

An Adaptive Sliding Mode Controller for Synchronized Joint Position Tracking Control of Robot Manipulators

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Abstract: A novel adaptive sliding mode control algorithm is derived to deal with synchronized joint position tracking control of robot manipulators. The proposed algorithm does not require the precise dynamic model, and is very practical. The cross-coupled technology is incorporated into the adaptive sliding mode control architecture through feedback of joint position errors and synchronization errors. Its robustness is verified by the Lyapunov stability theory. Simulation results obtained from a 3-link non-linear planer robot manipulator demonstrate the effectiveness of the approach under various disturbances.

1 INTRODUCTION

Problems of synchronized control are predominant in the modern manufacturing, which devices are accordingly required to have all machine axes move simultaneously to reduce work-in-progress (Sun, 2003). Performance of the synchronized control directly affects a multi-axes system's reliability and control accuracy, which result in low production efficiency and poor product quality. In a traditional multi-axes, each actuator does not receive information from other actuators. That is, the actuator only correct errors caused by its disturbance and do not respond to errors caused by other actuators (Sage and Mathelin, 1999). The overall performance of the system is related to all actuators' motion, so lack of synchronous coordination will reduce the overall performance.

The concept of the cross-coupled control was proposed to deal with the synchronization problem (Koren, 1980). In recent years, problems of synchronized control in robotics are also a focus of attention by researchers. The cross-coupling coordination scheme for two-manipulator systems is developed by maintaining certain kinematic relationship between manipulators using motion synchronization (Sun and Mills, 2002). A position synchronization sliding mode control based on low-pass filtering is applied to the operation of multiple robotic manipulator systems (Zhao and Li, 2011). Passivity-based control is incorporated into synchronization of networked robotic systems in

the task space (Liu and Chopra, 2012). Unfortunately, the methods above need a complex task space model to define the synchronization error, which makes it difficult to simulate and realize the controller. To simplify the design of the controller, the first actuator is selected as the reference to reduce the difficulty of the implementation (Yang and Su, 2008).

In order to realize the synchronized control in robotics, it needs to fully consider the relevant characteristics of robot manipulators. The robot manipulators are highly nonlinear, highly time-varying and highly coupled. Moreover, there always exist uncertainties in the system in the system model such as external disturbance, parameter uncertainty and sensor errors, which cause unstable performance of the robotic system (Guo and Woo, 2003).

Adaptive control is often adopted to deal with uncertainties (Slotine and Li, 1987). Assuming that the parameters of the linearised model change slowly, the adaptive control based on the computed torque method can separate all the uncertain parameters, which have a relationship with the system structure and the load, while it can cause robot parameter's values to jump, which is a challenge to the traditional adaptive control (Wang and Zhang, 2015). Sliding mode control is a powerful robust scheme to deal with the problem of uncertainties, as is insensitive to the system parameter variation or external disturbances (Slotine and Li, 1991). However, it is sometimes difficult to obtain the system models. Also, to achieve

robustness, it requires large uncertainty bound, which will often cause chattering (Ho and Wong, 2007).

In this paper, a novel adaptive sliding mode control algorithm is derived to deal with synchronized joint position tracking control of robot manipulators, during the process of large-scale structure component welding. The cross-coupled technology is incorporated into the adaptive sliding mode control architecture (Hu and Liu, 2014; Ge and Guan, 2012) through feedback of joint position errors and synchronization errors. The proposed algorithm's robustness is verified by the Lyapunov stability theory.

The layout of the paper is as follows. Section 2 presents the dynamic model of robot manipulator in joint space, and some relevant properties are discussed. In Section 3, a novel adaptive sliding mode controller is developed and analyzed for synchronized joint position tracking control of the robot manipulator. Simulation examples are given to demonstrate the performance of the proposed controller in Section 4. Finally, we offer brief conclusions.

2 DYNAMIC MODEL OF ROBOTIC MANIPULATORS IN JOINT SPACE

In general, the joint space dynamics of the 3-link welding robotic manipulator can be described as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + d(t) = u, \quad (1)$$

where $M(q) = M^T(q) \in R^{3 \times 3}$ is the symmetric positive definite inertia matrix; $q \in R^3$ denotes the joint position vector; $C(q, \dot{q}) \in R^{3 \times 3}$ is the Coriolis and centrifugal; $G(q) \in R^3$ is the vector of gravitational torques; $d(t) \in R^3$ denotes the bounded disturbance with respect to time t ; and $u \in R$ represents the torque input vector.

Several fundamental properties of the robot model in Eq. (1) can be obtained as follows.

Property 1. The matrix $\dot{M}(q) - 2C(q, \dot{q})$ is skew symmetric matrix, i.e.,

$$x^T (\dot{M}(q) - 2C(q, \dot{q}))x = 0, \forall x \in R^3.$$

Property 2. For arbitrary $a, v \in R^3$, we get that

$$M(q)a + C(q, \dot{q})v + G(q) = Y(q, \dot{q}, a, v)\theta,$$

where $Y(q, \dot{q}, a, v)$ denotes the regression matrix, θ is the constant unknown parameter vector.

Property 3. The unknown disturbance $d(t)$ is assumed to be unknown, but bounded, i.e., $\|d(t)\| < \eta$, where η is a positive constant.

3 CONTROLLER DESIGN

3.1 Synchronized Joint Position Tracking Control

The objective of a designed controller is to drive the joint position q to the desired trajectory position q^d . Define joint position tracking error as

$$e_i = q_i - q_i^d, \quad (2)$$

where q_i is the i -th actual joint position of n -link robot manipulator, q_i^d denotes the i -th desired joint position. In the synchronization control, besides $e_i \rightarrow 0$, it is also aimed to regulate motion relationship during the tracking so that

$$e_1 = e_2 = \dots = e_n. \quad (3)$$

Now define synchronization errors of a subset of all possible pairs of two joint positions from the n -link robot manipulator in the following way (Yang and Su, 2008).

$$\begin{aligned} \varepsilon_2 &= \frac{1}{l_2}e_2 - \frac{1}{l_1}e_1, \\ \varepsilon_3 &= \frac{1}{l_3}e_3 - \frac{1}{l_1}e_1, \\ &\vdots \\ \varepsilon_n &= \frac{1}{l_n}e_n - \frac{1}{l_1}e_1. \end{aligned} \quad (4)$$

Equation (4) can be rewritten in the matrix format.

$$\begin{aligned} \varepsilon &= Te \\ &= \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -s_1 & s_2 & 0 & \dots & 0 \\ -s_1 & 0 & s_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -s_1 & 0 & \dots & 0 & s_n \end{bmatrix} e. \end{aligned} \quad (5)$$

where $\varepsilon = [1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_n]^T$, $s_i = \frac{1}{l_i}$, $i = 1, \dots, n$.

Define coupled joint position error as (Sun and Shao, 2007):

$$\begin{aligned} e_1^* &= e_1 + \beta\varepsilon_1, \\ e_2^* &= e_2 + \beta\varepsilon_2, \\ &\vdots \\ e_n^* &= e_n + \beta\varepsilon_n. \end{aligned} \quad (6)$$

Equation (6) can be rewritten in the matrix format.

$$e^* = e + \beta\varepsilon = e + \beta Te = (I + \beta T)e. \quad (7)$$

3.2 Adaptive Sliding Model Controller

Let the sliding surface

$$s = \dot{e}^* + \Lambda e^*, \quad (8)$$

where $\Lambda = \text{diag}[\lambda_1, \lambda_2, \lambda_3]$ in which λ_i is a positive constant.

The objective of controller can be achieved by choosing the control input u , so that the sliding surface satisfy the sufficient condition (Slotine and Li, 1989; Slotine and Li, 1991). Let the reference state

$$\begin{aligned} \dot{q}_r &= \dot{q} - s \\ &= \dot{q} - (\dot{e}^* + \Lambda e^*) \\ &= \dot{q} - (I + \beta T)\dot{e} - \Lambda e^* \\ &= \dot{q}^d - \beta T\dot{e} - \Lambda e^*, \end{aligned} \quad (9)$$

and

$$\ddot{q}_r = \ddot{q} - \dot{s} = \ddot{q}^d - \beta T\ddot{e} - \Lambda \dot{e}^*. \quad (10)$$

Then the control law u is designed as (Hu and Liu, 2014)

$$u = \hat{M}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{G}(q) - K_r \text{sgn}(s)^\alpha, \quad (11)$$

where $\hat{M}(q)$, $\hat{C}(q, \dot{q})$ and $\hat{G}(q)$ are the estimations of $M(q)$, $C(q, \dot{q})$ and $G(q)$ respectively; $K_r = \text{diag}[K_{r11}, K_{r22}, K_{r33}]$ is a diagonal positive definite matrix; $\text{sgn}(s)^\alpha$ is defined as

$$\text{sgn}(x)^\alpha = [|x_1|^\alpha \text{sign}(x_1), |x_2|^\alpha \text{sign}(x_2), |x_3|^\alpha \text{sign}(x_3)]^T, \quad (12)$$

and $x \in R^3$, $0 < \alpha < 1$.

Then combining system in Eq. (1) with the control law in Eq. (11), we can conclude

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) &= \\ \hat{M}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{G}(q) - K_r \text{sgn}(s)^\alpha, \end{aligned} \quad (13)$$

and

$$\begin{aligned} (\hat{M}(q) - \tilde{M}(q))(s + \ddot{q}_r) + (\hat{C}(q, \dot{q}) - \tilde{C}(q, \dot{q}))(s + \dot{q}_r) \\ + (\hat{G}(q) - \tilde{G}(q)) &= \\ \hat{M}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{G}(q) - K_r \text{sgn}(s)^\alpha, \end{aligned} \quad (14)$$

$$\begin{aligned} M(q)\dot{s} + C(q, \dot{q})s &= \\ \tilde{M}(q)\ddot{q}_r + \tilde{C}(q, \dot{q})\dot{q}_r + \tilde{G}(q) - K_r \text{sgn}(s)^\alpha, \end{aligned} \quad (15)$$

By using Property 2, since the matrix $M(q)$, $C(q, \dot{q})$, $G(q)$ are linear in terms of the manipulator parameters, the system in Eq. (15) can be written as

$$\tilde{M}(q)\ddot{q}_r + \tilde{C}(q, \dot{q})\dot{q}_r + \tilde{G}(q) = Y(q, \dot{q}, \ddot{q}_r, \dot{q}_r)\tilde{\theta}, \quad (16)$$

and therefore

$$M(q)\dot{s} + C(q, \dot{q})s = Y(q, \dot{q}, \ddot{q}_r, \dot{q}_r)\tilde{\theta} - K_r \text{sgn}(s)^\alpha. \quad (17)$$

Based on the properties above, the adaptation law is designed as following:

$$\dot{\hat{\theta}} = -\Gamma Y^T s, \quad (18)$$

where Γ is a diagonal positive-definite control gain.

3.3 Stability Analysis

Consider the following Lyapunov function candidate for system in Eq.(17).

$$V = \frac{1}{2}s^T M(q)s + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta}, \quad (19)$$

where θ is a vector containing the uncertain manipulator and load parameters, $\hat{\theta}$ is its estimate, and $\tilde{\theta} = \hat{\theta} - \theta$ denotes the parameter estimation error vector. According to the Property 2, Eqs. (17), and (18), the derivative of the chosen Lyapunov function can be derived as:

$$\dot{V} = -s^T d(t) - s^T K_r \text{sgn}(s)^\alpha$$

By using Property 3, we can conclude

$$\begin{aligned} \dot{V} &\leq \|s\| \|d(t)\| - \lambda_{\min}(K_r) \|s\|^\alpha \\ &\leq \|s\| \eta - \lambda_{\min}(K_r) \|s\|^\alpha \\ &= -\|s\| (\lambda_{\min}(K_r) \|s\|^\alpha - \eta). \end{aligned} \quad (20)$$

Theorem 1. For system in Eq. (1) under controller in Eqs. (11) and (17), if $\lambda_{\min}(K_r) > 0$, $1 > \alpha > 0$, and $\Gamma > 0$, the synchronized joint position errors will converge to the neighborhood of $s = 0$ as

$$\|s\| \leq \left(\frac{\eta}{\lambda_{\min}(K_r)} \right)^{\frac{1}{\alpha}} \quad (21)$$

in finite time.

Proof. Notice that when Eq. (21) holds, from Eq. (20), we can conclude $\dot{V} \leq 0$. Then by the finite time stability theory, the neighborhood in Eq. (21) can be reached in finite time. This completes the proof. \square

The advantage of the proposed adaptive sliding mode control lies in maintaining better joint position tracking performance while synchronized control of

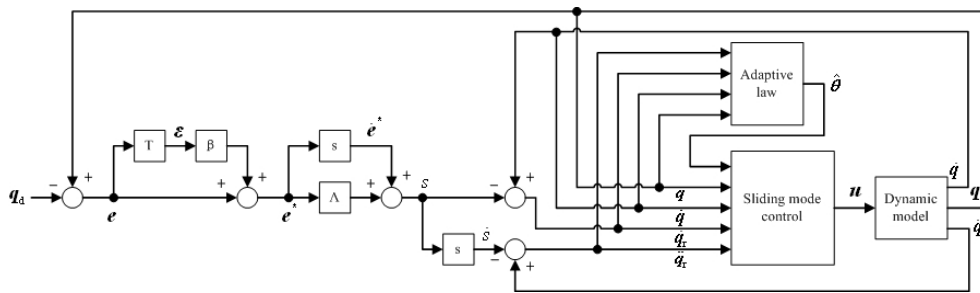


Figure 1: The Block diagram of the proposed controller.

robot manipulators. On one hand, the proposed control algorithm addresses the convergence of joint position tracking errors $e_i(t)$. On the other hand, it illustrates how these errors converge to zero, maintaining synchronization errors $\varepsilon_i(t) \rightarrow 0$. And the proposed controller can satisfy the transient performance of the joint position tracking. The proposed control algorithm can be depicted in Fig. 1.

4 SIMULATION

To validate effectiveness of the proposed approach, simulations were performed on a 3-link planar robot manipulator, as shown in Fig. 2, which dynamic model is derived by methods in (Spong and Hutchinson, 2006).

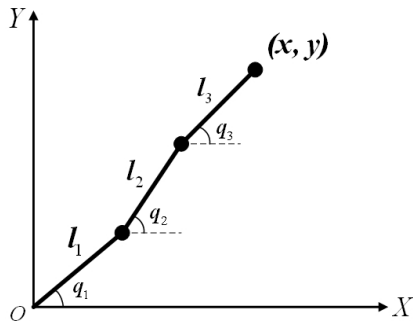


Figure 2: A 3-link planar robot manipulator.

The dynamic parameter model of 3-link non-linear planar robot manipulator in Eq. (1) is as follow:

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} + \begin{bmatrix} d_1(t) \\ d_2(t) \\ d_3(t) \end{bmatrix} = \mathbf{u} \quad (22)$$

where the parameters in the matrices above can be referred to the Eq. (23).

$$\begin{aligned} M_{11} &= m_1 l_1^2 + (m_2 + m_3) l_1^2 + i_1, \\ M_{12} &= M_{21} = (m_2 l_1 l_2 + m_3 l_1 l_2) \cos(q_2 - q_1), \\ M_{13} &= M_{31} = m_3 l_1 l_3 \cos(q_3 - q_1), \\ M_{22} &= m_2 l_2^2 + m_3 l_2^2 + i_2, \\ M_{23} &= M_{32} = m_3 l_2 l_3 \cos(q_3 - q_2), \\ M_{33} &= m_3 l_3^2 + i_3, \\ C_{11} &= C_{22} = C_{33} = 0, \\ C_{12} &= -\dot{q}_2 (m_2 l_1 l_2 + m_3 l_1 l_2) \sin(q_2 - q_1), \\ C_{13} &= -\dot{q}_3 m_3 l_1 l_3 \sin(q_3 - q_1), \\ C_{21} &= \dot{q}_1 (m_2 l_1 l_2 + m_3 l_1 l_2) \sin(q_3 - q_1), \\ C_{23} &= -\dot{q}_3 m_3 l_2 l_3 \sin(q_3 - q_2), \\ C_{31} &= \dot{q}_1 m_3 l_1 l_3 \sin(q_3 - q_1), \\ C_{32} &= \dot{q}_2 m_3 l_2 l_3 \sin(q_3 - q_2), \\ g_1 &= g [m_1 l_1 + (m_2 + m_3) l_1] \cos q_1, \\ g_2 &= g (m_2 l_2 + m_3 l_2) \cos q_2, \\ g_3 &= g m_3 l_3 \cos q_3. \end{aligned} \quad (23)$$

In the simulation, the robotic manipulator parameter values are $m_1 = 0.5 \text{ kg}$, $m_2 = 1.5 \text{ kg}$, $m_3 = 1.3 \text{ kg}$, $l_1 = l_2 = l_3 = 1 \text{ m}$, $r_1 = r_2 = r_3 = 0.5 \text{ m}$, where m_i is the link mass, l_i is the link length and r_i is the centroid length. The moment of inertia are $i_1 = 2 \text{ kg} \cdot \text{m}^2$, $i_2 = 2 \text{ kg} \cdot \text{m}^2$ and $i_3 = 2 \text{ kg} \cdot \text{m}^2$.

Table 1: The initial conditions of the robot manipulator.

$q_1(0)$	$q_2(0)$	$q_3(0)$	$\dot{q}_1(0)$	$\dot{q}_2(0)$	$\dot{q}_3(0)$
8	-9	1.5	8	-9	1.5

Table 2: The desired joint position.

$q_1^d(t)$	$q_2^d(t)$	$q_3^d(t)$
$\sin(3\pi t)$	$\cos(3\pi t)$	$\sin(3\pi t + \frac{1}{3}\pi)$

Table 3: The bounded disturbance.

$d_1(t)$	$d_2(t)$	$d_3(t)$
$5\sin(t)$	$2.5\cos(t)$	$5\sin(2t)$

The initial conditions and desired joint positions of the robot manipulator were selected in Table 1 and

Table 4: The control gains.

β	Λ	K_r	α
0.1	$diag[1, 1, 1]$	$diag[300, 300, 300]$	0.6

2. The bounded disturbance were set in Table 3. The control gains in Eqs.(9), (10) and (11) were chosen in Table 4.

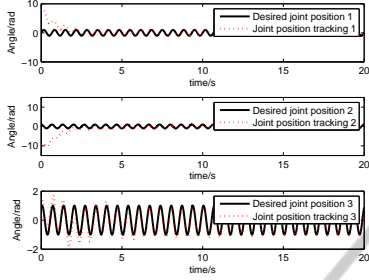


Figure 3: Joint position tracking response.

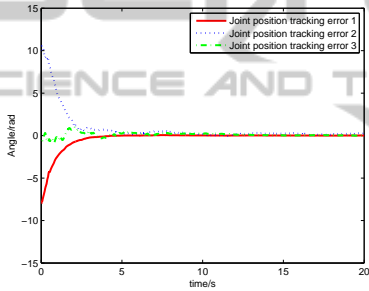


Figure 4: Joint position tracking error output.

Figs. 3 and 4 illustrate the synchronized joint position tracking response and error output of the robotic manipulator with the proposed adaptive sliding mode control ($\alpha \neq 0$) under various disturbances. The transient response and performance in joint position tracking is better than the preview work in (Hu and Liu, 2014), for comparison.

The synchronized joint position tracking errors in Fig. 4 will converge to the neighbourhood in finite time ($t = 3s$), which reflects that the proposed adaptive sliding mode control has a faster convergence speed than the traditional sliding mode controller. And it shows that the control gain Λ affects the convergence speed in simulations. Else, the proposed adaptive sliding mode control spends less time to perform the simulations, which is caused by the parameter α . The conclusion from Theorem 1 is not only proved by the analysis, but also by the simulations.

The synchronized joint position velocity tracking response and errors of the robotic manipulator with the proposed adaptive sliding mode control ($\alpha \neq 0$) under various disturbances are shown in Figs. 5 and 6. The proposed adaptive sliding control has a better

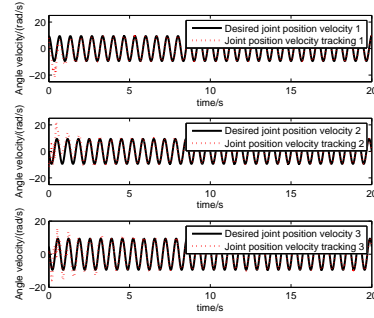


Figure 5: Joint position velocity tracking response.

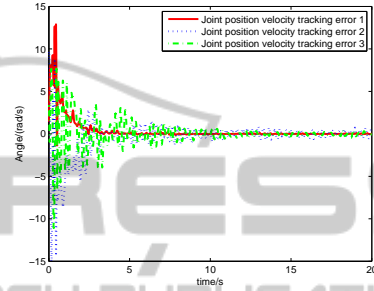


Figure 6: Joint position velocity tracking error output.

synchronized joint position velocity tracking transient response and performance than the results in (Hu and Liu, 2014). It can be seen that the neighbourhood or bound range is not only much smaller than the preview results, but also is getting smaller.

Besides, from the simulations above, the proposed adaptive sliding mode control does not require the precise dynamic model of the robot manipulator, which makes it very practical.

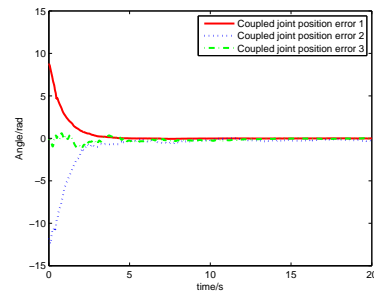


Figure 7: Coupled joint position error output.

Simulation results in Figs. 7, 8 and 9 display the coupled joint position error and synchronized joint position error ($\beta \neq 0$ and $\beta = 0$). These user-defined joint position errors will converge to the bound range in finite time. But the transient response and performance of synchronized joint position errors output in Figs. 8 and 9 is quite different, which is caused by the control gain parameter β . From the simulations in Fig. 9, synchronized joint position error $\epsilon_3(t)$ will con-

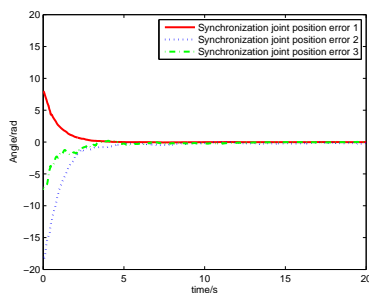


Figure 8: Synchronized joint position error ($\beta \neq 0$).

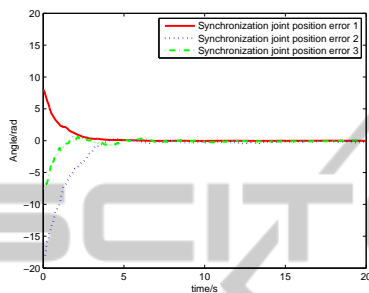


Figure 9: Synchronized joint position error ($\beta = 0$).

verge to the neighbourhood in advance. While, synchronized joint position errors $\varepsilon_1(t)$, $\varepsilon_2(t)$ and $\varepsilon_3(t)$ will converge simultaneously in Fig. 8.

5 CONCLUSIONS

We have proposed a novel adaptive sliding mode controller for synchronized joint position tracking control of robotic manipulator. The proposed algorithm does not require the precise dynamic model, and is very practical than the traditional sliding mode controller. On one hand, the proposed one addresses a better convergence to zero of both joint position tracking errors and joint position velocity tracking errors. On the other hand, it ensures the transient response and performance of synchronized joint position tracking. And, the proposed controller maintain the synchronized joint position errors will converge to the neighbourhood simultaneously. Simulation results obtained from a 3-link non-linear planer robot manipulator demonstrate the effectiveness of the approach under various disturbances.

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