

HVDC Line Parameters Estimation based on Line Transfer Functions Frequency Analysis

Jocelyn Sabatier, Toni Youssef and Mathieu Pellet

IMS Laboratory, UMR 5218 CNRS, Bordeaux University, 351 cours de la Libération, 33405 Talence Cedex, France

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Abstract: This paper proposes a method to estimate HVDC line parameters. After a reminder on the transfer functions that characterise the dynamic behaviour of a DC line, link between these transfer functions resonance frequencies and the line parameters is established. This link is then used to estimate the line parameters, the resonance frequencies being determined using the power spectral density of voltage signals at the input and output of the line. A numerical example highlights the efficiency of the proposed method.

1 INTRODUCTION

High-voltage direct current (HVDC) technology has become a credible alternative for transmitting power over long distances through submarine or underground cable crossings (Hammons et al, 2000). Indeed, the improvement of power electronics devices has opened new perspectives for transmission of electrical power through HVDC links, which offer extra means to control power flows in interconnected power systems or between non synchronous areas.

HVDC links offers numerous environmental benefits, including “invisible” power lines, neutral electromagnetic fields and compact converter stations. The power HVDC transmission line is one of the major components of an HVDC electric power system. Its major function is to transport electric energy, with minimal losses, from the power sources to the load centres, usually separated by long distances. Losses are only 3% per 1000 km at a standard cost (losses can be further reduced to 0.3% for 1000 km, but at a higher cost). Possible applications include:

- connecting wind farms to power grids,
- underground power links,
- providing shore power supplies to islands and offshore oil & gas platforms,
- connecting asynchronous grids.

To ensure proper operation of an HVDC grid, a control system must be implement with main objectives (Bahrman and Johnson, 2007):

- control basic system quantities such as DC line current, DC voltage, and transmitted power accurately and with sufficient speed of response
- control higher-level quantities such as frequency in isolated mode or provide power oscillation damping to help stabilize the AC network
- ensure stable operation with reliable commutation in the presence of system disturbances
- ensure proper operation with fast and stable recoveries during AC system faults and disturbances
- diagnose of the line integrity.

To reach these objectives, the controllers must have an accurate knowledge of the HVDC line that connect the grid node. This requires the implementation of line parameters estimation methods. Most of HVDC estimation methods found in literature are purely numerical or signal based methods that do not take into account the physical particularities of long transmission lines (Zhou et al 2006) (Eriksson and Söder 2010), (Chetty and Ijumba, 2011) (Cole, 2010) (Chakradhar and Ramu, 2007) (Xu and Fan, 2012) (Indulkar and Ramalingam, 2008) (Yuan, 2009) (Wilson et al, 1999) (Grobler and Naidoo 2006). The goal of this paper is to propose a different approach. After a reminder on the transfer functions that characterise the dynamic behaviour of a DC line, a relationship between these transfer functions resonance frequencies and the line parameters is established. This relationship is then used to estimate the line parameters, the resonance frequencies being determined using the power spectral density of

voltage signals at the input and output of the line. A numerical example highlights the efficiency of the proposed method.

2 HVDC LINE DYNAMICAL MODEL AND RESONANCE FREQUENCIES

2.1 Analytical Model of HVDC Lines

A HVDC transmission line can be characterized by the following parameters (Chakradhar and Ramu 2007):

- line length: L_{line}
- line series resistance r and inductance l
- line shunt capacitance c and conductance g

The series resistance relies basically on the physical composition of the conductor at a given temperature. The series inductance and shunt capacitance are produced by magnetic and electric fields around the conductors, and depend on their geometrical arrangement. The shunt conductance is due to leakage currents flowing across insulators and air. These parameters values determine the power-carrying capacity of the transmission line and the voltage drop across it at full load.

As shown by figure 1, the HVDC line dynamical behavior can be modelled by a quadropole $F(s)$, that links:

- V_e : line input voltage,
- I_e : line input current,
- V_s : line output voltage,
- I_s : line output current.

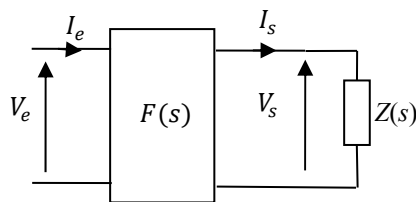


Figure 1: HVDC line model and connected load.

The HVDC line is connected to a load of impedance $Z(s)$ constituted of a resistance R_{ch} in parallel with a capacitor C_{ch} , and thus:

$$Z(s) = \frac{R_{ch}}{1 + R_{ch}C_{ch}s} \quad (1)$$

Solution of the Telegraphist's equation permits to show that the currents and voltages at the HVDC line terminals are linked by the relation

$$\begin{bmatrix} V_s(s) \\ I_s(s) \end{bmatrix} = F(s) \begin{bmatrix} V_e(s) \\ I_e(s) \end{bmatrix} \quad (2)$$

With

$$F(s) = \begin{bmatrix} F_{11}(s) & F_{12}(s) \\ F_{21}(s) & F_{22}(s) \end{bmatrix} = \begin{bmatrix} \cosh(\gamma(s)L_{line}) & -Z_c(s)\sinh(\gamma(s)L_{line}) \\ -\frac{\sinh(\gamma(s)L_{line})}{Z_c(s)} & \cosh(\gamma(s)L_{line}) \end{bmatrix} \quad (3)$$

and

$$\begin{aligned} \gamma(s) &= \sqrt{(g+cs)(r+ls)} \\ Z_c(s) &= \sqrt{(r+ls)/(g+cs)} \end{aligned} \quad (4)$$

If a load of impedance $Z(s)$ is connected to the line, then the following transfer functions can be computed (among others):

$$\begin{aligned} \frac{V_s(s)}{V_e(s)} &= \frac{\lambda(s)}{\lambda(s)\cosh(\gamma(s)L_{line}) + \sinh(\gamma(s)L_{line})} \\ \frac{I_s(s)}{V_e(s)} &= \frac{1}{Z_c(s)} \frac{1}{\lambda(s)\cosh(\gamma(s)L_{line}) + \sinh(\gamma(s)L_{line})} \end{aligned} \quad (5)$$

with

$$\lambda(s) = \frac{Z(s)}{Z_c(s)} = \frac{R_{ch}}{1 + R_{ch}C_{ch}s} \sqrt{\frac{g+cs}{r+ls}} \quad (6)$$

The gain of transfer functions $F_{21}(s)$ and $I_s(s)/V_e(s)$ are plotted in figure 2, using the following realistic numerical values for a HVDC line proposed in (Teppoz, 2005):

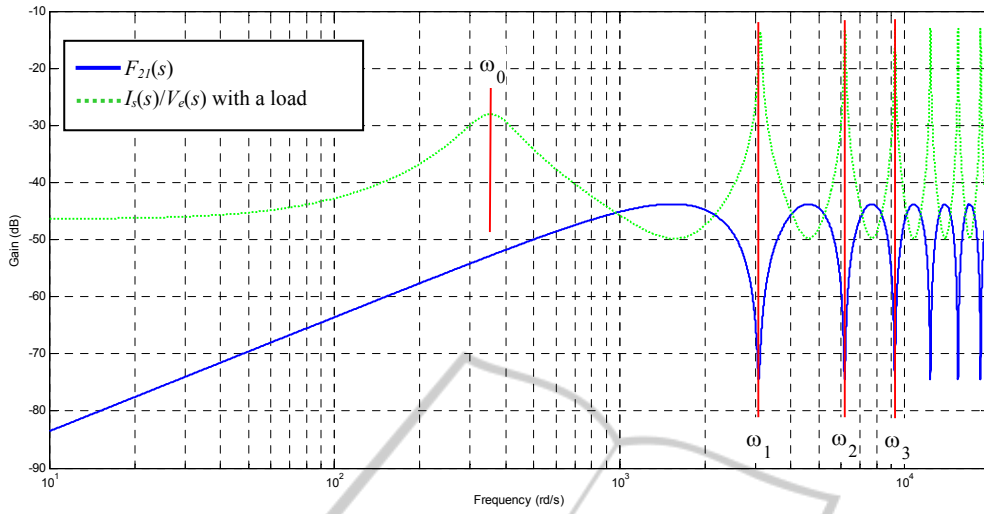
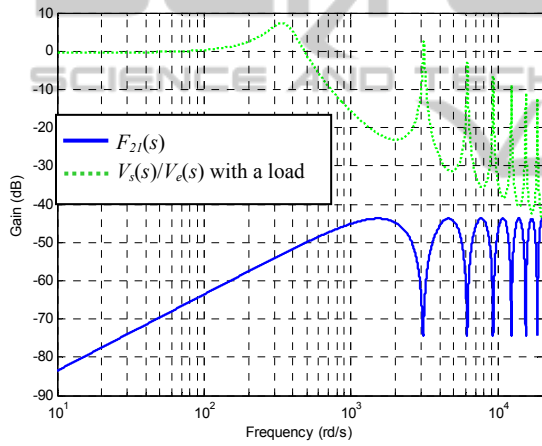
$$\begin{aligned} L_{line} &= 300 \text{ (km)}, & r &= 3e^{-2} \text{ (}\Omega \cdot \text{km}^{-1}\text{)}, \\ l &= 1.05e^{-3} \text{ (H} \cdot \text{km}^{-1}\text{)}, & c &= 11e^{-9} \text{ (F} \cdot \text{km}^{-1}\text{)}, \\ g &= 6.5e^{-9} \text{ (}\Omega^{-1} \cdot \text{km}^{-1}\text{)}, & R_{ch} &= 200 \text{ (}\Omega\text{)}, & C_{ch} &= 50e^{-6} \text{ (F)} \end{aligned}$$

Figure 2, but also Figure 3, permit to highlight a property: the zeros of $F_{21}(s)$ correspond to the resonance frequencies ω_k , $k \in \mathbb{N}^*$, of transfer functions $I_s(s)/V_e(s)$ and $V_s(s)/V_e(s)$. The first resonance ω_0 results in the load connected to the line. The separation between ω_0 and ω_k , $k \in \mathbb{N}^*$ tie in a well-built HVDC line since the bandwidth of the line must be higher than the one of the load. This property is now used to deduce a link between the ω_k , $k \in \mathbb{N}$.

2.2 Resonance Frequencies Link

The zero of transfer function $F_{21}(s)$ (or frequencies ω_k , $k \in \mathbb{N}^*$), are the values of $s = j\omega$ such that

$$\sinh(\gamma(s)L_{line}) = 0 \quad (7)$$


 Figure 2: Comparison of $F_{21}(s)$ and $I_s(s)/V_e(s)$ (with a load) gains.

 Figure 3: Comparison of $F_{21}(s)$ and $V_s(s)/V_e(s)$ gains.

or equivalently, the values of $s = j\omega$ such that

$$\begin{cases} \Re(\gamma(s)L_{line}) = 0 \\ \Im(\gamma(s)L_{line}) = k\pi \quad k \in \mathbb{Z} \end{cases} \quad (8)$$

Using

$$\gamma(s)L_{line} = \alpha + j\beta \quad (9)$$

and relation (4), then the following equality holds:

$$(g + cj\omega)(r + lj\omega)L_{line}^2 = \alpha^2 - \beta^2 + 2j\alpha\beta \quad (10)$$

or

$$\begin{cases} (gr - lc\omega^2)L_{line}^2 = \alpha^2 - \beta^2 \\ \omega(cr + gl)L_{line}^2 = 2\alpha\beta \end{cases} \quad (11)$$

thus leading to

$$\beta = \frac{\omega(cr + gl)L_{line}^2}{2\alpha} \quad (12)$$

and

$$4\alpha^4 - 4\alpha^2(gr - lc\omega^2)L_{line}^2 - \omega^2(cr + gl)^2L_{line}^2 = 0 \quad (13)$$

Solutions of equation (13) are:

$$\alpha = \pm \sqrt{\frac{(gr - lc\omega^2)L_{line}^2 \pm L_{line} \sqrt{(gr - lc\omega^2)^2 L_{line}^2 + \omega^2(cr + gl)^2}}{2}} \quad (14)$$

The first equation of relation (8) is met if

$$\begin{aligned} (gr - lc\omega^2)^2 L_{line}^2 \\ = (gr - lc\omega^2)^2 L_{line}^2 + \omega^2(cr + gl)^2 \end{aligned} \quad (15)$$

It can be deduced that $\omega = 0$ is a solution of relation (15). For the second equation of (8) with the condition $\alpha=0$, relation (10) can be rewritten as:

$$\beta^2 = (-gr + lc\omega^2)L_{line}^2, \quad (16)$$

and thus

$$\beta = L_{line} \sqrt{-gr + lc\omega^2}. \quad (17)$$

The frequencies ω_k that met the second equation of (8) are defined by

$$L_{line} \sqrt{-gr + lc\omega^2} = k\pi, \quad (18)$$

and thus

$$\omega_k = \sqrt{\frac{1}{lc} \left(\frac{\pi^2}{L_{line}^2} k^2 + gr \right)} \quad \text{with } k \in \mathbb{N}^+. \quad (19)$$

2.3 First Resonance Frequencies

To evaluate the first resonance frequency ω_0 , a one cell π -model of the line is considered. This model is represented by figure 4 with

$$\begin{aligned} R &= rL_{line} & G &= gL_{line} \\ C &= cL_{line} & L &= lL_{line} \end{aligned} \quad (20)$$

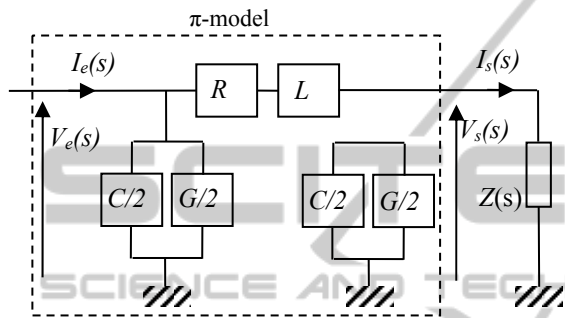


Figure 4: One cell π -model connected to a load $Z(s)$.

Using Kirchoff's laws, it can be demonstrated that:

$$\frac{V_e(s)}{I_s(s)} = \left(Z(s) + (R + Ls) \left(1 + \frac{G}{2} Z(s) + \frac{C}{2} s Z(s) \right) \right) \quad (21)$$

After simplifications, the transfer function $I_s(s)/V_e(s)$ is given by

$$\frac{I_s(s)}{U_e(s)} = C_0 \frac{1 + \frac{s}{\omega_1}}{1 + 2 \frac{\xi}{\omega_0} s + \left(\frac{s}{\omega_0} \right)^2} \quad (22)$$

with

$$\omega_0 = \sqrt{\frac{2R_{ch} + 2R + RGR_{ch}}{R_{ch}L(2C_{ch} + C)}}. \quad (23)$$

The first frequency resonance in figure 2 or figure 3 can thus be approximated by relation (23).

2.4 Steady State Analysis

To study the line steady state behavior, the π -model of figure 4 is considered again.

In steady state, the inductance L acts as a thread and the capacitor behaves like an open circuit. The linear resistance is of the order of $10^{-2} \Omega \cdot \text{km}^{-1}$ and the linear conductance is of the order of $10^{-9} \text{ S} \cdot \text{km}^{-1}$.

The product RG is thus very small and the following simplification can be done:

$$R \approx \frac{V_e - V_s}{I_e} \quad G \approx \frac{I_e - I_s}{U_e} \quad (24)$$

The lineic resistance and conductance of the line can be deduced using relations (20) using steady state (of low pass filtered) measures of line input and output currents and voltages.

3 LINE PARAMETERS ESTIMATION

The goal of this part is to show how to deduce line parameters r, l, c, g from the knowledge of frequencies $\omega_k, k \in \mathbb{N}$, resulting in the measures of voltages and currents at the line terminals.

With frequencies ω_k and ω_{k+1} given by relation (19), the following equations can be obtained:

$$lc\omega_{k+1}^2 - gr = \frac{\pi^2}{L_{line}^2} (1+k)^2 \quad (25)$$

and

$$lc\omega_k^2 - gr = \frac{\pi^2}{L_{line}^2} k^2. \quad (26)$$

Thus, using

$$K_k = \left(k \frac{\pi}{L_{line}} \right)^2 \quad (27)$$

the following products are obtained using relation (26) with $k=1$ and $k=2$:

$$gr = \frac{\omega_1^2 K_2 - \omega_2^2 K_1}{\omega_2^2 - \omega_1^2} \quad \text{and} \quad lc = \frac{K_2 - K_1}{\omega_2^2 - \omega_1^2}. \quad (28)$$

The line's length being considered known, if the second and the third resonance frequencies (ω_1 and ω_2) are known, the products lc and gr can be deduced from equation (28).

Let's consider the relation (23) which can be rewritten as

$$\omega_0^2 = \frac{2R_{ch} + 2rL_{line} + rgL_{line}^2 R_{ch}}{R_{ch}lL_{line}(2C_{ch} + cL_{line})} \quad (29)$$

and thus using relation (28) for the product lc :

$$\begin{aligned} 2\omega_0^2 R_{ch} C_{ch} l L_{line} + \omega_0^2 R_{ch} L_{line}^2 \frac{K_2 - K_1}{\omega_2^2 - \omega_1^2} \\ = 2R_{ch} + 2rL_{line} + rgL_{line}^2 R_{ch} \end{aligned} \quad (30)$$

The value of l is thus given by

$$l = \frac{2R_{ch} + 2rL_{line} + rgL_{line}^2 R_{ch} - \omega_0^2 R_{ch} L_{line}^2 \frac{K_2 - K_1}{\omega_2^2 - \omega_1^2}}{2\omega_0^2 R_{ch} C_{ch} L_{line}} \quad (31)$$

Now, from relation (28), the value of c is given by

$$c = \frac{1}{l} \frac{K_2 - K_1}{\omega_2^2 - \omega_1^2} \quad (32)$$

To summarize, the line length L_{line} being considered known, parameter r can be deduced from equations (24) and (20). Then from the first three resonance frequencies (ω_0 , ω_1 and ω_2), parameters g , c and l can be deduced from equations (28), (31) and (32).

4 APPLICATION USING POWER SPECTRAL DENSITY

Power Spectral Density (PSD) is now used to evaluate the first three resonance frequencies of the transfer function $V_s(s)/V_e(s)$. A 100 km length HVDC line is considered. This line is characterized by the parameters given in (Teppoz, 2005) and gathered in table 1.

A voltage pulse input is applied to the line (similar to a line disturbance). The resulting voltage output is represented by figure 5. Noise has been voluntarily added to the signals.

The PSD of the output voltage is then estimated and represented on figure 6. Figure 6 exhibits three spikes which correspond to the first three resonance frequencies of the Bode diagram of $V_s(s)/V_e(s)$ transfer function. The aim is now to obtain the value of the three frequencies. For that, the variance of the spectrum is calculated using a sliding window of 8 samples. A variance signal of the same length of the spectrum signal is obtained and plotted on figure 7.

Table 1: Considered HVDC line parameters.

Parameter	Value
L_{line}	100 (km)
r	$3e^{-2}$ ($\Omega \cdot \text{km}^{-1}$)
l	$1.05e^{-3}$ ($\text{H} \cdot \text{km}^{-1}$)
c	$11e^{-9}$ ($\text{F} \cdot \text{km}^{-1}$)
g	$6.5e^{-9}$ ($\Omega^{-1} \cdot \text{km}^{-1}$)
R_{ch}	200 (Ω)
C_{ch}	$50e^{-6}$ (F)

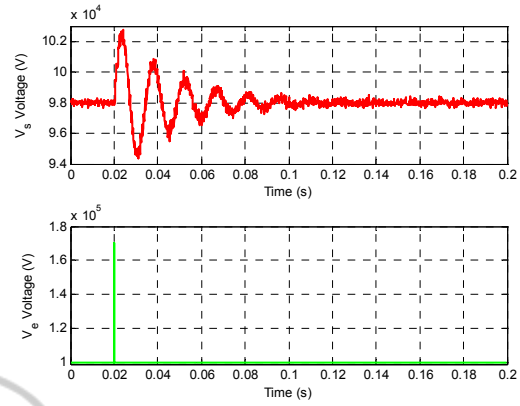


Figure 5: Time response of the system to a pulse input.

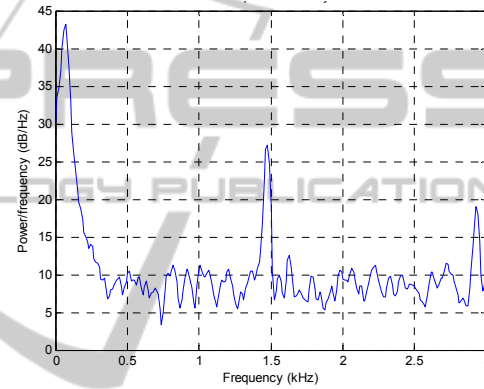


Figure 6: Power Spectral Density of the output of the system in response to a step input.

A threshold test is finally applied on the variance signal to obtain an estimation of the resonance frequencies. They are compared with the exact frequencies in table 2. In steady state, for $V_e(t) = 10^5 \text{V}$, then $V_s(t) \approx 9.57e^4 \text{V}$, $I_e(t) \approx 479.7 \text{A}$ and $I_s(t) \approx 478.6 \text{A}$. Using relations (23) and (19), the estimated line resistance r is thus $2.9e^{-2} \Omega \cdot \text{km}^{-1}$ leading to a relative error of 3% and g is estimated equal to $3.6779e-008 \Omega^{-1} \cdot \text{km}^{-1}$ thus estimated with an error of 464%. Finally, relations (31) and (32) provide $c = 1.156e^{-6} \text{F}$ and $l = 0.10207 \text{H}$. The estimation errors are respectively 4.3 % and 2.8%. The large estimation error for parameter g is a result of its very small value (very accurate voltages and currents measures with many decimals are required to estimate a so small value). But this error has no impact on the line dynamic behaviour.

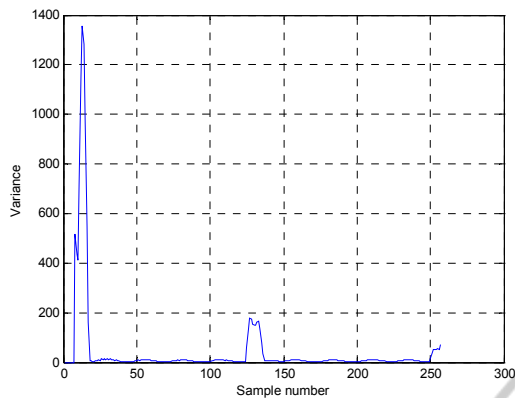


Figure 7: Variance signal of the spectrum of the output.

Table 2: Estimated resonance frequencies.

Frequencies	Exact (Hz)	Estimated (Hz)
ω_0	68.5	70.3
ω_1	1475	1477
ω_2	2945	2930

5 CONCLUSIONS

This paper proposes an approach for HVDC line parameters estimation. This method exploits the voltage information at the input and the output of the line. Using power spectral density computation the first three resonance frequencies of the transfer function linking the line input and the output voltage are obtained. Through a theoretical analysis of the line transfer functions, a link has been demonstrated between the resonance frequencies and the line parameters. The line steady state behaviour is finally used to obtain the numerical values of the line parameters the others being obtained using resonance frequencies estimation. This method differs from other methods presented in the literature by its frequency and physical approach. In future work, the authors intend now to propose another method permitting the computation of resonance frequencies using a fractional transfer function.

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