzy continuous resource allocation model invariant (the p-invariant property of the Petri net theory (Murata, 1989)). As a matter of fact, after realizing different activities, the resource's availability must go back to 100 %, even in the fuzzy case. To a certain extent, from the point of view of fuzzy continuous resource allocation mechanisms, the operation "⊞" can be seen as a kind of defuzzyfication operation. In particular, using this operation, it will be possible to find a linear expression of the fuzzy marking which will always be constant and which will correspond to the following expression:

$$M_{FCR}(R_{FC}) \boxplus (w_1 \otimes M_{FCR}(A1)) \boxplus (w_2 \otimes M_{FCR}(A2)) \boxplus \cdots \boxplus (w_{N_{FCR}} \otimes M_{FCR}(A_{N_{FCR}})) = CONST. \quad (7)$$

with $w_\alpha = Pre_{FCR}(R_{FC}, t_{in_\alpha}) = Pos_{FCR}(R_{FC}, t_{out_\alpha})$ for $\alpha = 1$ to $N_{FCR}$.

## 5 FUZZY TIME CONSTRAINT PROPAGATION MECHANISM

As the actual time required by an activity in a Workflow Management System is non-deterministic and not easily predicted, a fuzzy time interval can be assigned to every Workflow activity.

The static definition of a fuzzy time Workflow net is based on fuzzy static intervals $[a1, a2, a3, a4]s$ which represent the permanency duration (sojourn time) of a token in places. Before duration $a1$ the token is in the non-available state. After $a1$ and before $a4$, the token is in the available state for the firing of a transition. After $a4$, the token is again in the non-available state and cannot enable any transition: it therefore becomes a dead token. In a real time system case, the "death" of a token has to be seen as a time constraint that is not respected. A transition cannot be fired with dead tokens as this would correspond to an illegal action or behavior: a constraint violation. The dynamic evolution depends on the time situation of the tokens (date intervals associated with the tokens). For example, if the arrival date of the token in the place is $\delta = 3$, knowing that the fuzzy static interval of this place is $[5, 6, 7, 8]v$, then, the fuzzy visibility interval of this token is $[5+3, 6+3, 7+3, 8+3]v = [8, 9, 10, 11]v$.

In a Workflow Management System, a visibility interval depends on a global clock associated to the entire net which calculates the passage of time from date $= 0$, which corresponds to the start of the systems operation. In particular, the existing waiting times between sequential activities can be represented by visibility intervals whose minimum and maximum fuzzy boundaries will depend on the earliest and latest delivery dates of the considered case. Through correct knowledge of the beginning date of the process and the maximum duration of a case, it is possible to calculate estimated visibility intervals associated with each token in each waiting place using constraint propagation techniques very similar to the ones used in scheduling problems based on activity-on-arc graphs without circuits (Gondran et al., 1984).

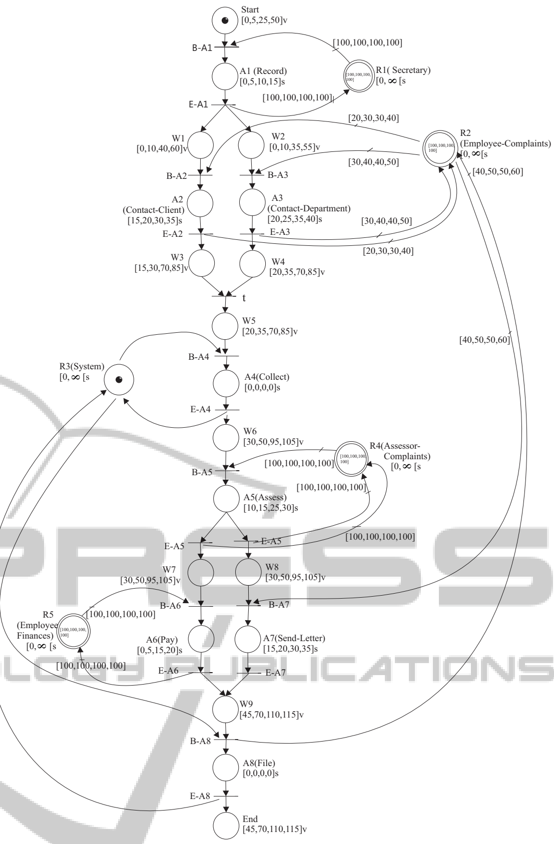

Figure 9: Visibility intervals of the "Handle Complaint Process".

Figure 9 shows the fuzzy static intervals (intervals of fuzzy durations) associated to the activity places of the process and the fuzzy visibility intervals (intervals of fuzzy dates) associated with the waiting places (condition places of the workflow net). It is important to note that there is no time restriction on resources - static interval defined for each resource is $[0, \infty[s$. The minimal fuzzy bounds of the estimated visibility intervals attached to the waiting places are calculated applying a forward constraint propagation technique applied to the different kinds of routings associated with the "Handle Complaint Process", and the maximum fuzzy bounds of the estimated visibility intervals are calculated by applying a backward constraint propagation techniques to the different kinds of routings considering the latest delivery dates of the case.

If the token appears in a place $p$ at date $\delta$ and if its visibility interval is given by $[a1, a2, a3, a4]$, then this token could be used for the firing of a transition at the earliest date $a1$ and at the latest date $a4$. The global state of the Workflow net will be then defined by the current marking of the net and by the time measured by the clock through the different visibility intervals. When a transition $t$ is fired at a date which belongs to its enabling interval, a new marking will be calculated, the tokens which will not be used for the firing of the transition will continue with their visibility interval, and new estimated visibility intervals will be associated to the tokens produced by the firing of $t$.

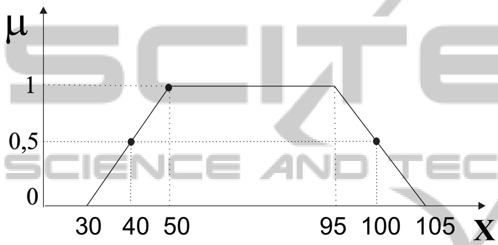


Figure 10: Possibilistic Distribution of W6.

For example, if a token is produced in place $W6$ at date $\delta = 50$, considering the possibilistic distribution shown in figure 10, the firing possibility measure of transition $B$-$A5$ will be equal to $\mu = 1$ and the activity associated to place $A5$ will be initiated normally and its visibility interval will be $[50, 50, 50, 50]$ (firing of $B$-$A5$) $\oplus$ $[10, 15, 25, 30]s$ (static interval associated to $A5$) $= [60, 65, 75, 80]v$. If the token in place $W6$ is produced earlier at date $\delta = 40$, for example, the firing possibility of transition $B$-$A5$ will be $\mu = 0.5$ (see figure 10) and its visibility interval will be $[40, 40, 40, 40]$ (firing of $B$-$A5$) $\oplus$ $[10, 15, 25, 30]s$ (static interval associated to $A5$) $= [50, 55, 65, 70]v$. However the firing could eventually be delayed until reaching a date corresponding to a possibility equal to $\mu = 1$. Finally, if the token in place $W6$ is produced at date $\delta = 100$, the firing possibility of transition $B$-$A5$ will be equal to $\delta = 0.5$ (see figure 10) but with a different meaning. This situation will correspond to a case where some of the previous activities on the process were delayed and its visibility interval will be $[100, 100, 100, 100]$ (firing of $B$-$A5$) $\oplus$ $[10, 15, 25, 30]s$ (static interval associated to $A5$) $= [110, 115, 125, 130]v$. It will be important then to immediately fire the transition $B$-$A5$ corresponding to the beginning of the next activity and to inform the responsible resource for executing this activity of the delay. Eventually, some of the next activities of this process will be executed with a high rank priority and the firing possibility of some of the last transitions in the process will reach a possibility $\mu = 1$ again, ensuring that the process deadline is respected.

## 6 FIRING RULES WITH FUZZY TIME AND FUZZY RESOURCE

If a transition has $n$ input places and if each one of these places has several tokens in it, then the enabling time interval $[a1, a2, a3, a4]$ of this transition is obtained by choosing for each one of these $n$ input places a token, the visibility interval associated with it. In this paper, there exists no time restriction on the resources (the static interval attached to the resource places is always $[0, \infty[s$ and, as a consequence, the enabling time interval of a transition will simply be equal to the visibility interval associated with the case to be treated by the corresponding transition. For example, knowing that the visibility interval attached to the case represented by a token in place $W1$ is equal to $[0, 10, 40, 60]v$, the enabling time interval of the transition $B$-$A2$ will be $[0, 10, 40, 60]v$ too.

For firing a transition, it is necessary that the arrival date of the token in the input place of the transition belongs to the fuzzy visibility interval associated with the input place of the transition ($\mu > 0$) and the resource availability (equation (1)) necessary to realize the activity initiated by the firing of the transition must be greater than 0 ($\Pi(a \leq b) > 0$). To evaluate the availability of resource and time simultaneously, the joint possibility presented in equation (3) must be calculated, where $\pi_X(x)$ corresponds to the resource availability and $\pi_Y(y)$ to the time possibility.

In order to understand the mechanism for transition firing in a Workflow net with fuzzy resources and fuzzy time, the authors consider a particular fragment of the "Handle Complaint Process", which is shown in Figure 11.

- At the fuzzy date $[0, 5, 25, 50]$:
  - the case in the *start* place becomes available to be treated by the resource in $R1$. We choose to fire the transition $B$-$A1$ at date 5 to reach the higher value possible (normal situation to treat the case) when considering the joint possibility $\pi_{(x,y)} = min(\pi_X(x), \pi_Y(y)) = 1$ with $\pi_X(x) = 1$ (time possibility equal to 1 when $x = 5$) and $\pi_Y(y) = \Pi(a \leq b) = 1$ (resource availability possibility). After the firing of the transition, a token is produced in place $A1$ with a visibility interval equal to $[5, 5, 5, 5]$ (firing of $B$-$A1$) $\oplus$ $[0, 5, 10, 15]s = [5, 10, 15, 20]v$.

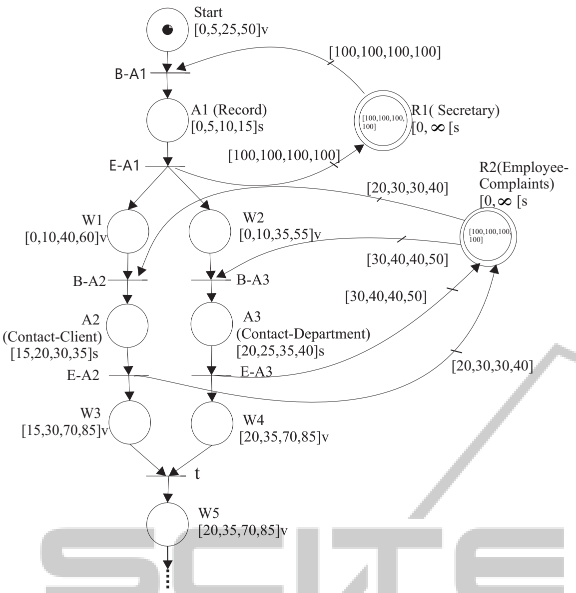


Figure 11: Fragment of the Workflow net - "Handle Complaint Process".

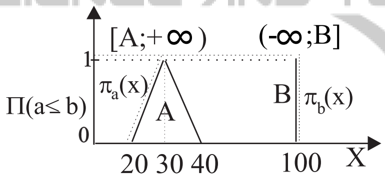


Figure 12: The possibility measure associated with $B$-$A2$ (resource $R2$).

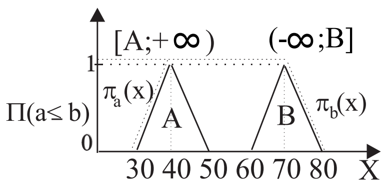


Figure 13: The possibility measure associated with $B$-$A3$ (resource $R2$).

- At the fuzzy date $[5,10,15,20]$:
  - if the activity $A1$ associated is finalized at date 10, the token becomes available in $A1$ , the transition $E$-$A1$ is fired because the joint possibility $\pi_{(}x,y) = min(\pi_X(x), \pi_Y(y)) = 1$ with $\pi_X(x) = 1$ (time possibility equal to 1 when $x = 10$) and $\pi_Y(y) = \Pi(a \leq b) = 1$ (resource availability possibility) and the resource is returned to $R1$. At the same time, tokens are produced in $W1$ and $W2$. To fire $B$-$A2$ and $B$-$A3$ it is necessary to evaluate the time and resource availability through equation (3). For $B$-$A2$, the joint possibility $\pi_{(x,y)} = min(\pi_X(x), \pi_Y(y)) = 1$ with $\pi_X(x) = 1$ (time possibility equal to 1 when $x = 10$) and $\pi_Y(y) = \Pi(a \leq b) = 1$ (resource availability possibility - see figure 12.) In the same manner, for $B$-$A3$, the joint possibility $\pi_{(x,y)} = min(\pi_X(x), \pi_Y(y)) = 1$ with $\pi_X(x) = 1$ (time possibility equal to 1 when $x = 10$) and $\pi_Y(y) = \Pi(a \leq b) = 1$ (resource availability possibility - see figure 13.) Thus, the transitions $B$-$A2$ and $B$-$A3$ are fired and a token is produced in $A2$ with a visibility interval of $[25,30,40,45]v$ and another in $A3$ with a visibility interval of $[30,35,45,50]v$. At this moment, $R2 = [10,30,30,50]$.
- At the fuzzy date $[25,30,40,45]$
  - if the activity for $A3$ is finalized at date 30, the token becomes available in $A3$, then transition $E$-$A3$ is fired ($\pi_{(}x,y) = min(\pi_X(x), \pi_Y(y)) = 1$ with $\pi_X(x) = 1$ (time possibility equal to 1 when $x = 30$) and $\pi_Y(y) = \Pi(a \leq b) = 1$ (resource availability possibility)) and the resource is returned to $R2$. At this moment, $R2 = [60,70,70,80]$. A token is produced in $W4$;
- At the fuzzy date $[30,35,45,50]$
  - if the activity for $A2$ is finalized at date 35, the token becomes available in $A2$, then the $E$-$A2$ transition is fired ($\pi_{(}x,y) = min(\pi_X(x), \pi_Y(y)) = 1$ with $\pi_X(x) = 1$ (time possibility equal to 1 when $x = 35$) and $\pi_Y(y) = \Pi(a \leq b) = 1$ (resource availability possibility)) and the resource is returned to $R2$. At this moment, $R2 = [100,100,100,100]$. A token is produced in $W3$. The transition $t$ is fired and a token is produced in $W5$.

## 7 CONCLUSIONS

This article presented how to model fuzzy hybrid resources in Workflow nets with fuzzy time intervals associated to the activities. Besides this, through the definition as well as use of a joint possibility distribution, it was possible to define a transition firing definition. This definition takes into consideration the time constraints associated to the cases of the process as well as the availability of the resources used to execute the activities.

Some advantages of this approach can be cited. For example, the event log will show the possibilities of firing each activity and may lead to a type of process quality analysis: if the activities, most of the time, are working with a possibility equal to 1, then the work resulting from the process will be of good quality. On the other hand, if a large number of the activities are associated with possibilities near to 0, then the quality of the process will be of poor quality. In addition, during the execution of process activities, the management of activities could suffer a certain influence according to the semantics associated with a low firing possibility. Finally, in the case of transitions in conflict, the information concerning the firing possibility can be used to make a decision: for example if the possibility is low because of delayed activities, we will give priority to the transition in relation to another that possesses a higher firing possibility.

As a future work proposal, it will be interesting to represent human behavior in a manner that is close to real life, a firing mechanism involving a conditional possibility, in such a way that the availability of the resource will be conditioned to time. Moreover, new firing rules based on a conditional possibility will then be defined and will be implemented at a business managing level through the use of a real time token player algorithm.

## ACKNOWLEDGEMENT

The authors would like to thank FAPEMIG for financial support.

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