Performance Analysis of a WDMA Protocol with a Multiple Tunable Receivers Node Architecture for High-speed Optical Fiber Lans

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Keywords: Wavelength Division Multiple Access, Multiple Tunable Receivers, Receiver Collision, Rejection Probability.

Abstract: A synchronous transmission WDMA protocol for high-speed optical fiber LANs of passive star topology is studied in this paper. The packet rejection at destination, referred as receiver collision, is extensively examined. A network interface with more than one tunable receivers per destination station is considered. This means that each station is capable of receiving more than one data packets during a time frame. The presented WDMA protocol expands previous studies about the receiver collision phenomenon that assume a single tunable receiver per station, while it provides an analytical framework about its effect on the total network performance. The average throughput and rejection probability are analytically derived, while the bandwidth utilization improvement provided by the use of the multiple tunable receivers station interface is estimated. The analysis considers Poisson arrivals and finite station population. Numerical results are comparatively studied for various numbers of data channels and stations.

1 INTRODUCTION

Latest technology achievements concerning the high-speed optical fiber networks deployment have introduced a variety of communication techniques in order to exploit the total fiber bandwidth provided. Wavelength Division Multiplexing (WDM) (Zheng and Mouftah, 2004) technique has been proven as the most preferable and widely used technique to divide the inefficient high fiber data rate into multiple parallel channels of lower data rates, each corresponding to a different optical wavelength. Moreover, the WDM technique utilization in conjunction with a variety of WDM access (WDMA) strategies that have been proposed for optical networks, have -without objection- given the opportunity to increase the total throughput achieved comparatively to the single channel system of the same bandwidth.

Similar to any multi-channel network, there are two main reasons for packet loss in WDM networks. First, packets are destroyed if two or more stations transmit a packet over the same WDM channel and the transmissions are overlapped in time. This phenomenon is referred as WDM channel collisions, while it is distinguished in two main categories: control channel collisions and data channel collisions, depending on the type of packet transmissions over each channel category (either control or data packet). Second, additional packets are aborted in case of the WDM receiver collisions phenomenon (Pountourakis, 1998). Particularly, a receiver collision occurs if a data packet that has been successfully transmitted over a WDM data channel cannot be picked up by the intended destination station since its tunable receiver is currently tuned to another WDM channel to receive a packet from another source station.

In literature, the WDM channel collisions have been extensively studied by means of analytical methods or simulations in local and metropolitan area scale (Zheng and Mouftah, 2004). On the other hand, the receiver collisions are not extensively studied in the majority of the WDMA protocols due to complexity reasons. Nevertheless, some studies take under consideration the receiver conflicts and provide the performance measures estimation via either analytical or simulation models. It is worth mentioning that the receiver collisions phenomenon can be evaluated in case that the destination station is capable of receiving packets transmitted over different channels, i.e. it is equipped with at least one tunable receiver (TR) or more than one fixed receivers (FR).

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A quick research about studies for passive star Local Area Networks (LANs) can show that there have been introduced some WDMA protocols which suffer from the receiver conflicts loss, while each station uses a tunable receiver that can be tuned over all WDM channels for reception. Especially, the receiver collisions effect on both synchronous and asynchronous transmission WDMA protocol cases with Poisson aggregated traffic is analytically examined by Pountourakis (1998), where a separate control channel is introduced to exchange control information in order to coordinate the data packets successful transmissions. Also, the receiver collisions impact on a synchronous transmission WDMA protocol is analytically explored by Baziana (2014) based on Poisson aggregated traffic scenario, while the Multichannel Control Architecture (MCA) is used in order to exchange the appropriate control information over multiple parallel control channels aiming to reduce the control loss probability. The use of the MCA is also introduced by Baziana and Pountourakis (2007 and 2012), where two synchronous transmission WDMA protocols are proposed assuming the receiver collisions effect for different access strategies on the MCA. In these studies, two different analytical Markovian models are extensively adopted for the rigorous analytical performance measures estimation.

On the other hand, in case of WDM Metropolitan Area Networks (MANs) the receiver collisions are considered in a slight different way. Thus, in order to face them many WDMA protocols assume a specific network and station configuration: according to it, each station around the ring may receive packets only from a dedicated channel especially assigned to it for reception, while it is equipped with a fixed receiver (FR) that is always tuned to the dedicated reception channel (Bengi and As, 2002), (Bengi, 2004), (Bregni *et al.*, 2006), (Herzog *et al.*, 2004) and (Yang *et al.*, 2004). Although this assumption aims to face the packet loss due to the receiver collisions, it provides bandwidth under-utilization. This is because it restricts the transmission of packets destined to a specific destination station over its dedicated reception channel, although there may exist other available channels for transmission in case that it is not currently free. In order to overcome the above drawback and to efficiently exploit the available fiber bandwidth, the use of a set of tunable transmitter and receiver (TT-TR) per station is proposed by Baziana and Pountourakis (2008 and 2010), by Turuk and Kumar (2004 and 2005) and by Turuk *et al.* (2004), while all WDM channels can be used for both transmission and reception. The transceivers tunability benefits to significantly reduce the dropping probability are given by MacGregor *et al.* (2002).

The up to now investigations about the receiver collisions effect in WDM networks performance mainly consider that each station is equipped with a single receiver, fixed or tunable. Since the recent technology evolutions provide us with reliable tunable receivers whose cost gradually decreases, the utilization of more than one tunable receivers per station appears as an interesting idea in order to reduce the packet loss at destination and consequently to increase the system performance. In other words, the utilization of a multiple tunable receivers station interface aims to provide gradual reduction of packet rejection probability at destination, improving the total throughput and eliminating the system delay.

This paper introduces a synchronous transmission WDMA protocol that takes under consideration the receiver collisions effect in a single-hop, passive star LAN that interconnects a finite number of stations. The single-hop architecture ensures that the communication between the source and the destination station is realized over the same channel without any wavelength conversion. The proposed network configuration uses a separate control WDM channel for the control information exchange prior to the data communication in order to coordinate the data packets successful transmission without data channel collisions. At each station a network interface is assumed that contains a tunable transmitter and a number x of tunable receivers (TT-TR^x). In this way, each station is capable of receiving at the end of each time frame more than one (and up to x) data packets that have been successfully transmitted over the data channels and are destined to it. In this way, the proposed protocol effectively faces the WDM receiver collisions phenomenon providing essential rejection probability reduction and total performance improvement, as compared to the singe tunable receiver per station case.

The present study expands previous studies, like this of Pountourakis (1998), about the impact of receiver collisions on the total network performance. Especially, in this study we provide an analytical model based on a Poisson arrival process in order to derive in close mathematical formulas the average throughput and the average rejection probability at destination. Numerical results for diverse finite numbers of stations and WDM channels are comparatively studied, giving the total performance improvement.

The proposed WDMA protocol performance depends on a number of key factors which are taken under consideration by the network configuration and the analysis. Some of these are the number of: station population, WDM data channels, and tunable receivers.

This study is organized as follows. The network model and the assumptions are described in Section 2. In Section 3 the protocol analysis is extensively described and the performance measures are analytically derived. Comparative numerical results and comments are discussed in Section 4. Finally, Section 5 outlines the concluding remarks. The Appendix explores some mathematical formulation.

2 NETWORK MODEL AND ASSUMPTIONS

We assume the passive star network presented in Figure 1. The total fiber bandwidth is divided into N+1 parallel WDM channels, each operating in a different wavelength $\{\lambda_0, \lambda_1, ... \lambda_N\}$. The channel λ_0 is called control channel and it transmits the control packets, while the remaining channels $\{\lambda_1, \lambda_2, ... \lambda_N\}$ are called data channels and they transmit the data packets. The passive star coupler interconnects a finite number M of stations (M > N). Each station network interface is equipped with a tunable transmitter and a set of x ($1 \leq x \leq N$) tunable receivers that can be tuned to all channels $\{\lambda_0, \lambda_1, ... \lambda_N\}$, as Figure 1 shows.

The control packet transmission time is defined as time unit reference and is called control slot or mini-slot. Thus, the data packet transmission time normalized in time units is L and is called data slot (L > 1). The control packet consists of the source address, the destination address and the data channel λ_k that belongs to the set of $\{\lambda_1, \lambda_2, ... \lambda_N\}$ and has been chosen for the corresponding data packet transmission. Both control and data channels use the same time reference which we call frame. We define as frame the time interval that includes N time units for the control packets transmissions plus the normalized data packet transmission time L, as Figure 2 depicts. Thus, the frame time duration F_d is:

$$F_d = N + L \text{ time units} \quad (1)$$

We assume a common clock to all stations. Time axis is divided into contiguous frames of equal length and stations are synchronized for transmission over the control and data channels

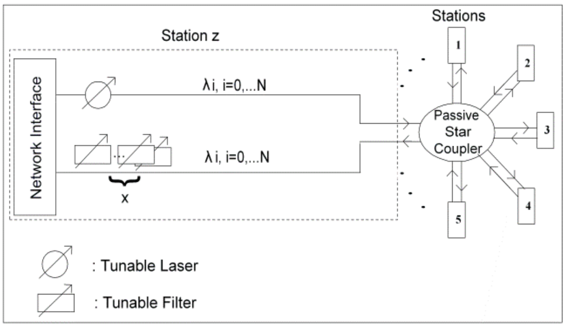

Figure 1: Network model.

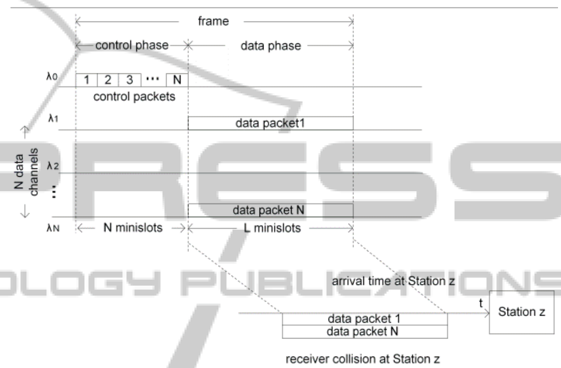

Figure 2: Frame duration. At the bottom: a receiver collision case.

during a frame. Each frame consists of the control and the data phase, as Figure 2 shows. The control phase consists of N time units, while the control packets transmissions occur. The data phase that follows lasts for L time units, while the data packets transmissions take place. At the beginning of a frame data phase, each station is able to transmit at a given wavelength λ_T and simultaneously receive from a set of wavelengths $\{\lambda_{R1}, \lambda_{R2,...} \lambda_{Rx}\}$. Finally, in our analysis we assume that the tunable transceivers have negligible tuning time and very large tuning range.

We assume that each station is equipped with a buffer with capacity of one data packet. If the buffer is empty the station is said to be free, otherwise it is backlogged. Packets are collectively generated in a Poisson stream. If a station is backlogged and generates a new packet, the packet is lost. Finally, the aggregated traffic from new generated and retransmitted packets obeys Poisson statistics.

The successfully transmitted data packets are uniformly distributed among the M stations, each randomly selected with equal probability (for the sake of generality we suppose that a station may send to and receive from itself). Thus, if more than x successfully transmitted over different data channels packets are destined to the same destination, the destination is able to receive only x packets of them with its tunable receivers and rejects all the others. This phenomenon is called receiver collision.

Especially at the beginning of each frame, each station tunes one of its tunable receivers to the control channel λ_0 in order to monitor the control packets transmissions from all stations during the control phase. Also at the beginning of each frame, if it has to send a data packet to another, first it tunes its tunable transmitter to the control channel λ_0. Then, it chooses randomly one of the data channels over which the data packet will be transmitted, let's say data channel i. Then, it informs the other stations about the i-th data channel selection, by transmitting a control packet during the i-th control mini-slot of the control phase with its tunable transmitter. The control packets from all stations compete according to the Slotted Aloha scheme to gain access over the N control mini-slots. Since the station continuously monitors the control channel with its tunable receiver during the control phase, by the end of this time period it is informed about the outcome of its control packet transmission. This means that, grace to the broadcast nature of the control channel, the station is aware of the data channel claims for transmission of all stations. Especially if one or more other stations have selected the same i-th data channel for transmission, the corresponding control packets have collided during the i-th control mini-slot and are all aborted, while all involved stations become backlogged. In the contrary if the station control packet has been successfully transmitted over the i-th control mini-slot, the station gains access to the i-th data channel for successful transmission during the frame data phase. This fact does not mean that the corresponding data packet will be correctly received by the destination. This fact depends on the number of the other data packets that are successfully transmitted over other data channels during the data phase and have the same destination. In this case, the destination station may receive up to x data packets with its tunable receivers, while the corresponding source stations become free. It is evident that the destination station aborts all the others packets destined to it due to the receiver collisions phenomenon, while the relative stations become backlogged. We may consider several arbitration rules for the selection of the data packets that are finally correctly received by the destination while the others are aborted, such as priority etc.

At the end of the control phase, the station is informed about the data packets that will be successfully transmitted over the N data channels and are destined to it. Based on this information and the above arbitration rules, the station decides which of these data packets it is going to receive, let's say z ($z \leq x$) of them. Thus at the beginning of the frame data phase, it tunes z of its tunable receivers to the corresponding data channels while the data packets reception immediately starts.

3 ANALYSIS

We denote as G the offered load, i.e. the average number of transmitted control packets per time unit on the control channel. According to Sudhakar *et al.* (1991), the probability P_{suc} of a successful data packet transmission over the data channel j (j=1,2,…, N) during a frame is given by:

$$P_{suc} = Ge^{-G} \tag{2}$$

Let S_N be a random variable representing the number of successful data packet transmissions over the N data channels during a frame, $0 \leq S_N \leq N$.

The probability $\Pr[S_N = s]$ of finding s successfully transmitted data packets over the N data channels during a frame conforms to the binomial probability law and is given by Pountourakis (1998):

$$\Pr[S_N = s] = \binom{N}{s} P_{suc}{}^{s} (1 - P_{suc})^{N-s} \tag{3}$$

Also, let $A_N(s)$ be the number of the correctly received data packets at destination given that s successful transmissions over the N data channels occurred during a frame, $1 \leq A_N(s) \leq S_N$ for $S_N > 0$.

The probability $\Pr[A_N(s) = r]$ of finding r correctly received data packets at destination from s successful transmissions over the N data channels during a frame is given by (see the Appendix):

$$\Pr[A_N(s) = r] = \sum_{\text{all sets}} \frac{M!s!}{M^s \prod_{i=0}^{s} k_i! \prod_{z=1}^{s} (z!)^{k_z}} \tag{4}$$

where: the sets of integers $\{k_0, k_1, k_2, ..., k_M\}$ $i, k_i \in \{0,1,2,..., M\}$ are defined in the Appendix.

Thus, the probability $S_{rc}(r)$ of finding r correctly received data packets at destination during a frame in steady state is given by:

$$S_{rc}(r) = \sum_{s=r}^{\min(M,N)} \Pr[S_N = s]\Pr[A_N(s) = r] \quad (5)$$

It is obvious that:

$$\min(s, x) \le r \le \min(s, Mx) \quad (6)$$

We define the throughput S_{rc} as the average number of correctly received data packets at destination during a frame in steady state. Thus:

$$S_{rc} = \sum_{r=1}^{\min(M,N)} r S_{rc}(r) \quad (7)$$

Also, we define the average rejection probability at destination P_{rej} in steady state, as the ratio of the average number of data packets rejected at destination due to the receiver collisions phenomenon to the average number of successfully transmitted data packets over the N data channels, during a frame. Thus, P_{rej} is given by:

$$P_{rej} = \frac{S - S_{rc}}{S} \quad (8)$$

where: S is the average number of successfully transmitted data packets over the N data channels, during a frame. In other words, S represents the average throughput per frame without the receiver collisions effect and it is given by Sudhakar *et al.* (1991):

$$S = NP_{suc} \quad (9)$$

while its maximum value S_{max} is provided for offered load G_{max}=1 and is given by Pountourakis (1998):

$$S_{max} = \frac{N}{e} \quad (10)$$

Finally, we define the normalized system throughput during a frame S_{nor} as:

$$S_{nor} = \frac{L}{F_d} S_{rc} \quad (11)$$

4 PERFORMANCE EVALUATION

In this Section, we present the numerical solution of the proposed protocol, for various numbers of stations M, data channels N, and tunable receivers per station x. In the following figures, we consider that the data packet length is L=100 time units.

Figure 3 illustrates the normalized throughput S_{nor} versus the offered load G for M=50 stations, N=3,8,10,13 data channels for x=2 tunable receivers per station. The curves provided are compared with the case of a single tunable receiver per station. Let's study in Figure 3 the S_{nor} value for N=13 data channels. As it is observed the network configuration with x=2 tunable receivers per station, as it is compared to the single tunable receiver case, provides higher S_{nor} value for a wide range of offered load conditions. Especially for low offered load (lower than G=0.2 control packets/control slot), the S_{nor} values provided in cases of x=1,2 are almost equal. This is because, in low offered load conditions the number of transmitted control packets over the control slots is low, while the consequent control channel collisions are not few. In this case, the number of successfully transmitted data packets over the N data channels is also low, introducing low number of rejection events at destination. In other words, for low offered load conditions the impact of the receiver collisions phenomenon is not significant, providing almost equal values of throughput. On the contrary, as the offered load increases up to almost G=2 control packets/control slot for values around G_{max}=1 control packets/control slot, the S_{nor} in case of the x=2 is essentially higher than in the x=1 case, while the maximum improvement is reached for G= G_{max}. This behavior is explained by the fact that for offered load around the G_{max} value, the system reaches maximum number of successfully transmitted data packets over the N data channels. Thus, under these offered load conditions the number of data packets that are distributed to the M destination stations is maximum, providing higher number of rejection events at destination. Thus, the utilization of x=2 instead of x=1 tunable receivers per station provides maximum throughput improvement, as it is observed in Figure 3. For high offered load conditions (higher than G=2 control packets/control slot), the throughput values for x=1,2 are almost equal. This is because, for this offered load the number of control channel collisions are getting higher, while the probability of a successful data packet transmission over the N data channels is getting lower. This is the reason why the impact of the receiver collisions phenomenon on the system throughput decreases too, while the use of higher number of tunable receivers per station (from x=1 to x=2) does not seem to improve the throughput achieved.

Also, the above behavior is noticed for N=10 and N=8. Thus, in Figure 3 it is shown that the S_{nor} maximum improvement provided by the utilization of x=2 instead of x=1 tunable receivers per station occurs for $G=G_{max}$, while it is analogous to the value of N. This means that as the number N of data channels decreases, the S_{nor} for $G=G_{max}$ decreases too. This can be understood since, as the N value decreases the number of successfully transmitted data packets over the N data channels decreases too, providing lower rejections at destination due to the receiver collisions. Consequently, the exploitation of more tunable receivers per station is not able to provide higher throughput values, as the number N decreases. This behavior can be representatively noticed when N=3, where the probability of a control channel collision is extremely high providing almost zero probability of a receiver collision. This is the reason why, the S_{nor} values for x=1, 2 tunable receivers per station are equal. The above remarks are validated by studying the S_{nor} improvement when increasing the number of tunable receivers per station from x=1 to x=2. For example for G=1.6 control packets/control slot, the S_{nor} increases about: 0.65% for N=3, 2.25% for N=8, 2.89% for N=10, and 3.84% for N=13.

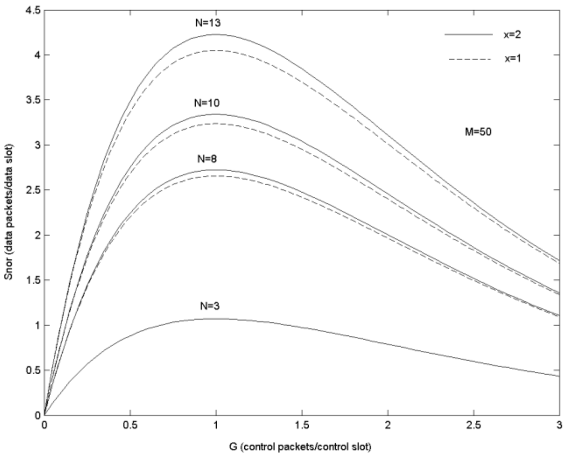

Figure 3: S_{nor} versus G, for M=50 stations, N=3,8,10,13 data channels and x=1,2 tunable receivers per station.

In Figure 4 the average rejection probability P_{rej} versus the offered load G is shown, for M=50 stations, N=8,10,13 data channels for x=2 tunable receivers per station, while the curves are compared with the case of a single tunable receiver per station. The previous results are validated in Figure 4. Particularly, it is illustrated that the increase of the number of tunable receivers per station from x=1 to x=2 provides significant P_{rej} reduction that reaches almost 100% in a wide range of offered load values, while it obtains its maximum value when $G=G_{max}$. Also, it is remarkable that the P_{rej} reduction is a decreasing function of N. This is understood since as N increases for a given number of stations, the probability of a destination conflict increases too, as previously described. As a direct result, the utilization of more tunable receivers per station provides lower rejection probability. For example, for G=1 control packets/control slot, the P_{rej} reduction when increasing the number of tunable receivers per station from x=1 to x=2, is: 98.5% for N=8, 98% for N=10, and 97.3% for N=13.

The proposed protocol performance is studying in Figure 5, when the station population varies. Especially, Figure 5 depicts the average rejection probability P_{rej} versus the offered load G for N=13 data channels, M=50,100,150 stations for x=2 tunable receivers per station, while the curves are compared with the case of a single tunable receiver per station. As in Figure 4, the utilization of x=2 instead of x=1 tunable receivers per station provides essential P_{rej} reduction that becomes maximum when $G=G_{max}$, while it is almost 100% in the whole offered load range. As Figure 5 illustrates, P_{rej} reduction is an increasing function of M for finite number of N. This is because, as M increases the offered load to the control channel is getting higher. This means that the probability of control channel collisions increases, while consequently the number of successfully transmitted packets that are distributed to the destination stations is getting lower. This is the reason why, the P_{rej} reduction provided by the high number of tunable receivers utilization increases as the station population increases. For example, for G=1 control packets/control slot, the P_{rej} reduction when increasing the number of tunable receivers per station from x=1 to x=2, is: 97.3% for M=50, 98.4% for M=100, and 99.1% for M=150.

It is obvious that in each network implementation, the determination of the number of tunable receivers per station has to take under consideration the desired performance level achieved (in terms of P_{rej}) in conjunction with the implementation cost. Figure 6 and 7 illustrate the rejection probability maximum value $P_{rej\text{-}max}$ for various numbers of stations M and data channels N, in the cases of number of tunable receivers per station x=2, 3. As expected, the increase of x from 2 to 3 provides significant performance improvement. For example for N=13, the $P_{rej\text{-}max}$ reduction when increasing from x=2 to x=3 is: 98.2% for M=50, 98.9% for M=100, and 99.4% for M=150. In other words, the $P_{rej\text{-}max}$ reduction is an increasing function

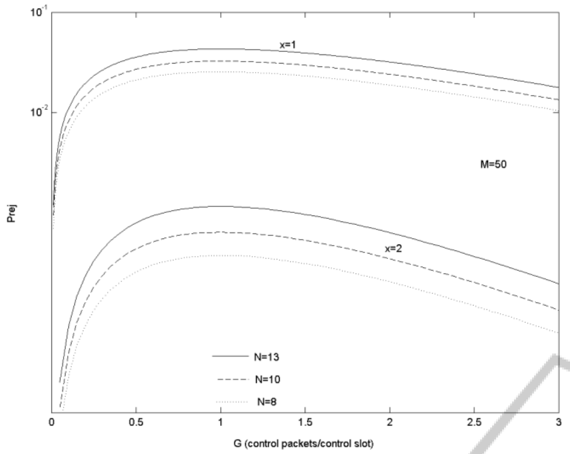

Figure 4: P_{rej} versus G, for N=3,8,10,13 data channels, M=50 stations and x=1,2 tunable receivers per station.

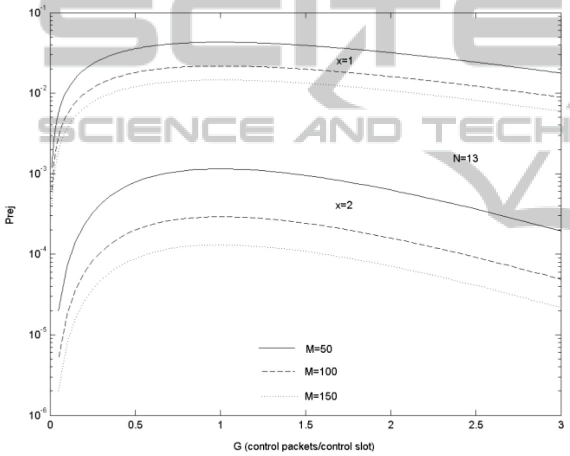

Figure 5: P_{rej} versus G, for M=50,100,150 stations, N=13 data channels and x=1,2 tunable receivers per station.

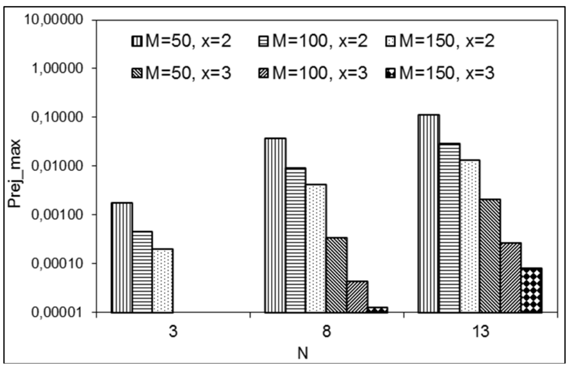

Figure 6: $P_{rej-max}$, for N=3,8,13 data channels, M=50,100,150 stations, and x=1,2 tunable receivers per station.

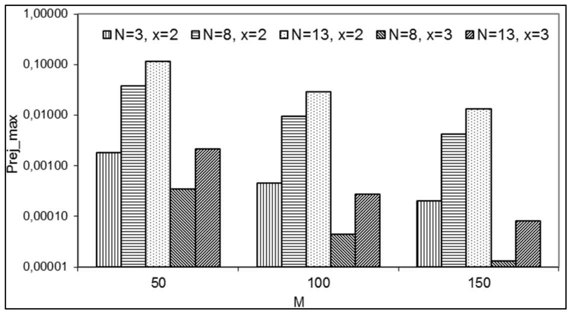

Figure 7: $P_{rej-max}$, for M=50,100,150 stations, N=3,8,13 data channels and x=1,2 tunable receivers per station.

of M. This is because as M increases for fixed N, the probability of a receiver collision decreases, fact that becomes noticeable with the concurrent increase of x. Similar, for M=50, the $P_{rej-max}$ reduction when increasing from x=2 to x=3 is: 99.1% for N=8, 98.5% for N=10, and 98.2% for N=13. This behavior is an immediate result of the above discussion. It is obvious that P_{rej}=0 when N=3 and x=3, since there is no receiver collisions probability.

5 CONCLUSIONS

This paper proposes a synchronous transmission WDMA protocol that examines the effect of receiver collisions in high-speed optical fiber LANs. As the cost of the optical tunable receivers gradually decreases, we exploit the idea to introduce at each station a network interface that consists of a number of tunable receivers. The utilization of more than one tunable receivers per station improves the network performance, since it provides essential rejection probability reduction at destination.

Also in this study, we provide an analytical framework for the performance measures evaluation, based on Poisson statistics. Thus, we derive analytical formulas for the estimation of both the system throughput and the rejection probability, considering a finite number of tunable receivers per station. The proposed protocol is general and expands previous studies that consider a single tunable receiver per station. Numerical results for various numbers of stations, WDM data channels, and tunable receivers per station depict that the increase of the number of tunable receivers about one significantly improves the total system performance and reduces almost 100% the probability of conflicts at destination. This result offers additional insights in WDM high-speed LANs.

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APPENDIX

We assume the model that consists of N data channels and M stations. We aim to analytically describe the distribution of the successfully transmitted data packets over the N data channels to the M stations. This model corresponds to the occupancy problem of the distribution of indistinguishable balls (data packets) to cells (destination stations), supposing that the arrangements should have equal probabilities. We consider indistinguishable packets transmitted to indistinguishable destination stations using Maxwell-Boltzman statistics (Feller, 1968).

We are interested in the probability $\Pr[A_N(s) = r]$ of r correctly received data packets at destination when s data packets have been successfully transmitted over the N-channel system, during a time frame, $1 \leq s$.

Let's suppose that each station may transmit to any of the M stations (for the sake of generality we suppose that a station may send to and receive from itself). According to the Maxwell-Boltzman statistics, there are M^s possible arrangements of the s successfully transmitted data packets to the M destination stations, each with equal and constant probability: $1/M^s$.

We consider that the distribution of s data packets to M stations provides the following result by the end of a frame:

- there are k_0 of M destination stations, $k_0 \in \{0,1,2,...,M\}$: for each of them there is no successfully transmitted data packet destined to it,
- there are k_1 of M destination stations, $k_1 \in \{0,1,2,...,s\}$: for each of them there is 1 successfully transmitted data packet destined to it, and so on. In general, there are k_i of M

destination stations, $i, k_i \in \{1,2,...,s\}$: for each of them there are i successfully transmitted data packets destined to it.

It is obvious that:

$$\sum_{i=0}^{s} k_i = M \tag{12}$$

and:

$$\sum_{i=0}^{s} ik_i = s \tag{13}$$

Since each destination station is capable of receiving up to x data packets per frame, it is:

$$\sum_{i=0}^{x-1} ik_i + \sum_{i=x, k_i \neq 0}^{s} i\ x = r \tag{14}$$

For each set of integers $\{k_0, k_1, k_2, ..., k_M\}$ that satisfy (12), (13), and (14) and it is $k_0, k_1, ... k_M \in \{0,1,2,...,M\}$, the probability P_{ki} that: no data packet is destined to k_0 stations, one data packet is destined to k_1 stations, and so on; and generally, i data packets are destined to k_i stations is given by Wentzel and Ovcharov (1986):

$$P_{ki} = \frac{M!s!}{M^s \prod_{i=0}^{s} k_i! \prod_{z=1}^{s} (z!)^{k_z}} \tag{15}$$

Thus, the probability $\Pr[A_N(s) = r]$ is defined as the sum of the probabilities P_{ki}, for all possible sets of integers $\{k_0, k_1, k_2, ..., k_M\}$ that satisfy (12), (13), and (14) and it is $k_0, k_1, ... k_M \in \{0,1,2,...,M\}$, i.e.:

$$\Pr[A_N(s) = r] = \sum_{\text{all sets}} \frac{M!s!}{M^s \prod_{i=0}^{s} k_i! \prod_{z=1}^{s} (z!)^{k_z}} \tag{16}$$