Flatness based Feed-forward Control of a Flexible Robot Arm under Gravity and Joint Friction

Elisha Didam Markus

Department of Electrical, Electronic and Computer Engineering, Central University of Technology, Free State, South Africa

Keywords: Flexible Joint Robot, Differential Flatness, Trajectory Planning, Friction Control, Open Loop Control.

Abstract: This paper discusses the open loop control problem of a flexible joint robot that is oriented in the vertical plane. This orientation of the robot arm introduces gravity constraints and imposes undesirable nonlinear behavior. Friction is also added at the joints to increase the accuracy of the model. Including these dynamics to the robot arm amplifies the open loop control problem. Differential flatness is used to propose a feed-forward control that compensates for these nonlinearities and is able to smoothly steer the robot from rest to rest positions. The proposed control is achieved without solving any differential equations which makes the approach computationally attractive. Simulations show the effectiveness of the open loop control design on a single link flexible joint robot arm.

1 INTRODUCTION

Flexible robots are applied in situations where speed, dexterity and maneuverability are required. Most research done in flexible joint robots do not take unwanted gravitational and frictional effects of the model into consideration (Kandroodi et al., 2012; Pereira et al., 2007; Markus et al., 2012b; Jiang and Higaki, 2011; Tokhi and Azad, 2008; Markus et al., 2012a; Ozgoli and Taghirad, 2006; Ider and zgren, 2000; Rodriguez et al., 2014). This is because the flexible robot at its joint already has a complex structure (Dwivedy and Eberhard, 2006; Kandroodi et al., 2012). Firstly, they are underactuated (non-holonomic) as they possess one control input with two outputs. Secondly, flexible joint robots are nonminimum phase systems. This is due to the effect of the link deflection whose movements acts in opposition to the motor response (Tokhi and Azad, 2008). This structure already poses problems in the design of their control. Therefore additional effects of gravity and friction which are nonlinear dynamics that improve the accuracy of the model based control are often circumvented because of their complicated control requirements (Palli et al., 2009; Cambera et al., 2014). In practice, the effects cannot be completely neglected or ignored if accurate control is to be achieved.

This study hereby proposes an open loop control technique for a single link robot under gravity, with flexibility and friction at the joints. The feedforward control is useful in applications where fast point to point movements of the robot are required as is the case with flexible robots. These type of robot movements tend to induce vibrations at the elastic joints. This is common with industrial robots as shown in figure 1. In order to obtain fast and precise point to point motion at the joint, the vibrations have to be compensated for in the feedforward control law. The motion is planned over a finite time period and executed based on the dynamic model.

Differential flatness has been applied to feed-forward control laws of many nonlinear systems among which are braking control (De Vries et al., 2010), magnetic levitation (Hagenmeyer and Delaleau, 2003), diesel air system (Kotman et al., 2010),wind turbines (Schlipf and Cheng,), crane rotator (Bauer et al., 2014), control of a parking car (Muller et al., 2006), distributed control (Kharitonov and Sawodny, 2006) and robotic systems (Morandini et al., 2012) just to mention a few. Differential flatness or flatness (for short) has been used in these applications due to its ease in trajectory planning and execution (Fliess et al., 1997; Fliess et al., 1999; Levine, 2006; Levine, 2009; Markus et al., 2013). In this study, the motion planning problem for the flexible robot with friction and gravity is shown to be easily achieved without closing the loop if we can measure a fictitious output of the robot, use that parameter to parameterise all of the system states and then design nominal trajectories along the measured parame-

Didam Markus E..
Flatness based Feed-forward Control of a Flexible Robot Arm under Gravity and Joint Friction.
DOI: 10.5220/0005502701740180
In *Proceedings of the 12th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2015), pages 174-180
ISBN: 978-989-758-123-6

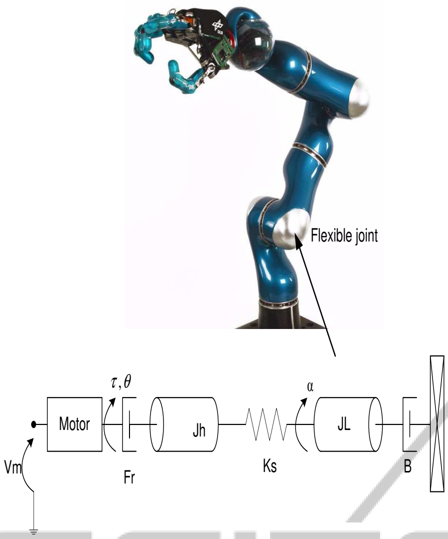

Figure 1: Model of flexible joint.

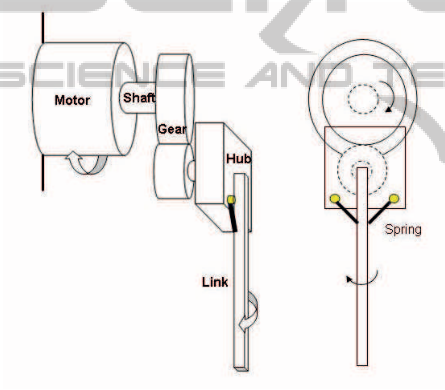

Figure 2: The schematic diagram of a single link flexible joint arm.

ter. For the single link flexible joint robot, we choose this fictitious parameter as the tip position of the robot link. Defining this as the flat output, all the states of the robot and control inputs can be estimated and used in the feedforward control.

The paper is organized as follows: Section 2 gives the mathematical model of the flexible joint robot arm. The differential flatness analysis of the robot and trajectory planning are explained in section 3. The feedforward controller design is explained in section 4. Section 5 discusses simulations and results. The paper is concluded in section 6.

2 FLEXIBLE JOINT ROBOT DYNAMIC MODEL

The model used for the study is the standard Quanser flexible joint manipulator platform (Quanser, 2015) and the schematic representation is shown in Fig. 2. The robot is oriented vertically which introduces gravity in the spring. Friction is also added to improve the accuracy of the model in a practical scenario. The robot arm is attached to the motor by two linear springs in a tendon-like fashion. This results in flexibility at the joint. Figure 3 shows the coordinates of operation of the arm. We define θ as the motor angular displacement and α as the joint twist or link deflection. The position of the the arm end effector is given as the sum of the two angles $(\theta+\alpha)$ which is our generalized coordinate. The dynamic model of the flexible joint robot is already reported in (Markus et al., 2012a).

$$\begin{aligned} J_L(\ddot{\theta}+\ddot{\alpha})+K_s\alpha-mghsin(\theta+\alpha)&=-B\dot{\alpha} \\ (J_L+J_h)\ddot{\theta}+J_L\ddot{\alpha}-mghsin(\theta+\alpha)&=\tau \end{aligned} \tag{1}$$

where J_h and J_l are the motor and link inertia respectively. m is the link mass, h is the height of the center of mass of the link. K_s and g represents the spring stiffness and gravity constant respectively. We assume that the viscous damping B at the link is negligible so $B\dot{\alpha}=0$.

We now add viscous and coulomb friction defined by equation 2 (Reyes and Kelly, 2001) to obtain equation 3:

$$Fr(\dot{\theta})=b\dot{\theta}+f_c sgn(\dot{\theta}) \tag{2}$$

b and f_c are the coefficients of Viscous and Coulomb friction at the joint θ.

$$\begin{aligned} J_L(\ddot{\theta}+\ddot{\alpha})+K_s\alpha-mghsin(\theta+\alpha)&=0 \\ (J_L+J_h)\ddot{\theta}+J_L\ddot{\alpha}-mghsin(\theta+\alpha)&=\tau-Fr\dot{\theta} \end{aligned} \tag{3}$$

The model parameters are presented in table 1.

Mathematically, the control problem studied in this paper can be defined as: According to Figure 2, let us define the output of the arm as $y=\theta+\alpha$. We want to design a control $u(t)=f(y)$ that will steer the flexible robot joint from one state to another i.e. $x(t)=f(y)$ such that $x(t_1)<x(t)<x(t_2)$ and $y(t)$ are defined as functions of a so called flat output up

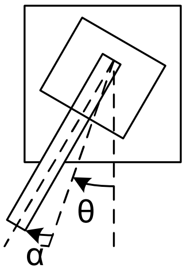

Figure 3: Coordinates of the flexible joint arm.

Table 1: Model parameters.

parameter	Notation	Value	Unit
link mass 1	m	0.403	Kg
Link Stiffness	K_s	1.61	N/m
Height of C.M.	h	0.06	m
Motor Constant	K_m	0.00767	N/$rads$/s
Gear Ratio	K_g	70	
Inertia of hub	J_h	0.0021	Kgm^2
Motor Resistance	R_m	2.6	Ω
Inertia of load	J_L	0.0059	Kgm^2
Viscous coefficient	b_1	0.175	Nms
Coulomb coefficient	f_c	1.734	Nm
Gravity	g	-9.81	m/s^2

to a certain order. The presence of nonlinearities including gravitational forces, frictional torques opposing the torque at the motors and joint flexibility have to be overcome to achieve the required motion. For the dynamic equations of 1-2, we define a boundary $y = L\ \forall t$ from start to finish. L is a finite value over time that defines the displacement of the robot.

3 DIFFERENTIAL FLATNESS ANALYSIS OF MANIPULATOR

Given a nonlinear system of the form:

$$\dot{\mathbf{x}} = f(x,u) \tag{4}$$

where: $x \in \Re^n$ is the state vector and $u \in \Re^m$ is the input vector.

The system in (4) is said to be differentially flat if there exists a variable or set of variables $y \in \Re^m$ called the flat output of the form:

$$y = h(x,u,\dot{u},\ddot{u},......,u^{(p)}) \tag{5}$$

such that:

$$x = \alpha(y,\dot{y},\ddot{y},......,y^{(p)}),$$

and

$$u = \beta(y,\dot{y},\ddot{y},......,y^{(q+1)}) \tag{6}$$

p and q being finite integers, and the system of equations

$$\frac{d}{dt}\alpha(y,\dot{y},\ddot{y},......,y^{(q+1)})$$
$$f(\alpha(y,\dot{y},\ddot{y},......,y^{(q)}),\beta(y,\dot{y},\ddot{y},......,y^{(q+1)}) \tag{7}$$

are identically satisfied(Rouchon et al., 1993).
The flat output is defined as:

$$y = (\theta + \alpha) \tag{8}$$

The transformation between the flat output and the robot states are easily derived in equation 9 (we define the robot states as: $(\theta,\dot{\theta},\alpha,\dot{\alpha})$
In terms of the flat output;

$$\begin{aligned} \theta &= y + \frac{-mgh\sin(y)+J_L\ddot{y}}{K_S} \\ \dot{\theta} &= \dot{y} - \frac{-\dot{y}mgh\cos(y)+J_Ly^{(3)}}{K_S} \\ \alpha &= \frac{mgh\sin(y)-J_L\ddot{y}}{K_S} \\ \dot{\alpha} &= \frac{\dot{y}mgh\cos(y)-J_Ly^{(3)}}{K_S} \end{aligned} \tag{9}$$

Using the flatness theory and after some computations, the feedforward control law for the tip position of the flexible joint arm is derived as (Markus et al., 2012a):

$$\begin{aligned} &u_{ff} = \frac{1}{K_sK_mK_g}mgh(-K_sR_msin(y)+sin(y)\dot{y}^2\zeta_3) \\ &+A_2sgn(\frac{\dot{y}K_s-\dot{y}\zeta_2+y^{(3)}J_l}{K_s})K_s\zeta_3+A_1y^{(3)}J_l\zeta_3 \\ &+A_1\dot{y}K_s\zeta_3+K_sR_m\ddot{y}J_l-A_1\dot{y}\zeta_2\zeta_3+K_s\ddot{y}\zeta_3 \\ &+y^{(4)}J_l\zeta_3-\ddot{y}\zeta_2\zeta_3+\zeta_1y^{(3)}J_l+\zeta\dot{y}K_s \\ &-\zeta_1\dot{y}\zeta_2 \end{aligned} \tag{10}$$

where

$$\zeta_1 = K_m^2K_g^2, \zeta_2 = mghcos(y), \zeta_3 = J_hR_m$$

3.1 Trajectory Generation and Motion Planning

Having designed the flatness based control law, the reference trajectories are then generated for the flexible robot tip movements from point to point (see (Levine, 2009)). The trajectory is obtained by using an interpolation polynomial. This interpolation enables us to find polynomial coefficients based on some initial and final conditions. These conditions are imposed on the position, speed, acceleration and jerk of the robot arm. Assuming that there is no obstacle, the flexible robot trajectories are designed with a ninth order polynomial. This polynomial covers all the constraints of the fourth order dynamics for the position, speed, acceleration and the control torque required in the feed-forward control.

Ideally, in the absence of external disturbances, the designed open loop control is expected to steer the robot smoothly through the reference path without any errors. This is because, frictional effects, gravity and other nonlinearities are naturally accounted for using the flat output. Possibilities of saturation and singularities are also taken care of because the design follows the constraints set by the dynamic model.
For the robot represented by equation(1),assume that the initial time for the robot to move from an initial point is given as t_1 with initial conditions $y(t_1) = y_1$

and the final time as t_2 with final conditions $y(t_2) = y_2$, it is required to generate trajectories for every state of the manipulator and the corresponding feed-forward control satisfying $u(t_1, t_2)$. We see that this problem can be solved for the robot arm without resorting to solving differential equations.

In the general case, the reference trajectory of the robot tip is formulated as (Anene, 2007; Markus et al., 2012a):

$$y^*(\delta) = \alpha_0 + \alpha_1\delta + \alpha_2\delta^2 + \alpha_3\delta^3 + ... + \alpha_{2n+1}\delta^{2n+1} \tag{11}$$

with $n = 4$ and

$$\delta = \frac{t - t_1}{t_2 - t_1} \tag{12}$$

differentiating 12, we obtain:

$$\frac{d\delta}{dt} = \frac{1}{t_2 - t_1} \tag{13}$$

substituting $t = t_1$ into equation 12, $\delta = 0$ and $t = t_2$ results in $\delta = 1$. Doing some manipulations, the coefficients can be obtained as:

$$\alpha_i = \frac{1}{i!} y^i(t_1)(t_2 - t_1)^i \tag{14}$$

for $i = 0 \rightarrow n$ and $i = n \rightarrow 2n + 1$,

$$[\alpha_{n+1} \;\; \alpha_{n+2} \;\; ...\alpha_{2n+1}]^T = A * B \tag{15}$$

The expressions for A and B are given in the appendix due to their long length.

Using the values obtained from A and B above, the reference trajectory y^* for the state variable of interest around the time boundaries t_1 and t_2 is hereby given by:

$$y^*(t) = y^*(t_1) + (y^*(t_2) - y^*(t_1)) \sum_{j}^{2n+1} \alpha_j \left(\frac{t - t_1}{t_2 - t_1}\right)^j \tag{16}$$

and the derivatives of the trajectories can be written as:

$$\dot{y}^*(t) = \alpha_r + \sum_{j=r+1}^{2n+1} (j).\alpha_j \tau^{j-r} \left(\tfrac{1}{t_2 - t_1}\right), \; for \, r = 1$$
$$\dot{y}^*(t_1) = \alpha_r\left(\tfrac{1}{t_2 - t_1}\right);$$
$$\dot{y}^*(t_2) = \sum_{j=r+1}^{2n+1} (j).\alpha_j \tau^{j-r} \left(\tfrac{1}{t_2 - t_1}\right)$$
$$\dot{y}^*(t_2) - \dot{y}^*(t_1)(t_2 - t_1) = \sum_{j=r+1}^{2n+1} (j).\alpha_j$$

The remaining computations are too long and may be found in (Anene, 2007)

4 FEEDFORWARD CONTROLLER DESIGN

As earlier stated, the importance of the feed-forward controller design for the flexible joint robot is to accomplish, with the same controller, high precision positioning, fast oscillation-free displacements, and robustness against modeling errors and nonlinearities brought about by friction and gravitational effects. The feed-forward control u_{ff} is as derived in equation 10. The trajectories for each state in the control are easily interpolated using the linear polynomials already described in section 3. It is assumed that the position of the motor is available for measurements. All other states are easily interpolated using the measured parameter.

Reference trajectories are derived for the flat output using the boundary time between $t_1 = 0$ and $t_2 = 1s$; for fast point to point position from $y = 0$ to $y = 0.06rads$. The corresponding equation of the tip position based on equation 16 will be:

$$y^*(t) = 7.56t^5 - 25.2t^6 + 32.4t^7 - 18.9t^8 + 4.2t^9 \tag{17}$$

5 SIMULATIONS

Using the Matlab/SIMULINK platform, simulations were carried out to test the designed feedforward control. The simulation block used for the study is shown in figure 4.

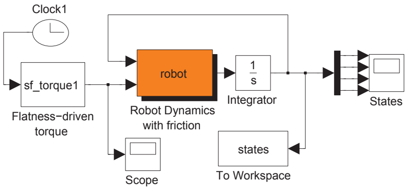

Figure 4: Feedforward control in Simulink.

Initial experiments on the test bed involves checking for the robot behavior under no friction, with friction, zero voltage and under a defined voltage. The open loop behavior of the flexible joint robot without any control can be seen in figures 5 and 6. Figure 5 indicates the motor position dropping from a position of $1rads$ under gravity and no voltage applied to the motors. The motor completely drops the flexible link to a zero position under gravity in about $2seconds$. The link deflection shows a deflection of about $-0.12rads$ before returning to zero.

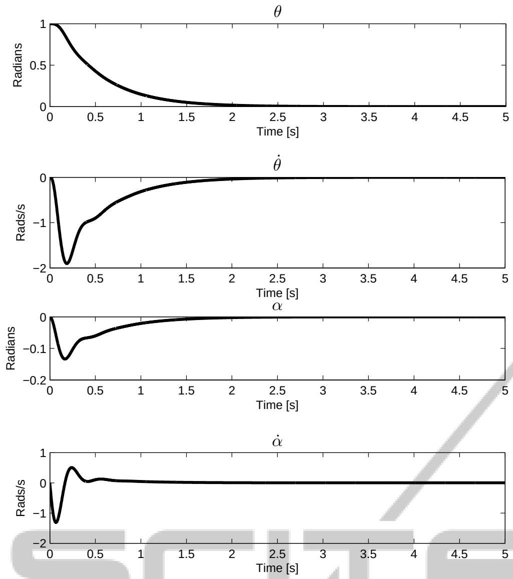

Figure 5: Open loop robot states under no friction.

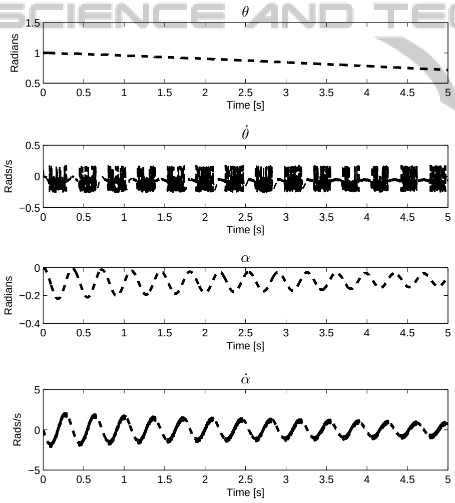

Figure 6: open loop robot states under friction.

With friction at the joint, figure 6 shows that the friction slows down the movement of the link towards the zero position. Within the 5*minutes* simulation time, the motor position was still at above the 0.5*rads* mark. It takes a longer time to dampen to zero. The link deflection α shows a lot of oscillations and the velocities also show strong oscillatory behavior. This kind of scenarios are undesirable and are a motivation for designing the flatness based feedforward control to overcome these nonlinear effects.

Using the flat output equation 17, rest to rest trajectories are generated for the flexible robot arm as shown in figure 7. These trajectories are now used to define the feedforward control while compensating for the undesirable nonlinear effects that were seen in the open loop behavior of figures 5 and 6. The torque and control voltage generated by the flat output to drive the flexible robot arm under gravity and friction from a zero position to a position of 0.06rads without any oscillations are shown in figures 8 and 9 respectively. The proposed flatness based feedforward control designed in equation 10 is then applied to drive the robot from a position of 0 to 0.06rads. The result in figure 10 shows that the robot is easily driven along this path without having to integrate any equations. The robot arm is able to follow its reference position without errors. It is assumed that disturbances are non existent or are kept at a minimum.

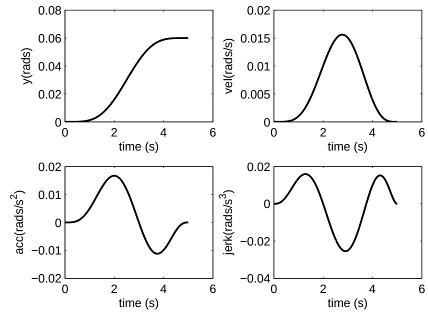

Figure 7: Rest to rest trajectories for the flexible joint robot.

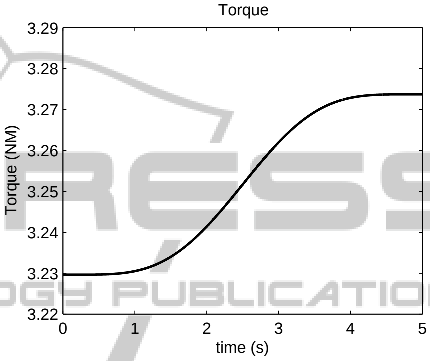

Figure 8: Open loop torque.

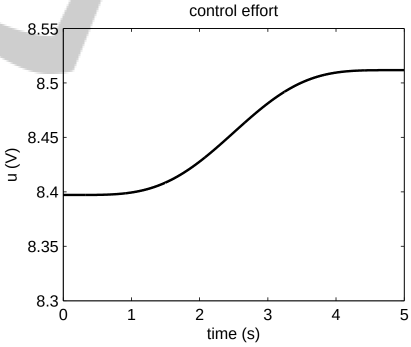

Figure 9: Open loop voltage.

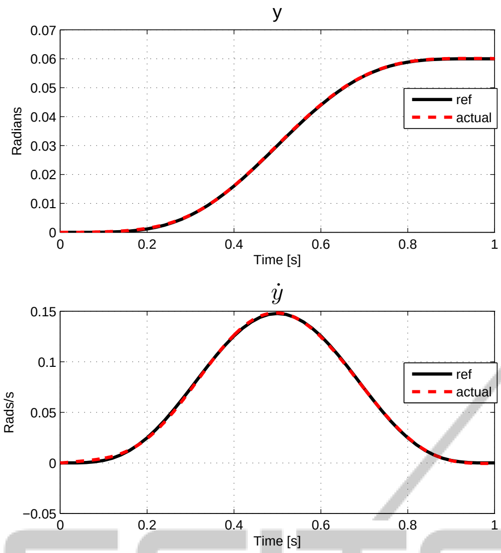

Figure 10: Trajectories driven by flatness based feedforward control.

6 CONCLUSION

The feedforward control for the single link flexible joint robot arm under the influence of gravity and frictional effects is solved using differential flatness theory. The control was accomplished for point to point position movements in finite time. The technique does not require any solution of differential equations despite the highly nonlinear dynamics of the robot. The proposed control has great potential for carrying out fast and precise point to point movements without any oscillations for the flexible robot arm. The proposed approach can be extended to the case of multi-link robot control where elasticity is considered at each joint and the effects of gravity taken into account. This will be studied in future works.

REFERENCES

Anene, E. (2007). Flat control of a synchronous machine. *Unpublished Thesis*.

Bauer, D., Schaper, U., Schneider, K., and Sawodny, O. (2014). Observer design and flatness-based feedforward control with model predictive trajectory planning of a crane rotator. In *American Control Conference (ACC), 2014*, pages 4020–4025. IEEE.

Cambera, J. C., Chocoteco, J. A., and Feliu, V. (2014). Feedback linearizing controller for a flexible single-link arm under gravity and joint friction. In *ROBOT2013 First Iberian Robotics Conference*, pages 169–184. Springer.

De Vries, E., Fehn, A., and Rixen, D. (2010). Flatness-based model inverse for feed-forward braking control. *Vehicle System Dynamics*, 48(S1):353–372.

Dwivedy, S. and Eberhard, P. (2006). Dynamic analysis of flexible manipulators, a literature review. *Mechanism and Machine Theory*, 41(7):749–777.

Fliess, M., Levine, J., Martin, P., Ollivier, F., and Rouchon, P. (1997). Controlling nonlinear systems by flatness. *Systems and Control in the Twenty-first Century*, page 137154.

Fliess, M., Levine, J., Martin, P., and Rouchon, P. (1999). A lie-backlund approach to equivalence and flatness of nonlinear systems. *Automatic Control, IEEE Transactions on*, 44(5):922–937.

Hagenmeyer, V. and Delaleau, E. (2003). Exact feedforward linearization based on differential flatness. *International Journal of Control*, 76(6):537–556.

Ider, S. and zgren, M. (2000). Trajectory tracking control of flexible-joint robots. *Computers and Structures*, 76(6):757–763.

Jiang, Z. and Higaki, S. (2011). Control of flexible joint robot manipulators using a combined controller with neural network and linear regulator. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 225(6):798–806.

Kandroodi, M., Mansouri, M., Shoorehdeli, M., and Teshnehlab, M. (2012). Control of flexible joint manipulator via reduced rule-based fuzzy control with experimental validation. *International Scholarly Research Network ISRN Artificial Intelligence*.

Kharitonov, A. and Sawodny, O. (2006). Flatness-based feedforward control for parabolic distributed parameter systems with distributed control. *International Journal of Control*, 79(07):677–687.

Kotman, P., Bitzer, M., and Kugi, A. (2010). Flatness-based feedforward control of a two-stage turbocharged diesel air system with egr. In *Control Applications (CCA), 2010 IEEE International Conference on*, pages 979–984. IEEE.

Levine, J. (2006). On necessary and sufficient conditions for differential flatness. *Arxiv preprint math/0605405*.

Levine, J. (2009). *Analysis and control of nonlinear systems: A flatness-based approach*. Springer.

Markus, E., Agee, J., and Jimoh, A. (2013). Differentially flat trajectory control of a 6dof industrial robot manipulator. In *International Association for Science and Technology (IASTED)Control and Application conference*, pages 215–221. Iasted.

Markus, E., Agee, J., Jimoh, A., and Tlale, N. (2012a). Nonlinear control of a single-link flexible joint manipulator using differential flatness. In *Robotics and Mechatronics Conference (ROBMECH)*. IEEE.

Markus, E., Agee, J., Jimoh, A., Tlale, N., and Zafer, B. (2012b). Flatness based control of a 2 dof single link flexible joint manipulator. In *2nd International Conferenceon Simulation and Modeling Methodologies, Technologies and Applications*, pages 437–442. SciTePress.

Morandini, M., Masarati, P., Bargigli, L., and Vaccani, L. (2012). Feedforward control design from general-purpose multibody analysis for an original parallel robot concept. In *Proceedings of the 2nd Joint International Conference on Multibody System Dynamics, P. Eberhard and P. Ziegler, eds., Stuttgart, Germany*.

Muller, B., Deutscher, J., and Grodde, S. (2006). Trajectory generation and feedforward control for parking a car. In *Computer Aided Control System Design, 2006 IEEE International Conference on Control Applications, 2006 IEEE International Symposium on Intelligent Control, 2006 IEEE*, pages 163–168. IEEE.

Ozgoli, S. and Taghirad, H. (2006). A survey on the control of flexible joint robots. *Asian Journal of Control*, 8(4):332–344.

Palli, G., Borghesan, G., and Melchiorri, C. (2009). Tendon-based transmission systems for robotic devices: Models and control algorithms. In *Robotics and Automation, 2009. ICRA'09. IEEE International Conference on*, pages 4063–4068. IEEE.

Pereira, E., Becedas, J., Payo, I., Ramos, F., and Feliu, V. (2007). Control of flexible manipulators. theory and practice.

Quanser (2015).

Reyes, F. and Kelly, R. (2001). Experimental evaluation of model-based controllers on a direct-drive robot arm. *Mechatronics*, 11(3):267–282.

Rodriguez, A. S.-M., Ibanez, J. C. C., and Battle, V. F. (2014). Online algebraic identification of the payload changes in a single-link flexible manipulator moving under gravity.

Rouchon, P., Fliess, M., Levine, J., and Martin, P. (1993). Flatness, motion planning and trailer systems. pages 2700–2705 vol. 3. IEEE.

Schlipf, D. and Cheng, P. W. Flatness-based feedforward control of wind turbines using. In *Proceedings of the 19th World Congress of the*. Universität Stuttgart.

Tokhi, M. and Azad, A. (2008). Flexible robot manipulators: modelling, simulation and control. In *IET Control Series*, volume 68. London Institution of Engineering and Technology.

APPENDIX

The equations for A and B are given here in the appendix due to their size:

$$A=\begin{bmatrix} 1 & 1 & ... & 1 \\ n+1 & n+2 & ... & 2n+1 \\ n.n+1 & n+1.n+2 & ... & 2n.2n+1 \\ n-1.n.n+1 & n.n+1.n+2 & ... & 2n-1.2n.2n+1 \\ . & . & ... & . \\ . & . & ... & . \\ . & . & ... & . \\ (n+1)! & \frac{(n+2)!}{2!} & ... & \frac{(2n+1)!}{(n+1)!} \end{bmatrix}^{-1}$$

and

$$B=\begin{bmatrix} y(t_2)-y(t_1)-\sum_j \frac{1}{i!}y^i(t_1)(t_2-t_1)^i; for\, i=1-n \\ y^{(i-1)}(t_2)-y^{(i-1)}(t_1)(t_2-t_1)^{i-1}-\sum_j \frac{1}{(i-1)!}y^i(t_1)(t_2-t_1)^i; for\, i=2-n \\ y^{(i-1)}(t_2)-y^{(i-1)}(t_1)(t_2-t_1)^{i-1}-\sum_j \frac{1}{(i-2)!}y^i(t_1)(t_2-t_1)^i; for\, i=3-n \\ . \\ . \\ . \\ y^i(t_2)-y^i(t_1)(t_2-t_1)^i; for\, i=n \end{bmatrix}$$