Algorithms for the Hybrid Fleet Vehicle Routing Problem

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Keywords: Vehicle Routing Problem, Heterogeneous Feet, Heuristics.

Abstract: In the classical Vehicle Routing Problem (VRP) literature, as well as in most VRP commercial software packages, it is commonly assumed that all vehicles are identical in their characteristics. In real-world problems however, this is often not true. In many cases, fleets are made up of different vehicle types, which may vary by size, engine/fuel type, and other performance-impacting factors. Even in a homogeneous fleet, vehicles often differ by age and condition, which can greatly impact performance. Our research was specifically motivated by cases where the fleet contains vehicles that not only vary in performance, but this variation is a function of the arc type, such that a given vehicle might have lower cost on some arcs but higher cost on others. We refer to this as the Hybrid Fleet Vehicle Routing Problem (HFVRP). We propose two heuristic methods that take into account the vehicle-specific cost structures. We provide computational results to demonstrate the quality of our solutions, as well as a comparison with a Genetic Algorithm (GA) based method seen in the literature.

1 INTRODUCTION

In this paper, we present models and algorithms for solving the Hybrid Fleet Vehicle Routing Problem (HFVRP). In the traditional VRP, a collection of identical vehicles must be routed so as to visit every customer in a given set while minimizing transportation cost; it is assumed that the cost to traverse an arc between any pair of customers is the same for all vehicles in the fleet. The HFVRP is a variation of VRP in which vehicles in the fleet may differ, and we allow the arc cost to vary by vehicle type.

HFVRP has applicability in many real-world contexts. It is quite common, for example, that vehicles within a given fleet will vary in age and thus in fuel efficiency (and associated cost). Moreover, fleet managers in many industries are gradually moving towards more fuel-efficient, environmentally-friendly vehicle types within their fleet. As older vehicles with traditional combustion-based engines are retired, they are being replaced with hybrid or electric vehicles. Not only do these vehicle types vary in efficiency, but this variation may depend on driving conditions – one vehicle may be more efficient in city driving, for example, while another is more efficient in highway driving. Furthermore, in some countries, electric vehicles are given privileges like being allowed to pay less toll. In such cases, no one vehicle type will be Pareto-dominant over the others. Thus, the extension from VRP to HFVRP is not only in determining which vehicle type to place on which routes but also in actually designing the routes to leverage the strengths of the different vehicle types (Gusikhin et al., 2010).

VRP is known not only to be NP-hard in theory (the Traveling Salesman Problem is a special case of VRP, in which there is only one vehicle to be scheduled), but also to often be computationally challenging in practice as well. It is therefore frequently solved with heuristics, as we discuss in Section 2. However, these heuristics often rely on approaches that target the minimization of total mileage traveled in the system. In HFVRP, circuitous mileage may lead to better matching of vehicle types to driving conditions, and thus the solution with minimum total distance traveled may not be the minimum-cost solution.

In this research, we begin by posing an explicit mathematical programming approach to solving HFVRP, and demonstrate the computational challenges of this approach. We then introduce two heuristics for solving this problem, one which can be solved very quickly and without the use of any commercial solvers, and the other which can be used in contexts where there is more run time – and access

Peng F., M. Cohn A., Gusikhin O. and Perner D..
Algorithms for the Hybrid Fleet Vehicle Routing Problem.
DOI: 10.5220/0005422600690077
In *Proceedings of the 1st International Conference on Vehicle Technology and Intelligent Transport Systems* (VEHITS-2015), pages 69-77
ISBN: 978-989-758-109-0

to a commercial mixed integer programming solver – available. We then provide comparison with a GA based algorithm, and experiments to gain insights into both the computational performance and the solution quality.

The contribution of this research is in: investigating an important variation of the classical VRP with real-world relevance; identifying structural challenges that impact the tractability of this problem; presenting heuristic approaches to find quality solutions in tolerable run times; and conducting computational experiments to assess performance and solution characteristics.

2 MOTIVATION AND LITERATURE REVIEW

The VRP is a classical problem that has been studied extensively in the literature. Given a fleet of identical vehicles and a group of customers that need to be served, VRP seeks the best assignment of routes to vehicles such that all customers are covered while the overall cost (usually measured in distance traveled) is minimized. This problem was first introduced some fifty years ago (Dantzig and Ramser, 1959). Since then, many aspects of the problem have been extensively studied (Toth and Vigo, 2001; Baldacci et al., 2008; Laporte, 2009). VRP is not only theoretically interesting but also has broad applicability in real-world practices, from transportation, distribution and logistics to scheduling (Baldacci et al., 2010; Tahmassebi, 1999).

In practice, exact solutions to VRP can typically only be obtained for relatively small sized problems (Hasle and Kloster, 2007), with problems having even as few as a couple of hundred customers often not guaranteed to be solvable. Therefore a myriad of construction heuristics have been developed to tackle this problem, including the savings heuristic (Clark and Wright, 1964; Desrochers and Verhoog, 1991), giant-tour based heuristic (Golden et al., 1984), and many augmented methods (Li et al., 2007; Salhi and Rand, 1993).

Many variants of the classical VRP have been investigated as well. For example, in the capacitated version of VRP, each vehicle can only carry a limited amount of goods (Baldacci and Mingozzi, 2009; Campos and Mota, 2000; Ralphs et al., 2003). In the VRP with time windows, some or all of the customers/depot can accept delivery only during a specified time period (de Oliveira and Vasconcelos, 2010; Kim et al., 2006; Kritikos and Ioannou, 2010; Li et al., 2010). In VRP with stochastic supply/demand, supply/demand are not deterministic but have some variability (Novoa and Storer, 2009).

We are interested in a variation of VRP that we call the Hybrid Fleet Vehicle Routing Problem (HFVRP). Unlike VRP, HFVRP does not assume that all vehicles have identical characteristics. Variants of HFVRP have appeared in the literature in various forms and other various names, such as the heterogeneous vehicle routing problem (Choi and Tcha, 2007), the fleet size and mix vehicle routing problem (Golden et al., 1984), mix fleet vehicle routing problem (Wassan and Osman, 2002), etc. These usually differ in whether fixed cost is considered, and whether the fleet size is limited. Various solution heuristics have been proposed for this family of problems. For example, Taillard (Taillard, 1999) first solved the VRP problem for each vehicle type, then use a column generation-based approach to generate and store an augmented set of routes, then within this set of routes solve a set partitioning problem to obtain the final solution. Choi and Tcha (Choi and Tcha, 2007) performed column generation on the case where a customer can be visited more than once, where a dynamic programming approach was used to solve the sub-problems and find new columns. Meta-heuristics have also been explored: Ochi et al. (Ochi et al., 1998) used the petal genetic algorithm; Wassan and Osman (Wassan and Osman, 2002) and Tarantilis et al. (Tarantilis et al., 2008) applied tabu search method to this problem.

Our interest is in the specific version where the cost of traversing an arc varies by vehicle, and the time/length of the route is limited (this problem is sometimes referred to as the Heterogeneous VRP with Vehicle Dependent Routing Costs (Baldacci et al., 2008). These conditions are almost always the situation in practice – different vehicles have different engines and thus different fuel efficiency; even within a fleet of the same vehicles, age can sometimes cause fuel efficiency to vary substantially. Such problems are particularly challenging because cost is no longer so directly tied to distance. Instead, we might be willing to travel circuitous mileage if that excess mileage led to the use of lower-cost arcs.

Being a generalization of VRP, HFVRP is an NP-hard problem, and in industrial applications it is in general solved heuristically. One approach would be to start by assuming a common cost, across all vehicles, for each given arc, and solve the traditional VRP using known heuristics. Then, in a second phase, the true cost of assigning each vehicle to each of the chosen routes could be calculated and the actual matching of vehicles to routes could be done optimally. However, this can lead to sub optimal solutions, because of the fact that the shortest distance routes are no longer necessarily the cheapest, and that the routes good for one vehicle may be bad for another.

Therefore we need to develop new heuristics that specifically address the challenges of HFVRP. In the remainder of the paper we present first an exact formulation, then two heuristics: a randomized greedy heuristics and a set-partitioning based approach. Finally we present computational results to compare the heuristics and identify appropriate contexts for the use of each.

3 FORMULATIONS AND HEURISTIC METHODS

3.1 Connection Based Formulation

We begin by presenting a connection based approach to solving HFVRP. Note that the same technique can also be used in formulating problems such as capacitated vehicle routing (Golden et al., 1984; Baldacci and Mingozzi, 2009). Let T be the set of vehicle type indices, N be set of nodes, where node 0 is the depot, and let $N_0 = N \backslash \{0\}$ be the set of all customer nodes. We use variables x_{ij}^t to denote whether a vehicle of type t travels directly from node i to j, $i, j \in N, t \in T$. Moreover, let M_t be the number of routes vehicles of type $t \in T$ can serve. This restriction stems from the fact that drivers have only limited work time available in a given period. M_t is simply the number of vehicles of type t if each driver can drive only one trip over the time horizon under consideration. We use Q_t to represent the length of time a driver can spend working for vehicle $t \in T$, and use d_{ij}^t to denote the time it takes for vehicle of type t to travel from node i to j. Finally, let c_{ij}^t be the cost of traveling from customer i to j, $i, j \in N$. $c_{ii}^t = 0\ \forall i \in N$. HFVRP can be formulated as:

$$\text{(C)} \quad \text{minimize} \sum_{t\in T}\sum_{i\in N}\sum_{j\in N} c_{ij}^t x_{ij}^t$$

subject to:

$$\sum_{t\in T}\sum_{i\in N} x_{ij}^t = 1 \qquad j \in N_0 \qquad (1)$$

$$\sum_{j\in N_0} x_{0j}^t \leq M_t \qquad t \in T \qquad (2)$$

$$\sum_{i\in N} x_{ij}^t = \sum_{i\in N} x_{ji}^t \qquad t \in T, j \in N \qquad (3)$$

$$\sum_{i\in S}\sum_{j\in S} x_{ij}^t \leq |S| - 1 \qquad \forall S \subset N_0, |S| \geq 2 \qquad (4)$$

$$x_{ij}^t \in \{0,1\} \qquad i, j \in N, t \in T \qquad (5)$$

Here Constraints (1) ensures that each customer is covered. Constraints (2) specify that no more than the maximum number of each vehicle type can be used. Flow conservation constraints (3) specify that for each node, any vehicle entering this node must also leave. Here (4) helps eliminate subtours in the solution. In addition, to impose the route length restriction, consider a *path* p of k^p nodes:

$$n_1^p \to n_2^p \to \ldots, \to n_{k^p}^p,$$

if for vehicle $t \in T$, a route starting from the depot that traverses the path and returns to the depot violates the route length restriction, in other words:

$$\sum_{j=1}^{k^p-1} d_{n_j^p, n_{j+1}^p}^t > Q_t,$$

but removing either end points does not, we call such path a *minimal violating path* (MVP). We include one constraint:

$$\sum_{j=1}^{k^p-1} x_{n_j^p, n_{j+1}^p}^t \leq k^p - 1 \qquad (6)$$

for each MVP to ensure no route takes longer than Q_t. Note that here we implicitly assume it is always faster to go directly between two nodes than through a third node).

This formulation can also be applied to HFVRP with vehicle-dependent capacities, in which case we can use Q_t to represent the capacity of vehicle $t \in T$, and d_{ij} to represent the load at node j for all i. As is often the case with VRP, we have observed significant fractionality in our computational experiments, leading to very slow convergence of the branch-and-bound tree and/or incompletion due to running out of memory. For example, we tested an instance with 58 customers and 4 vehicle types. After solving for more than 15 hours with Cplex version 12.1 on a Mac Pro with two 2.8GHz Intel Xeon CPU and 10Gb of RAM, 161400 nodes of the branch-and-bound tree had been explored with 160031 nodes still pending, and no integer-feasible solutions had yet been found.

3.2 Greedy Heuristic

The lack of efficient methods to solve the mixed integer programming (MIP) formulation for all but fairly small problem instances forces us to investigate in heuristic approaches. We next present a method that can find high quality solutions quickly for many problem instances. As illustrated in Figure 1, this heuristic (Heuristic 1) can be summarized as a randomized greedy algorithm. The algorithm works as follows:

1. if no customer is outstanding, stop; otherwise identify the smallest cost from all vehicles to all outstanding customers, without loss of generality, say customer i, vehicle number j with cost c

2. find the second smallest cost from i to all vehicles, record vehicle number j', and cost c'
3. for j, accept customer i with probability $P_1 = 1 - c/2c'$ and reject i with probability $1 - P_1$, so that the smaller c is compared to c', the bigger chance j will be selected to serve i. On the other hand if $c = c'$, or the costs of using j and j' are equivalent, the probability becomes $1/2$;
4. if customer rejected in 3), it is then assigned to vehicle j' with probability $P_2 = c/2d$
5. if the customer is assigned to neither vehicles, the process restarts, and in the immediate next iteration, i will not be considered for selection to avoid repeating the same situation; otherwise, move the chosen vehicle to i and go to 1).

This approach has certain resemblance to the greedy heuristic, because priority is always given to the assignment with the smallest arc cost. In every stage of the algorithm, we keep track of the cumulative service time of each vehicle, and returns a vehicle to the depot if no more assignment is possible.

Each iteration of the algorithm takes very little time, so we can run it many times and keep the best solution. We will provide empirical studies in Section 4. We also emphasize that this is an approach that can be implemented fairly easily, and does not require the use of an underlying MIP solver, thus has great value for many small fleet managers with limited resources for sophisticated implementations.

Nonetheless, the randomized approach clearly runs the risk of sub-optimal solutions. In particular, the number of feasible routes is exponentially large, with only a small fraction of these being generated in the randomized runs. This therefore motivates us to consider a hybrid approach, blending the route-based approach with an underlying MIP structure for combining routes effectively, with the aim of being able to find higher-quality solutions when more sophisticated implementations are viable.

3.3 Route Based Formulation

In some cases, when more time is allowed to find good solutions and more sophisticated technology is available, we can expand our idea to leverage the strength of an optimization-based approach, and take advantage of the randomized approach to reduce the problem size and improve tractability. We present the route-based model (also referred to as the composite variable model (Armacost et al., 2002; Barlatt et al., 2009)). We need the following additional notations:

- R_t: set of possible routes of customers that can be served by a vehicle of type t;

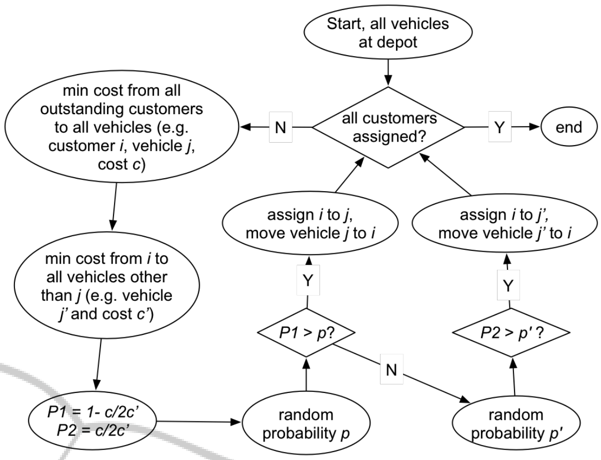

Figure 1: Flow chart for the greedy heuristic.

- w_{rt}: binary variable that equals 1 if and only if route $r \in R_t$ is served by a vehicle of type $t \in T$; 0 otherwise
- C_{rt}: cost associated with assigning route $r \in R_t$ to a vehicle of type $t \in T$
- δ_{ri}: binary indicator of whether a node $i \in N_0$ is covered by a route $r \in R_t$;

The route-based model can then be formulated as follows:

$$\text{(S)} \quad \text{minimize} \sum_{t \in T} \sum_{r \in R_t} C_{rt} w_{rt}$$

subject to:

$$\sum_{t \in T} \sum_{r \in R_t} \delta_{ri} \cdot w_{rt} = 1 \qquad i \in N_0 \qquad (7)$$

$$\sum_{r \in R_t} w_{rt} \leq M_t \qquad t \in T \qquad (8)$$

$$w_{rt} \in \{0, 1\} \qquad r \in R_t, t \in T$$

Here constraint (7) ensures that each customer is covered by one and only one route. Constraint (8) forces the number of vehicles of each type used to be no more than the total number of that type.

Observe that if we were to enumerate all routes, solving this problem would lead to an optimal solution. In reality, this is impractical for all but very small instances. Techniques such as column generation and branch-and-price (Barnhart et al., 1998) may be applied to problems like this, and iteratively generate only routes that appear promising. However, the problems for finding such routes are essentially VRP type and NP-hard. In the Appendix, we show that formulation (S) is stronger than (C), and it is therefore advantageous to work with (S).

We leverage the speed of the randomized approach from Section 3.2 with the power of the route-based formulation above to improve the solution quality. First we run the randomized heuristic for a large number of iterations. Instead of keeping only the set of routes with the smallest cost, all routes generated in each iteration are put into a pool of routes. Once the route generation is finished, we will have many candidate routes that are cost efficient. Then we formulate problem (S) with the poll of candidate routes, and solve it to find the best route combination.

4 RESULTS AND DISCUSSION

4.1 Baseline — Genetic Algorithm

In order to provide a comparison for the proposed algorithm against an existing class of algorithms, a genetic algorithm (GA) (Holland, 1975) was developed. GA is suitable for a wide range of combinatorial problems, including VRP. Recent applications of the GA to various VRP problems have shown its competitiveness with other heuristic techniques in both computation time and quality (Baker and Ayechew, 2003). Our implementation of the GA is based on the "pure GA" described in Baker and Ayechew (Baker and Ayechew, 2003). In this approach, the problem is decomposed into two steps. The first, controlled by the GA, assigns customers to individual vehicles and over time moves the solution space to be less infeasible and lower in cost. Since this does not establish the order in which customers assigned to a vehicle are to be visited, a 2-opt algorithm (Lin et al., 1965) is employed to solve the corresponding TSP for each individual vehicle.

Our GA implementation is performed in Matlab on a Windows 7 machine with an Intel Core i5-2520M processor at 2.5GHz and 8GB RAM. In order to test the quality of the developed algorithm, several test datasets were chosen from *branchandcut.org*, themselves cases from published articles. The results for these experiments, with the alphanumeric naming convention given, are shown in Table 1. Moreover, we considered one more dataset that was extracted from the actual daily service routes of a food-gathering company. We randomly selected five of their existing routes (spanning 58 customers) as well as the vehicles serving these routes, calculated the travel time between each pair of locations and compiled this into a travel-time matrix. To obtain cost information, we used average costs per mile for each individual vehicle (this information can be obtained by multiplying the fuel consumption per mile and corresponding fuel price. One of the methods for accurate fuel consumption estimation is presented in Kolmanovsky et al. (Kolmanovsky et al., 2011). We assume the maximum work time for a vehicle/driver is eight hours.

Table 1: Performance of GA solution vs. the best known solution on test cases.

Case	Best sol.	GA sol.	Difference
A-n37-k5	669	731.9	9.41%
E-n23-k3	569	569.7	0.13%
E-n30-k3	534	557.8	4.47%
P-n16-k8	450	451.9	0.43%
P-n55-k8	588	677.8	15.27%
58-Node	N/A	187.1	N/A

4.2 The Value of Explicitly Considering Heterogeneous Arc Costs, and Our Heuristics Compared with the GA

In current practices, HFVRP is often solved using a single cost across all vehicles for each arc, for example, the minimum cost, maximum cost, or average cost. Once the routes have been constructed, they are then assigned to individual vehicles and the true costs of assigning vehicles to routes can be accurately calculated. In this experiment, we consider the value of considering the true (vehicle-specific) costs explicitly when designing the vehicle routes. The following experiments were performed with c++ on a Mac Pro with two 2.8GHz Intel Xeon CPU and 10Gb of RAM.

The food-gathering company data is a simplification that assumes Pareto dominance across vehicles – that is, if Vehicle A is more cost-effective on one arc than Vehicle B, then it will be more cost-effective on all arcs. This is often not the case and, in fact, the motivation for our research comes from cases where this is not true. Thus, we generated three additional instances that are not Pareto-dominant. Cost perturbation was done by multiplying each entry in the cost matrix by a random number. For slight cost perturbation, the random number was uniformly taken between 0.8 and 1.2; for medium cost perturbation, the random number was uniformly taken between 0.5 and 1.5; and for large perturbation, the random number was uniformly taken between 0.3 and 1.7.

For each of these four instances, we solve the HFVRP four times. In the first, we apply the solver-free heuristic from Section 3.2 (H1), using the average cost data (i.e., for each arc, we take the average across all vehicle types for that arc). After constructing the routes, we then assign them to specific vehicles: the most fuel-efficient vehicle gets the longest route, the second most fuel-efficient vehicle gets the

Table 2: Absolute and relative costs of solutions from different cost structure between VRP and HFVRP.

	Pareto	Low None-Pareto	Medium	High
H1 with	150	152	153	122
avg. cost	104%	107%	124%	132%
H1 with	149	145	126	95
true cost	103%	102%	103%	103%
H2 with	146	145	142	123
avg. cost	101%	102%	116%	133%
H2 with	145	142	123	93
true cost	100%	100%	100%	100%

second longest route, etc. Fuel efficiency is measured by average fuel cost across all arcs. Finally, we compute the final cost using each vehicles specific arc costs. In the second approach, we apply Heuristic 1 using the true arc costs within the heuristic. In both of these two approaches, we set a one-hour time limit on run time. In the third approach, we use the solver-based heuristic from Section 3.3 (H2) with the average cost data. In the fourth, we use the true cost data. For both solver-based heuristic runs, we use 100,000 randomly generated columns.

Results appear in Table 2. Observe that in all four instances, for either heuristic, incorporating true costs in the initial route construction reduces cost (by as much as 33% in one instance). This shows that, with the same algorithm, explicitly considering the cost of individual vehicles provides an advantage over using an uniform cost for all vehicles in the route generation process. Moreover, Heuristic 2 always achieves better solutions compared with Heuristic 1 on the same datasets, which is no surprise given the much larger set of routes available for selection in the set partitioning model. Furthermore, compared to results in Table 1, all four implementations of our heuristics outperformed the GA, sometimes by as much as 22%.

It is not surprising that the additional value of applying our approaches improves as the data becomes less Pareto-dominant. We therefore focus on the most highly non-Pareto instance for the remainder of our computational experiments.

4.3 The Effect of Randomness/heuristic Parameters on Performance

Because our two heuristics both have significant random components, we next test to see the variation in outcomes.

Returning to the complete data set (again with the least-Pareto dominant arc costs), we apply solver-free Heuristic 1 fifty times using a run time limit of one hour for each run. Figure 2 shows the variation in objective value. Note the limited range in solution values. We repeat the previous experiment using solver-based Heuristic 2 with a limit of 100,000 columns. Results appear in the Figure 3.

Recognizing that the solution quality of the heuristics depends in part on the algorithmic parameters, we conduct the following experiments: we run the solver-free heuristic with five different time limits: 1 minute, 15 minutes, 30 minutes, 45 minutes, and 60 minutes. For each time limit, we run ten random instances of the heuristic. Results are displayed in Figure 4. Observe that increased runtime improves performance both in reducing objective value and variation between individual tests.

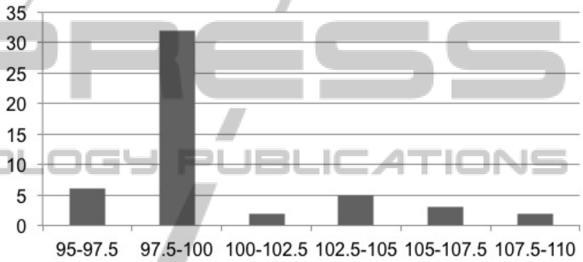

Figure 2: Histogram of objective values from 50 runs of heuristic 1 using true cost.

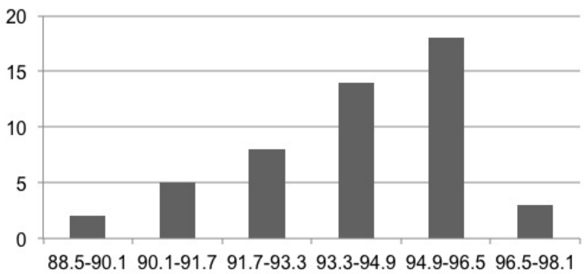

Figure 3: Histogram of objective values from 50 runs of heuristic 2 using true cost.

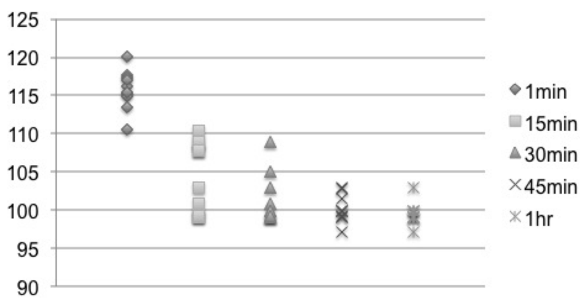

Figure 4: Objective values from heuristic 1 under different time limits, 10 runs for each instance.

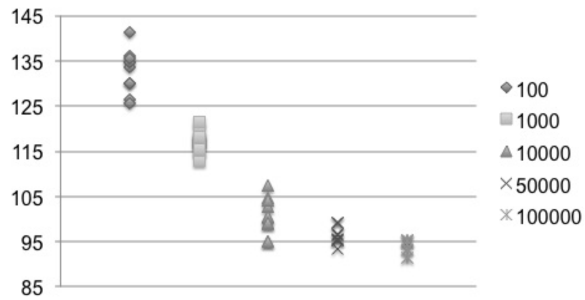

Figure 5: Objective values from Heuristic 2, with different number of starting routes, 10 runs for each instance.

Finally, we run the solver-based heuristic with 100, 1,000, 10,000, 50,000, and 100,000 columns. For each column limit, we run ten random instances of the heuristic. Results are displayed in Figure 5. Clearly, there is benefit in increasing the number of columns, with both minimum value, the average value, and the variance all decreasing as the number of columns increases.

5 CONCLUSIONS AND FUTURE RESEARCH

Variations in vehicle type and efficiency are found in virtually every vehicle fleet in operations. With the current push towards more fuel-efficient and environmentally-friendly vehicles, even greater variations are being observed, as fleet operators are gradually replacing older vehicles with new vehicles that that vary substantially from the original fleet composition.

Even the homogeneous-fleet version of VRP is often a difficult problem to solve in practice, and heuristics must be employed to find high-quality solutions quickly. We have observed that many of the heuristic approaches used in traditional VRP (which are often focused on minimizing total distance traveled within the solution) are not well-suited to HFVRP, where explicitly considering the differences in the cost structure and matching vehicle types to driving conditions is critical (especially when no one vehicle type demonstrates Pareto dominance over all the others).

We have therefore identified, implemented, and analyzed three different approaches to gain insight into solving this challenging and important real-world problem. We start with an exact approach; not surprisingly, this approach is only tractable for problem instances of very limited size or special structure. We next consider a greedy approach that can quickly and easily be implemented, as well as having very fast run times. This approach shows great promise for those environments in which the time to solve problem instances is limited, as are the resources and IT/optimization capabilities of the fleet manager. Finally, we extend this randomized approach into a hybrid heuristic that incorporates the randomization within an optimization-based framework, typically leading to higher-quality solutions without a significant increase in run-time. Both of our heuristic approaches outperformed the genetic algorithm in solving a real-world HFVRP problem in our experiments.

This research is only an initial foray into the study of this complex problem. Several promising avenues of investigation remain. The first is to replace the randomization component of the route-based problem with an optimization-based column generation routine, where the subproblems are solved with specialized VRP algorithms (this is also referred to as price-and-branch). The second is to extend HFVRP to some of the other characteristics commonly observed in real-world applications of VRP such as time windows and capacity constraints. Finally, we observe that VRPs are typically assumed to be additive – i.e. the cost of a route is simply the sum of the individual costs of the arcs comprising that route. In the case of certain vehicle types, this assumption is not realistic. Consider a vehicle that picks up different loads at different locations, as the loads accumulate the mass of the vehicle and correspondingly fuel consumption change, which depends on the arcs traversed thus far. We thus propose to investigate a variation of HFVRP in which the cost of traversing a route may be non-additive relative to the individual arc costs.

ACKNOWLEDGEMENTS

This research was supported by the National Science Foundation under Grant SBIR-I 1013832.

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APPENDIX

We denote the LP relaxation to problem (S) as (SL), and the LP relaxation to problem (C) as (CL).

Theorem. *The lower bound generated by problem (SL) is at least as tight as the lower bound generated by problem (CL)*

Proof. We first show that, for any feasible solution to (SL), there is a corresponding feasible solution to (CL) with the same objective function value. For any feasible solution, say $\hat{w}$, to problem (SL), let $\hat{x}^t_{ij} = \sum_{r \in R_t} \delta_{ri} \hat{w}_{rt}$. It is easy to see that (1) and (2) are satisfied since (7) and (8) are. Moreover, this solution also satisfies (3), since each route $r \in R_t$ already guarantees the flow conservation constraint (3) for all its nodes. Assuming the connection costs are additive, the objective function value corresponding to $\hat{x}$ will equal the objective value for $\hat{w}$. To see that this solution also satisfies (6), first note that since any route in (SL) satisfies this constraint, the maximum number of legs a route r can overlap with a MVP p is $k^p - 1$. Therefore along the direction of r, there will be at least one leg on p, either right before the start or after the end point of the overlapping section or both, that is exposed (i.e., not covered by r). Because of (7), the sum of the flow on the exposed leg is at most $1 - \hat{w}_{rt}$. The sum of all "lost" flows on all exposed legs along p is therefore $\geq \sum_{r \in R_t} \hat{w}_{rt} \geq 1$. The right inequality holds whenever at least one vehicle of type t is used. We can show that (4) is satisfied by $\hat{x}$ in a similar way.

To complete the second half of this proof, we show that not all solutions to (SL) are feasible to (CL). Consider a simple problem with 3 customers besides the depot and 3 vehicles of the same type. Assuming that the trip length limit is large enough, a solution to (CL) can make a round-trip between each pair of customers with "half" a vehicle (i.e., the weight of the connection is 0.5 for both legs of the trip). This solution will satisfy all the constraints in (CL), but will not have a corresponding solution in (SL) since it skips the depot altogether. $\square$