Experimental Modal Analysis based on a Gray-box Model of Flexible Structures

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Abstract: The main objective of this paper is to propose an experimental modal analysis procedure, based on the use of a gray-box model for flexible structures. The described approach presents interesting advantages with respect to commercial solutions: ease of use due to the low number of parameters to set for an identification session; no need for expert users, even in the presence of particular cases such as double modes, since it does not use a stabilization diagram to be elaborated; use of a gray-box model whose unknown parameters have a clear physical meaning. All these characteristics are discussed in the paper, and the performance of the proposed procedure has been evaluated by using experimental data available from a non-trivial standard benchmark. The results have been compared with those obtained by using a commercial tool.

1 INTRODUCTION

Experimental modal analysis has become a key technology in structural dynamic analysis and in mechanical products development. Modal analysis is the process of extracting the dynamics characteristics of a vibrating system from experimental data (Ewins, 1984). When all of these parameters have been obtained, a complete mathematical model of the system can be defined. The mathematical model may then be used in computer simulations to predict the response of the system when it is modified, so that a designer can evaluate different design modifications without building prototypes (Ewins, 1984; Heylen et al., 1995) or to design an active vibration and noise control for a flexible structure (Cavallo et al., 2008; Cavallo et al., 2010).

This paper presents a new frequency domain procedure for experimental modal analysis, based on the use of a gray-box model of flexible structures. The main objective of the paper is to show how, with the proposed approach, it is possible to obtain experimental results similar to those obtained with the commercial solutions, but with some advantages in terms of experience required to the user and physical meaning of the model unknown parameters. The procedure uses the results of the identification method proposed in (Cavallo et al., 2007). Differently from (Cavallo et al., 2007), where the identification procedure was developed only for model-based control applications, in this paper for the first time the same procedure has been adapted, as regards the estimation of the mode shapes, for experimental modal analysis. The proposed procedure is compared with the polyreference Least Squares Complex Frequency domain method (pLSCF) (Guillaume et al., 2003). The comparison is made with this algorithm since the pLSCF is one of newest approach presented in literature and moreover there are some papers in which the pLSCF is compared with the others classical methods (Guillaume et al., 2003; Peeters et al., 2004), obtaining an indirect comparison also with these last ones. The pLSCF algorithm has been applied by using the modal analysis estimator module integrated in the commercial software FEMtools^{TM} (FEMtools Version 3.6, 2012).

The proposed procedure, similarly to the pLSCF method, separates the estimation of the system poles from the mode shapes. However, the pLSCF method uses a polynomial form and generally it starts with a model order very high, trying to fit models that contain much more modes than those actually present in the experimental data. Then, the true physical modes are separated from the spurious ones by interpreting the so-called "stabilization diagram" by an expert user. The poles corresponding to a certain model order are compared to the poles of a one order lower model and if their differences are minor than pre-set limits, the pole is labelled as a stable one. The spu-

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rious poles will not be stable during this comparison and will be sorted out of the modal parameters. Finally, an eigenvalue decomposition of the companion matrix, associated with the polynomial coefficients identified, is implemented to compute the real modal parameters (Guillaume et al., 2003). The number of parameters to set by the user before starting the optimization procedure is high and can lead to wrong results if the user experience is not adequate.

Differently from the commercial solutions, the procedure here proposed estimates the eigenvalues by using a subspace approach, and the user is required only to fix the number of modes to identify and to select the frequency range in which to perform the identification. Spurious modes outside the frequency range of interest are not extracted. Moreover, the proposed approach does not use a stabilization diagram to select stable poles in the selected frequency range, since the stability of the estimated poles is guaranteed by the identification procedure of the dynamic matrix detailed in (Cavallo et al., 2007). As a consequence, the interaction of an expert user is not necessary, even in the presence of particular cases such as double modes. This feature makes the proposed approach very interesting to be automated for applications where iterative modal analysis is necessary (e.g., health monitoring, model-based control, optimal positioning of actuators and sensors for control).

Another advantage of the proposed procedure is the use of a gray-box model whose unknown parameters have a clear physical meaning. In particular, for all the applications where a numerical–experimental correlation between a finite element model and experimental data is necessary, the use of a gray-box model can simplify the user task of tuning the numeric parameters in order to match the experimental data.

2 MODELLING OF FLEXIBLE STRUCTURES

Consider a $n-$degree-of-freedom mechanical system whose generalized coordinates are represented by a $n \times 1$ vector q and with $n \times n$ mass matrix M and stiffness matrix K; both matrices are positive definite and symmetric. The equation of motion can be written in the form (Cavallo et al., 2010)

$$M\ddot{q} + Kq = f \tag{1}$$

where f is the $n \times 1$ vector of generalized forces. It is evident that the state of this dynamic system is constituted by the $2n \times 1$ vector $z = (q^{\mathrm{T}}\dot{q}^{\mathrm{T}})^{\mathrm{T}}$. With this choice, the state space representation of the system is

$$\begin{aligned} \dot{z} &= \begin{pmatrix} 0 & I \\ -M^{-1}K & 0 \end{pmatrix} z + \begin{pmatrix} 0 \\ M^{-1} \end{pmatrix} u \\ y &= \begin{pmatrix} 0 & I \end{pmatrix} z \end{aligned}$$

if the observed output is the velocity $\dot{q}$. Since the matrix $M^{-1}K$ have all real and positive eigenvalues, it can be diagonalized by solving the following eigenvalue problem

$$M^{-1}KU = U\Omega \tag{2}$$

where $\Omega = \mathrm{diag}\{\omega_{n_1}^2, \ldots, \omega_{n_n}^2\}$ is the diagonal matrix of the eigenvalues of $M^{-1}K$ and U is the matrix whose columns are the eigenvectors of $M^{-1}K$, the so-called "mode shapes". It is well-known that the eigenvectors are orthogonal with respect to the mass matrix and thus they can be normalized so as

$$U^{\mathrm{T}}MU = I \tag{3}$$

Now, define the transformation in the state space

$$z = \begin{pmatrix} U & 0 \\ 0 & U \end{pmatrix} x, \tag{4}$$

where x are the so-called "modal coordinates", and consider a number m of input forces f_i, $i \in \mathcal{F} \subseteq \{1, \ldots, m\}$ applied to the structure along a set of degrees of freedom, so that, defining the unit vector e_i as the vector with 1 at entry i and 0 elsewhere, a selection matrix S can be defined whose rows are e_i^{T}, $i \in \mathcal{F}$. Assuming to observe the velocity along the same degrees of freedom, the state space representation assumes the form

$$\dot{x} = Ax + Bu = \begin{pmatrix} 0 & I \\ -\Omega & -\Lambda \end{pmatrix} x + \begin{pmatrix} 0 \\ U^{\mathrm{T}}S^{\mathrm{T}} \end{pmatrix} f \tag{5}$$

$$y = Cx = \begin{pmatrix} 0 & SU \end{pmatrix} x \tag{6}$$

where the diagonal matrix $\Lambda = \mathrm{diag}\{2\zeta_1\omega_{n_1}, \ldots, 2\zeta_n\omega_{n_n}\}$, being $\zeta_i \in (0,1)$ the damping coefficient of the $i-$th mode, has been introduced to take into account the unavoidable damping present in the physical system. Notice, that this corresponds to add in Eq. (1) the friction forces $d = D\dot{q}$ acting in the material, with a proportional damping matrix $D = \alpha_1 K + \alpha_2 M$. This choice to add a proportional damping in the model Eq. (1) changes the finite-dimensional, complex eigenvalue problem. In particular, the solution yields n pairs of complex conjugate eigenvalues, named "modes"

$$\lambda_i = -\zeta_i\omega_{n_i} \pm j\omega_{n_i}\sqrt{1-\zeta_i^2} \qquad i = 1, \ldots, n \tag{7}$$

where λ_i is the $i-$th complex eigenvalue of the structure. The Eq. (7) represents the relation between the complex eigenvalue, the damping coefficient ζ_i and the natural frequency ω_{n_i} of the $i-$th mode.

Furthermore, this choice for the viscous damping has also the following implications: it does not affect the mode shapes that are the eigenvectors ($\psi_i \in \mathbb{C}^n$) of the matrix $M^{-1}K$; the mode shapes are real and orthonormal; each mode shape is described by a standing wave that contains fixed stationary node points. In contrast, if the matrix D is not representative of a proportional damping, the mode shape are complex valued and results in modes, which are described as complex modes, having different characteristics; each mode shape is described by a travelling wave and appears to contain a moving node point on the structure; in such a case, the mode shape from the undamped case does not diagonalise the damping matrix D. Since in the typical case of experimental modal analysis the available data are Frequency Response Functions (FRFs), also the relations between the gray-box model and the FRFs data are useful. In particular, applying the Fourier transform, the frequency response matrix corresponding to the model (5)–(6), is symmetric and has the form

$$G(j\omega) = U\Phi(j\omega)U^{\mathrm{T}} \tag{8}$$

with $\Phi(j\omega) = \mathrm{diag}\{\phi_1(j\omega),\ldots,\phi_n(j\omega)\}$ and $\phi_l(j\omega) = j\omega/(\omega_{n_l}^2 - \omega^2 + j2\zeta_l\omega_{n_l}\omega)$, $l = 1,\ldots,n$ and thus for an applied force at a spatial position k (as input) and a velocity measurement at a spatial position h (as output), the FRF $g_{hk}(j\omega)$ in physical coordinates is

$$g_{hk}(j\omega) = \sum_{l=1}^{n} \phi_l(j\omega)\psi_{h_l}\psi_{k_l} \tag{9}$$

where ψ_{h_l} is the *h-th* column of the matrix U of the mode shape corresponding to the *l-th* mode.

3 MODAL PARAMETERS ESTIMATION

The objective of experimental modal analysis (Ewins, 1984) is to derive natural frequencies and damping coefficients as well as mode shapes. In the proposed approach, the unknown parameters to identify are explicitly present in the gray-box model reported in Eqs. (5)–(6) and used in the identification procedure. Note that if the number of excitation inputs is different from the number of measured outputs, the selection matrix in Eq. (5) is different from the one in the Eq. (6). In particular, if the number of inputs is equal to m and the number of outputs is p ($p > m$ is the typical case of experimental modal analysis) the gray-box model can be rewritten as

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & I \\ -\Omega & -\Lambda \end{pmatrix} x + \begin{pmatrix} 0 \\ B_2 \end{pmatrix} u && (10) \\ y &= \begin{pmatrix} 0 & C_2 \end{pmatrix} x && (11) \end{aligned}$$

where $\Lambda, \Omega \in \mathbb{R}^{n\times n}$, $B_2 \in \mathbb{R}^{n\times m}$ and $C_2 \in \mathbb{R}^{p\times n}$ have to be estimated. With this choice, the rows of B_2 represent spatial samples of the mode shapes corresponding to excitation points, while the columns of C_2 represent spatial samples of the mode shapes corresponding to measurement points.

All the parameters to be identified for the model (10)–(11) can be estimated by using the procedure detailed by the same authors in (Cavallo et al., 2007). The only choices that the user has to do, as discussed in the introduction, are: to select the frequency range in which to perform the identification and to fix the number of modes to identify.

The selection of the working frequency range $[f_1, f_2]$ Hz, in which to extract the modes of interest, has to take into account that the real system is infinite dimensional. In fact, for flexible structures the FRFs in a given frequency range depend not only on the modes with natural frequencies within the considered range but also on the lower and higher frequency modes. This means that near the extremes of the range the experimental data are not sufficient for an accurate identification of modal parameters and thus the FRFs should be measured on a working frequency range $[f_1', f_2']$ Hz wider than the interesting one $[f_1', f_2'] \supset [f_1, f_2]$. Fixed the frequency range, denote with $G_i \in \mathbb{C}^{p\times m}$, $\quad i = 1,\ldots,M$ the experimental measured samples of the frequency response matrix in the frequency range $[f_1', f_2']$ Hz (not necessarily uniformly spaced). The choice of M depends on the modal density of the flexible structure and obviously on the width of the frequency range of interest. The number of the modes to be identified (n) can be fixed on the basis of a classical indicator used in experimental modal analysis (Ewins, 1984), i.e.

$$H(\omega_i) = \sum_{h=1}^{p}\sum_{k=1}^{m} |g_{hk}(j\omega_i)|, \quad i = 1,\ldots,M \tag{12}$$

by counting (approximately and lightly overestimating) its number of peaks by simple inspection.

By following the procedure in (Cavallo et al., 2007), in a first step, the M measured samples of the FRFs are used to construct the matrices detailed in the cited paper, from which, via a subspace identification technique, the eigenvalues of the dynamic matrix A are estimated. At the end of this step, the n fixed modes are completely estimated in terms of natural frequencies and damping ratios. The second stage consists in determining an estimate of B_2 and C_2. In particular, in view of (9), the frequency response matrix can be written as

$$G(j\omega) = \sum_{l=1}^{n} \phi_l(j\omega) c_l b_l^{\mathrm{T}} = \sum_{l=1}^{n} \phi_l(j\omega) R_l \qquad (13)$$

where $R_l = c_l b_l^{\mathrm{T}} \in \mathbb{R}^{p\times m}$ is the residue matrix corresponding to the l-th mode, with c_l and b_l^{T} the l-th column of C_2 and the l-th row of B_2, respectively. From Eq. (13) it is evident that the unknown parameters R_l can be easily computed by resorting to a least mean square technique, being $G(j\omega)$ the measured data and the generic $\phi_l(j\omega)$ known from the first step. From the estimated R_l residues it is possible to construct the matrices B_2 and C_2 in different ways. The interested reader can refer to (Cavallo et al., 2007) and (Cavallo et al., 2010) for more details. For the experimental modal analysis, objective of this paper, the estimation of residues is sufficient. In fact, the estimated residues allows an easy estimation of the so called *modal constants* from which any normalization can be chosen to estimate the mode shapes (details can be found in any textbook on experimental modal analysis, e.g., (Ewins, 1984; Heylen et al., 1995)). Recalling that the modal constants are the constant matrices A_l appearing in the following decomposition of the FRF matrix

$$G(j\omega) = \sum_{l=1}^{n} \phi_l(j\omega) R_l = j\omega \sum_{l=1}^{n} \left(\frac{A_l}{j\omega - \lambda_l} + \frac{A_l^*}{j\omega - \lambda_l^*} \right), \qquad (14)$$

it is straightforward to see that the modal constants are related to the residues as

$$A_l = \frac{1}{2j\omega_{n_l}\sqrt{1-\zeta_l^2}} R_l. \qquad (15)$$

Equation (15) shows the classical property that modal constants are purely imaginary for flexible structures exhibiting only stationary waves. For the evaluation of the experimental results in the remainder of the paper, the normalization selected to compute the normalized mode shape from the modal constant, corresponding to the l-th mode, is

$$\phi_l = \frac{Im(A_l)}{\max|(A_l)|}. \qquad (16)$$

4 EXPERIMENTAL RESULTS

The proposed approach for the experimental modal analysis has been tested by using data for the Modal Parameter Estimation (MPE) Round Robin. This round robin is an initiative of the Michigan State Technical University (MTU) and was presented at the recent IMAC XXIX conference. More than twenty participants from several countries has taken part in the exercise, performed on simulated and experimental data. The data are available for research activities on the web (IMAC website, 2011). In particular, the calibration data set with proportional damping and the experimental data of the plexiglass plate available online have been considered for this work. For each data set a comparison with the FEMtools^{TM} commercial solution is also presented.

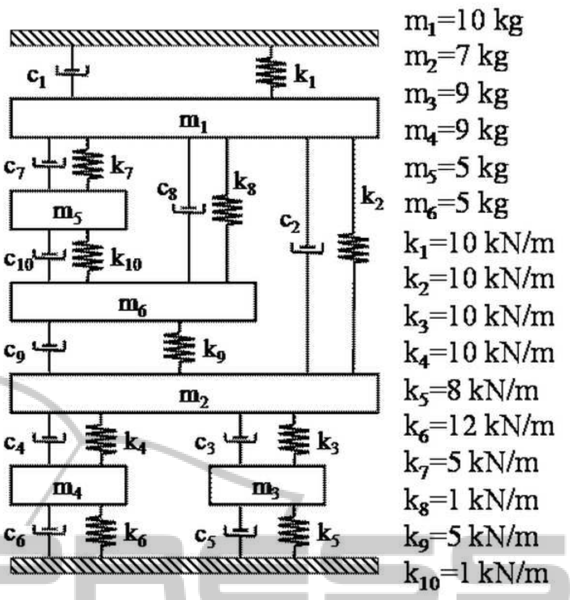

Figure 1: Lumped parameter model used to generate calibration data set for IMAC MPE Round Robin.

4.1 The Calibration Data

The calibration data sets, available from IMAC MPE Round Robin, were generated using a lumped parameters model, constituted by 6 masses, 10 springs and 10 dampers. The details about the model are reported in Fig. 1 (see also the web site (IMAC website, 2011)). As detailed above, the proposed approach is based on a gray-box model with proportional damping. According to this hypothesis, the data set with $D = 0.0001K + 0.05M$ has been selected. As described above, the only choice, that the proposed approach asks the user, is to set the number n of modes to identify. For a lumped parameter model the number of mode is finite, and in particular is $n = 6$ for the selected model. With this choice of n, the proposed approach has been applied, starting from the FRF samples of the considered calibration data set, to estimate the modal parameters of the lumped parameter model. The results are reported in Table 1, where

Table 1: Calibration data with proportional damping: modes extracted by using the proposed approach.

Mode #	ω_n^a [Hz]	ω_n^e [Hz]	E_{ω_n} [%]	ζ^a [%]	ζ^e [%]	E_ζ [%]	MAC_1 alias [%]	MAC_2 [%]
1	3.395	3.506	3.269	0.224	0.224	0	3.25	99.67
2	5.238	5.493	4.868	0.240	0.245	2.083	2.72	98.13
3	6.423	6.427	0.062	0.264	0.264	0	3.05	99.84
4	7.547	7.552	0.066	0.290	0.290	0	0.51	99.86
5	8.305	8.388	0.999	0.309	0.311	0.647	3.25	98.78
6	12.639	12.641	0.016	0.428	0.429	0.234	1.03	99.98

the natural frequencies and the damping ratios of the estimated modes are compared to the actual ones, analytically obtained by the numeric matrices M, K and D. The percentage errors for the estimated natural frequencies have been quantified as

$$E_{\omega_n} = \frac{|\omega_n^a - \omega_n^e|}{\omega_n^a} 100, \tag{17}$$

where ω_n^a and ω_n^e are the actual and the estimated values of the natural frequencies, respectively. Similarly, for the damping ratios the following percentage error has been evaluated

$$E_\zeta = \frac{|\zeta^a - \zeta^e|}{\zeta^a} 100, \tag{18}$$

where ζ^a and ζ^e represent the actual and the estimated values of the damping ratios, respectively. The results, reported in Table 1, show the good estimation obtained for all the 6 modes of the considered model. The percentage errors are always less than 5% and often less than 1%. For the mode shapes, the results have been evaluated by using the classical Modal Assurance Criterion (MAC). The MAC index between two modes ψ_i and ψ_j is defined as:

$$MAC = \frac{|\psi_i^H \psi_j|^2}{(\psi_i^H \psi_i)(\psi_j^H \psi_j)} 100, \tag{19}$$

where the symbol $(\cdot)^H$ indicates the transpose conjugate. The MAC index can be used to evaluate the correlation between: two mode shapes of the same estimated model; an estimated mode shape with the corresponding actual mode shape; two corresponding mode shapes estimated by using two different procedures. Based on this observations, the first significant analysis is the evaluation of the correlation between the mode shapes of the estimated model. In particular, since the mode shapes define an orthogonal basis, they are uncorrelated and the corresponding MAC value should be zero. As a consequence, computing the MAC values for a set of mode shapes of the same model should, in theory, provide a diagonal matrix, i.e. diagonal elements with a value of 100 and zero-valued off-diagonal elements. Hence, the MAC_1 index values have been computed by using Eq. (19), where ψ_i and ψ_j (with $i, j = 1, \ldots, 6$) are the normalized mode shape obtained from Eq. (16). The results are reported in Fig. 2, where the off-diagonal elements are clearly very low. The maximum MAC_1 alias, which is the highest off-diagonal value for the considered modes, is reported in Table 1 to simplify its evaluation. The comparison of the estimated mode shapes with the actual ones, analytically obtained by the numeric matrices M, K and D, has been carried out by computing the MAC_2 index values by using Eq. (19). In particular, the correlation between the estimated ψ_i and the corresponding actual ψ_j normalized mode shapes (with $i = j = 1, \ldots, 6$) has been computed for the 6 modes of the considered model. The excellent results are reported in the last column of Table 1. In order to better appreciate the results obtained with the proposed approach, the MPE has been also carried out by using the commercial solution FEMtools^{TM}. The same calibration data have been used to implement the same analysis. Hence, the same [illegible] a fair

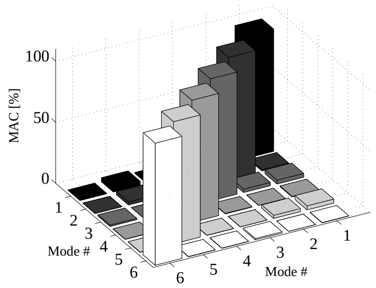

Figure 2: Calibration data with proportional damping: MAC_1 for the modes extracted with the proposed approach.

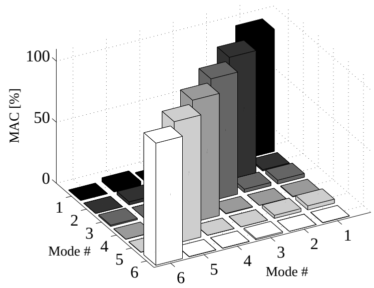

Figure 3: Calibration data with proportional damping: MAC_1 for the modes extracted by using FEMtools.

Table 2: Calibration data with proportional damping: modes extracted by using FEMtools.

Mode #	ω_n^a [Hz]	ω_n^e [Hz]	E_{ω_n} [%]	ζ^a [%]	ζ^e [%]	E_ζ [%]	MAC_1 alias [%]	MAC_2 [%]
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2	5.238	5.493	4.868	0.240	0.245	2.083	2.69	98.14
3	6.423	6.427	0.062	0.264	0.264	0	3.06	99.84
4	7.547	7.552	0.066	0.290	0.290	0	0.49	99.86
5	8.305	8.388	0.999	0.309	0.311	0.647	3.21	98.78
6	12.639	12.641	0.016	0.428	0.428	0	1.00	99.99

comparison. The estimated natural frequencies and damping ratios have been compared with the actual ones and the mode shapes have been evaluated by using the MAC indices. The corresponding results are reported in Table 2 and Fig. 3. The obtained results are practically identical. These results, obtained in a standard benchmark, allow to conclude that the proposed method is technically sound and with the same performance of a well assessed method, but with the advantage that the number of parameters to set for an identification procedure is lower than a commercial solution since this procedure does not use a stabilization diagram.

4.2 The Plexiglass Plate

The plexiglass plate data, available from IMAC MPE Round Robin, were acquired from a plexiglas plate that measures $53 \times 32 \times 1.5$ cm. The plexiglass material has been chosen in order to obtain high damped modes. The plate has been designed to have a double mode at the first resonance peak in order to obtain non-trivial experimental data. The identification of these double mode is the greater difficulty on which to assess the performance of the proposed approach. The plate was tested in a free-free boundary condition. The measurements were made by using triaxial accelerometers, mounted with the wax and a random excitation signal as source. Hence, the available FRF data have accelerations as outputs and forces as inputs. Since the proposed approach uses a gray-box model with velocities as outputs, a preliminary division by $j\omega$ has been applied to the data in order to compute the necessary FRF samples. The proposed approach has been applied by fixing the number of modes $n = 20$, by using the indicator in Eq. (12). The first 10 extracted modes are reported in Table 3. In particular the natural frequencies and the damping ratios are reported in the first two columns. Concerning the mode shapes, as made for the calibration data, the MAC_1 index has been evaluated for the 10 identified modes and the results are reported in Fig. 4, while the MAC_1 alias values are reported in the 4-th column of the Table 3. In this case, since no actual values are available, only a comparison with the results carried out by using FEMtools is possible. The results about the first 10 modes extracted with the commercial tool are reported in Table 4 and in Fig. 5. In this experimental case, the differences between the proposed approach and the commercial solution are very low, in terms of natural frequencies and damping ratios for the extracted modes. Concerning the mode shapes, separate discussions for the first two modes, that represent the double mode, and the remaining ones have to be done. In particular, Fig. 4 shows that the mode shapes associated to the first two modes, evaluated by using the proposed approach, are clearly two distinct modes, since the MAC_1 index between mode 1 and mode 2 is about 7%. Instead, from Fig. 5, it is evident that the first two modes, evaluated by using FEMtools, are more similar to each other, showing a MAC_1 index between them of about 28%. For the re-

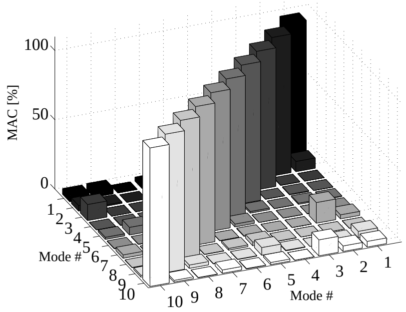

Figure 4: Plexiglass plate data: MAC_1 for the modes extracted by using the proposed approach.

Table 3: Plexiglass plate: first 10 modes extracted by using the proposed approach.

Mode #	ω_n^e [Hz]	ζ^e [%]	MAC_1 alias [%]	MAC_3[%]
1	99.660	4.85	7.8	69.23
2	99.717	6.07	14.7	71.25
3	226.792	3.66	10.7	97.88
4	279.570	3.53	3.3	92.30
5	291.574	3.48	8.5	94.30
6	356.267	3.25	8.6	96.77
7	417.760	3.36	14.7	93.81
8	502.296	3.03	25.9	82.95
9	569.037	3.01	11.2	88.72
10	667.035	4.07	34.34	56.10

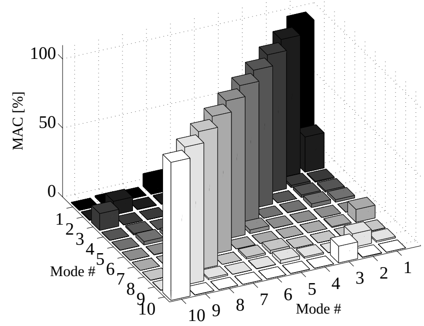

Figure 5: Plexiglass plate data: MAC_1 for the modes extracted by using FEMtools.

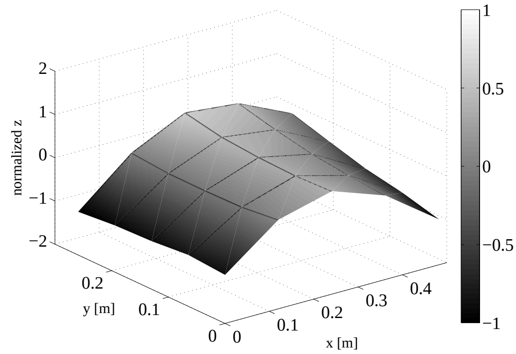

Figure 6: Plexiglass plate data: normalized mode shape at 99.660 Hz (Mode #1), estimated with the proposed approach.

maining modes, the differences between the proposed approach and FEMtools in terms of MAC_1 and MAC_1 alias, are not significant since the MAC index values are sometimes in favor of the proposed approach and other in favor of the commercial tool, but always sufficiently low. It is important to remark that the extraction of the double mode, by using FEMtools, has required iterated attempts by an expert user. Without the expertise there is the risk of confusing, during the identification procedure, the double mode with a simple mode. On the contrary, with the proposed approach, there are not differences due to the presence/absence of double modes. An additional MAC_3 index has been computed by using Eq. (19) to evaluate the correlation between the ψ_i and the corresponding ψ_j normalized mode shapes (with $i = j = 1,\ldots,10$), estimated with the proposed approach and the commercial tool, respectively. These correlation index values, computed for the first estimated 10 modes, are reported in the last column of the Table 3. The results show that the mode shapes #1, #2 and #10 have discrete differences.

To further highlight the quality of the results obtained, the spatial reconstruction of some mode shapes are reported, both for the proposed approach and the commercial solution. First off all, the mode shapes associated to the double mode are reported, since they represent the more interesting problem for the considered experimental data. In particular, Fig. 6 and 8 reports the normalized mode shapes for the double mode, obtained with the proposed approach, while Fig. 7 and 9 the corresponding mode shapes, obtained with FEMtools^{TM}.

An additional mode shape for the remaining modes is reported for completeness of the work. In particular, Fig. 10 and 11 show the normalized mode shapes corresponding to mode #6. From the MAC index values reported in previous tables it is evident that the quality of all other modes is very similar to that of the mode #6 here plotted.

Table 4: Plexiglass plate: first 10 modes extracted by using FEMtools.

Mode #	ω_n^e [Hz]	ζ^e [%]	MAC_1 alias [%]
1	99.917	4.53	28.18
2	100.130	4.80	28.18
3	226.633	3.71	12.2
4	279.301	3.70	28.2
5	290.900	3.44	6.1
6	355.853	3.40	34.3
7	417.651	3.24	35.1
8	503.163	3.25	4.5
9	568.682	2.93	10.1
10	672.990	3.17	23.3

5 CONCLUSIONS

In this paper an experimental modal analysis, based on the use of a gray-box model, has been presented. A first advantage of the proposed approach is the low number of parameters needed to be set: the user is required only to fix the number of modes to identify

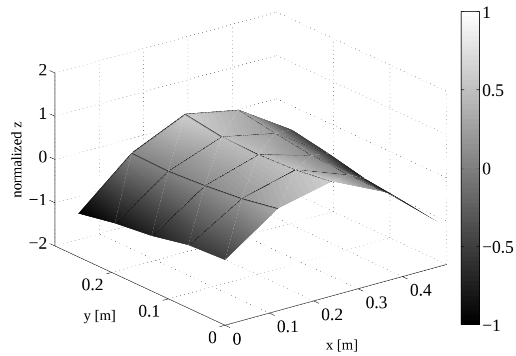

Figure 7: Plexiglass plate data: normalized mode shape at 99.917 Hz (Mode #1), estimated by using FEMtools.

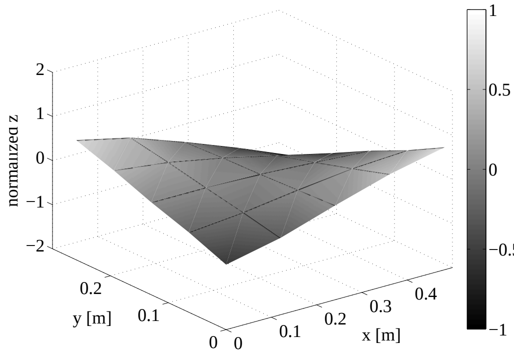

Figure 8: Plexiglass plate data: normalized mode shape at 99.717 Hz (Mode #2), estimated by using the proposed approach.

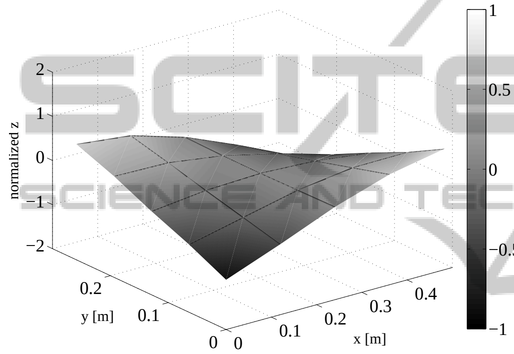

Figure 9: Plexiglass plate data: normalized mode shape at 100.130 Hz (Mode #2), estimated by using FEMtools.

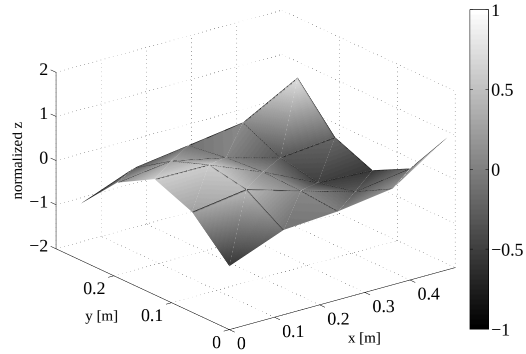

Figure 10: Plexiglass plate data: normalized mode shape at 356.267 Hz estimated by using the proposed approach.

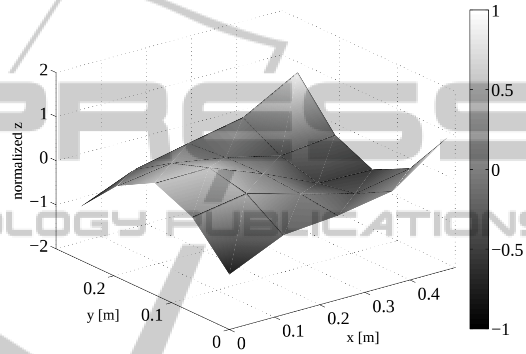

Figure 11: Plexiglass plate data: normalized mode shape at 355.853 Hz estimated by using FEMtools.

and to select the frequency range in which to perform the identification. Moreover, the proposed approach does not use a stabilization diagram to select stable poles in the selected frequency range and, as a consequence, the interaction of an expert user is not necessary, even in the presence of particular cases such as double modes. Another advantage of the proposed procedure is the use of a gray-box model whose unknown parameters have a clear physical meaning: for all the applications where a numerical–experimental correlation between a finite element model and experimental data is necessary, the use of a gray-box model can simplify the user task of tuning the numeric parameters in order to match the experimental measurements. The experimental results, obtained in a standard benchmark, are practically identical to those obtained with commercial solutions, with some advantages for double mode identification. These results allow to conclude that the proposed method is technically sound and with the same performance, of a well assessed method, but with the advantages just summarized. Future activities are planned in order to test the proposed approach to more complex flexible systems.

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