Implementation of Evolving Fuzzy Models of a Nonlinear Process

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Abstract: This paper presents details on the implementation of evolving Takagi-Sugeno-Kang (TSK) fuzzy models of a nonlinear process represented by the pendulum dynamics in the framework of the representative pendulum-crane systems. The pendulum angle is the output variable of the TSK fuzzy models that are obtained by online identification. The rule bases and the parameters of the TSK fuzzy models are continuously evolved by an online identification algorithm (OIA) that adds new rules with more summarization power and modifies the existing rules and parameters. The OIA is associated with an input selection algorithm that guides the modelling in terms of ranking the inputs according to their importance factors. Three TSK fuzzy models evolved by the OIA are exemplified. The performance of the new evolving TSK fuzzy models is illustrated by experimental results conducted on pendulum-crane laboratory equipment.

1 INTRODUCTION

As shown in (Angelov, 2002; Sayed Mouchaweh et al., 2002; Lughofer, 2011, 2013; Precup et al., 2015), the evolving Takagi-Sugeno-Kang (TSK) fuzzy models are characterized by the continuous online learning for rule base learning. In this regard, an online identification algorithm (OIA) generally continuously evolves the rule bases and the parameters of the TSK fuzzy models, and the models are built online by adding new or removing old local models (i.e., the adding mechanism). A useful classification of OIAs dedicated to evolving TSK fuzzy models is given in (Dovžan et al., 2014), where the OIAs are organized in three categories. First, the adaptive algorithms must start with the initial structure of the TSK fuzzy model (given by other algorithms or by the experience of the specialist), the number of space partitions/clusters does not change over time, and only the parameters of the membership functions (m.f.s) and the local models are adapted. Second, the incremental algorithms, represented by RAN (Platt, 1991), SONFIN (Juang and Lin, 1998), SCFNN (Lin et al., 2001), NeuroFAST (Tzafestas and Zikidis, 2001), DENFIS (Kasabov and Song, 2002), eTS (Angelov and Filev, 2004), FLEXFIS (Lughofer and Klement, 2005) or PANFIS (Pratama et al., 2014), implement only adding mechanisms. Third, the evolving algorithms, besides the adding mechanism, implement removing and some of them also merging and splitting mechanisms.

Building upon the recent results on evolving TSK fuzzy models given in (Precup et al., 2012c, 2014), this paper gives details on the implementation of evolving TSK fuzzy models of a representative nonlinear process represented by the pendulum dynamics in the framework of pendulum-crane systems. As shown in (Precup et al., 2014), the pendulum-crane systems are important as translational electromechanical systems. The crane control systems can carry out either the cart position control or the position control of the cart and the downward or upward angle control of the pendulum as well. The process models for crane systems can give the cart position (Precup et al., 2014) or the pendulum angle (Precup et al., 2012c).

Some recent examples of TSK fuzzy models for the pendulum dynamics, i.e., the pendulum angle is the output variable, are presented in the literature

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with focus on fuzzy control. The parameters of TSK fuzzy models are tuned in (Al-Hadithi et al., 2012) by the parameters' weighting method that exhibits low computational effort. Fuzzy state observers are combined with TSK fuzzy models in (Kolemishevska-Gugulovska et al., 2012). Type-2 TSK fuzzy models that alleviate the noise of training data and that account for mismatched m.f.s are proposed in (Li and Sun, 2012; Li et al., 2014). TSK fuzzy models with perturbations and state multiplicative noises are suggested in (Chang and Huang, 2014). The quasi-Linear Parameter Variation formulation of TSK models is discussed in (Allouche et al., 2014). The dynamic decoupling concept is introduced in (Chiu, 2014) by the virtual input dynamics, which decouples the system uncertainty and the control signal in each rule. The modelling errors between nonlinear dynamic systems and TSK fuzzy models are analyzed in (Tsai and Chen, 2014). The so-called universal TSK fuzzy models for discrete-time non-affine nonlinear systems are proposed in (Gao et al., 2015).

Three evolving TSK fuzzy models are proposed in this paper, namely models with one, two and three inputs. These models are derived by an OIA that belongs to the incremental algorithms according to the classification given in (Dovžan et al., 2014). The OIA adds new rules with more summarization power and modifies the existing rules and parameters, and it is associated with an input selection algorithm that guides the modelling in terms of ranking the inputs according to their importance factors.

This paper offers twofold new contributions with respect to the previously discussed state-of-the-art, expressed as the functionalities of the OIA. First, the OIA is inspired from (Angelov and Filev, 2004), and it offers rule bases and parameters that continuously evolve by adding new rules with more summarization power, the existing rules and parameters are modified in terms of using the potentials of new data points. Second, an input selection algorithm is inserted in the OIA.

These contributions are advantageous compared to the state-of-the-art because, as shown in (Precup et al., 2014) but for crane control systems, the OIA ensures a relatively simple and transparent implementation. In addition, the OIA derives TSK fuzzy models with improved performance proved for a complex nonlinear process represented by the pendulum dynamics. This paper applies and adapts the results obtained in (Precup et al., 2014) for the cart position models to the pendulum angle models.

The new functionalities of the OIA and the TSK fuzzy models proposed in this paper are compared with the TSK fuzzy models obtained by three OIAs: the adaptive algorithm ANFIS (Jang, 1993) and the incremental algorithms DENFIS and FLEXFIS. The comparison shows that the proposed evolving TSK fuzzy models ensure the performance enhancement on the validation data.

This paper is structured as follows: an overview on the OIA is presented in the next section. The case study concerning the derivation and validation of the new TSK fuzzy models for the pendulum dynamics in the framework of pendulum-crane systems are treated in Section 3. The comparison of model performance is included. The conclusions are highlighted in Section 4.

2 ONLINE IDENTIFICATION ALGORITHM

The steps of the OIA are obtained by the relatively simple reformulation of the results given in (Angelov and Filev, 2004; Precup et al., 2014) focusing on the cost-effective implementation of the recursive procedure. The OIA consists of the following steps that can be organized in terms of the flowchart, omitted here for the sake of simplicity:

Step 1. The rule base structure is initialized, i.e., the parameters in the rule antecedents are initialized. This is carried out such that to have a single rule, $n_R = 1$, where n_R is the number of rules. The subtractive clustering (Takagi and Sugeno, 1985) is applied to compute the parameters of the TSK fuzzy models using the first data point $\mathbf{p}_1$, where the expression of the data point $\mathbf{p}$ at the discrete time step k is

$$\mathbf{p}_k = [p_k^1 \quad p_k^2 \quad \dots \quad p_k^{n+1}]^T, \tag{1}$$

T indicates the matrix transposition, the data point in the input-output data space $\mathbf{R}^{n+1}$ is

$$\begin{aligned}\mathbf{p} &= [\mathbf{z}^T \quad y]^T = [z_1 \quad z_2 \quad \dots \quad z_n \quad y]^T \\ &= [p^1 \quad p^2 \quad \dots \quad p^n \quad p^{n+1}]^T \in \mathbf{R}^{n+1},\end{aligned} \tag{2}$$

the rule base of the affine-type TSK fuzzy models is

$$\begin{aligned}&\text{Rule } i: \text{IF } z_1 \text{ IS } LT_{i1} \text{ AND} \dots \text{AND } z_n \text{ IS } LT_{in} \\ &\text{THEN } y_i = a_{i0} + a_{i1} z_1 + \dots + a_{in} z_n, i = 1 \dots n_R,\end{aligned} \tag{3}$$

where z_j, $j = 1 \dots n$, are the input variables, n is the number of input variables, $LT_{ij}, i = 1 \dots n_R, j = 1 \dots n$, are the input linguistic terms, y_i is the output of the local model in the rule consequent of rule

$i, i = 1...n_R$, and $a_{il}, i = 1...n_R, l = 0...n$, are the parameters in the rule consequents.

Using the algebraic product t-norm to model the AND operator and the weighted average defuzzification method in the TSK fuzzy model structure, the output y of the TSK fuzzy model is

$$y = \left[\sum_{i=1}^{n_R} \tau_i y_i\right] / \left[\sum_{i=1}^{n_R} \tau_i\right] = \sum_{i=1}^{nR} \lambda_i y_i \\ = \sum_{i=1}^{n_R} \lambda_i [1 \quad \mathbf{z}^T]^T \boldsymbol{\pi}_i, \tag{4}$$

where the firing degree of the rule i is

$$\tau_i(\mathbf{z}) = \text{AND}(\mu_{i1}(z_1), \mu_{i2}(z_2), ..., \mu_{in}(z_n)) \\ = \mu_{i1}(z_1) \cdot \mu_{i2}(z_2) \cdot ... \cdot \mu_{in}(z_n), i = 1...n_R, \tag{5}$$

the normalized firing degree of the rule i is

$$\lambda_i = \tau_i / \left[\sum_{i=1}^{n_R} \tau_i\right], i = 1...n_R, \tag{6}$$

and the vector $\boldsymbol{\pi}_i, i = 1...n_R$, in (4) is the parameter vector of the rule i (Precup et al., 2014)

$$\boldsymbol{\pi}_i = [a_{i0} \quad a_{i1} \quad a_{i2} \quad ... \quad a_{in}]^T, i = 1...n_R, \tag{7}$$

The parameters are initialized in terms of (Angelov and Filev, 2004)

$$\hat{\boldsymbol{\theta}}_1 = [(\boldsymbol{\pi}_1^T)_1 \quad (\boldsymbol{\pi}_2^T)_1 \quad ... \quad (\boldsymbol{\pi}_{n_R}^T)_1]^T \\ = [0 \quad 0 \quad ... \quad 0]^T, \mathbf{C}_1 = \Omega\, \mathbf{I}, r_s = 0.4, \\ k = 1, n_R = 1, \mathbf{z}_1^* = \mathbf{z}_k, P_1(\mathbf{p}_1^*) = 1, \tag{8}$$

where $\mathbf{C}_k \in \mathbf{R}^{n_R(n+1) \times n_R(n+1)}$ is the covariance matrix, $\mathbf{I}$ is the $n_R(n+1)^{\text{th}}$ order identity matrix, $\Omega = \text{const}, \Omega > 0$, is a large number, $\hat{\boldsymbol{\theta}}_k$ is an estimation of the parameter vector in the rule consequents at the discrete time step k, and r_s, $r_s > 0$, is the spread of all Gaussian input m.f.s $\mu_{ij}, i = 1...n_R, j = 1...n$, of the fuzzy sets of the input linguistic terms LT_{ij}

$$\mu_{ij}(z_j) = \exp[-(4/r_s^2)(z_j - z_{ij}^*)^2], \\ i = 1...n_R, j = 1...n, \tag{9}$$

and $z_{ij}^*, i = 1...n_R, j = 1...n$, are the centres of these m.f.s. $\mathbf{p}_1^*$ in (8) is the first cluster centre, $\mathbf{z}_1^*$ is the centre of the rule 1 and also a projection of $\mathbf{p}_1^*$ on the axis $\mathbf{z}$ defined in (2), and $P_1(\mathbf{p}_1^*)$ in (8) is the potential of $\mathbf{p}_1^*$.

The input selection algorithm suggested in (Precup et al., 2014) is next applied in order to select the important input variables from all possible input variables. This algorithm consists of the following steps that are organized as sub-steps of this step 1 of the OIA:

Sub-step 1.1. The algorithm is initialized by setting the values of the $\lambda, 0 < \lambda < 1$, that represents the importance threshold, and $\tau, 0 < \tau < 1$, that stands for the significance threshold.

Sub-step 1.2. The input variable z_j, $j = 1...n$, is applied to the initial TSK fuzzy model, the outputs $y_{j,k}$ of the initial TSK fuzzy model at the discrete time moment k, $k = 1...D$, are read, where D is the number of input-output data points. The change range R_{z_j} for the input variable $z_j, j = 1...n$, is calculated

$$R_{z_j} = \max_{k=1}^{D} y_{j,k} - \min_{k=1}^{D} y_{j,k}, \tag{10}$$

and the importance factor I_{z_j} of the input variable $z_j, j = 1...n$, is calculated as well

$$I_{z_j} = R_{z_j} / \left[\max_{j=1}^{n} R_{z_j}\right]. \tag{11}$$

The most important input variable is characterized by $I_{z_j} = 1$. As shown in (Precup et al., 2014), large values of R_{z_j} and I_{z_j} indicate a big influence of the input variable $z_j, j = 1...n$, and small values of R_{z_j} and I_{z_j} indicate a relatively unimportant input variable $z_j, j = 1...n$.

Sub-step 1.3. The importance of all input variables is ranked according to the values of the importance factors $I_{z_j}, j = 1...n$.

Sub-step 1.4. All input variables that fulfil the condition

$$I_{z_j} < \lambda \tag{12}$$

are removed. The condition (12) points out that the input variable $z_j, j = 1...n$, is unimportant, so it is justified to remove it. This sub-step gives the set of remaining n_r input variables, which are selected out of the initial n input variables, $n_r < n$.

Sub-step 1.5. The closely related input variables are recognized to carry out the independent input variable testing by the calculation of the correlation functions $Corr(z_i, z_j)$, $0 \le Corr(z_i, z_j) \le 1$, between the selected input variables z_i and z_j, $i, j = 1...n_r$

$$Corr(z_i, z_j) = \left\{ \sum_{k=1}^{D} [(z_{i,k} - \bar{z}_i)(z_{j,k} - \bar{z}_j)] \right\} / \left\{ D \sqrt{\phi_{z_i} \phi_{z_j}} \right\}, \tag{13}$$

where $\bar{z}_i$ and $\bar{z}_j$ are the means of vectors z_i and z_j, $i, j = 1...n_r$, respectively, and ϕ_{z_i} and ϕ_{z_j} are the variances of z_i and z_j, $i, j = 1...n_r$, respectively. respectively. If the following condition is fulfilled:

$$Corr(z_i, z_j) > \tau, \tag{14}$$

then the input variable z_i is closely related with the input variable z_j. The condition (14) is used in keeping the independent input variables among the n_r selected input variables. The condition (14) also helps in removing one of the two input variables z_i or z_j. Therefore, this sub-step leads to the set of remaining n_i independent input variables out of the n_r selected input variables, $n_i < n_r$.

Step 2. At the next time step, k is set to $k = k + 1$, and the next data sample $\mathbf{p}_k$ is read.

Step 3. The potential of each new data sample is computed in terms of (Precup et al., 2014)

$$\begin{aligned} & P_k(\mathbf{p}_k) = (k-1)/[(k-1)(\vartheta_k + 1) + \sigma_k \\ & - 2\nu_k], \vartheta_k = \sum_{j=1}^{n+1} (p_k^j)^2, \\ & \sigma_k = \sum_{j=1}^{n+1} \sum_{l=1}^{k-1} (p_l^j)^2, \nu_k = \sum_{j=1}^{n+1} (p_k^j \sum_{l=1}^{k-1} p_l^j)., \end{aligned} \tag{15}$$

Step 4. The potentials of the centres of existing rules (clusters) are recursively updated by (Angelov and Filev, 2005)

$$\begin{aligned} & P_k(\mathbf{p}_l^*) = (k-1) P_{k-1}(\mathbf{p}_l^*) \\ & / \left[k - 2 + P_{k-1}(\mathbf{p}_l^*) + P_{k-1}(\mathbf{p}_l^*) \sum_{j=1}^{n+1} (d_{k(k-1)}^j)^2 \right], \end{aligned} \tag{16}$$

where $P_k(\mathbf{p}_l^*)$ is the potential at the discrete time step k of the cluster centre, which is a prototype of the rule l.

Step 5. The possible modification or upgrade of the rule base structure is carried out using, as described in (Angelov and Filev, 2004; Precup et al., 2014), the potential of the new data compared to the potential of existing rules' centres. The rule base structure is modified if certain conditions are fulfilled.

Step 6. The parameters in the rule consequents are updated using the Recursive Least Squares (RLS) algorithm (Takagi and Sugeno, 1985; Chiu, 1994)

$$\begin{aligned} & \hat{\boldsymbol{\theta}}_k = \hat{\boldsymbol{\theta}}_{k-1} + \mathbf{C}_k \boldsymbol{\psi}_{k-1} (y_k - \boldsymbol{\psi}_{k-1}^T \hat{\boldsymbol{\theta}}_{k-1}), \\ & \mathbf{C}_k = \mathbf{C}_{k-1} - \frac{\mathbf{C}_{k-1} \boldsymbol{\psi}_{k-1} \boldsymbol{\psi}_{k-1}^T \mathbf{C}_{k-1}}{1 + \boldsymbol{\psi}_{k-1}^T \mathbf{C}_{k-1} \boldsymbol{\psi}_{k-1}}, k = 2...D, \end{aligned} \tag{17}$$

where the initial conditions are given in (4), and the output of the TSK fuzzy model in (4) is expressed in terms of the vector form

$$\begin{aligned} & y = \boldsymbol{\psi}^T \boldsymbol{\theta}, \boldsymbol{\theta} = [\boldsymbol{\pi}_1^T \quad \boldsymbol{\pi}_2^T \quad \dots \quad \boldsymbol{\pi}_{n_R}^T]^T, \\ & \boldsymbol{\psi} = [\lambda_1 [1 \ \mathbf{z}^T] \quad \lambda_2 [1 \ \mathbf{z}^T] \quad \dots \quad \lambda_{n_R} [1 \ \mathbf{z}^T]]. \end{aligned} \tag{18}$$

Step 7. The output of the evolving TSK fuzzy model at the next discrete time step $k+1$ is predicted using the particular form of (18)

$$\hat{y}_{k+1} = \boldsymbol{\psi}_k^T \hat{\boldsymbol{\theta}}_k. \tag{19}$$

The algorithm continues with the step 2 until all data points from the set of input-output data

$$\{\mathbf{p}_k \mid k = 1...D\} \tag{20}$$

are read. The step 1 is conducted offline, and the steps 2 to 7 are conducted online.

3 FUZZY MODELS AND EXPERIMENTAL VALIDATION

A laboratory setup that contains a pendulum-cart system described in (Turnau et al., 2008) has been used in the development and validation of the evolving TSK fuzzy models. The state equations of the process in the pendulum-cart system are presented in (21).

The variables in (21) are: x_1 – the cart position (the distance between the cart and the centre of the rail), x_2 – the angle between the upward vertical and the ray pointing at the centre of mass cart, x_3 – the cart velocity, x_4 – the pendulum angular velocity, u – the control signal represented by a constrained PWM voltage signal, $|u| \le u_{\max} > 0$, m_c – the equivalent mass of the cart, m_P – the mass of the pole and load, and l_d – the distance from the axis of rotation to the centre of mass. The parameters in (21) are: J_p – the moment of inertia of the pendulum-cart system with respect to the axis of rotation, p_1 – the ratio between the control force and the control signal, p_2 – the ratio between the control force and x_3, f_c – the dynamic cart coefficient, and f_p – the rotational friction coefficient. The

$$\begin{aligned}
&\dot{x}_1 = x_3, \\
&\dot{x}_2 = x_4, \\
&\dot{x}_3 = \left\{ \frac{J_p}{(m_c + m_p) l_d} [\frac{p_1 u}{(m_c + m_p) l_d} - x_4{}^2 \sin x_2 - \frac{(f_c - p_2) x_3}{(m_c + m_p) l_d}] + [g \sin x_2 - \frac{f_p x_4}{(m_c + m_p) l_d}] \cos x_2 \right\} \\
&/[\frac{J_p}{(m_c + m_p) l_d^2} - \cos^2 x_2], \\
&\dot{x}_4 = \left\{ [\frac{p_1 u}{(m_c + m_p) l_d} - x_4{}^2 \sin x_2 - \frac{(f_c - p_2) x_3}{(m_c + m_p) l_d}] \cos x_2 + \frac{1}{l} [g \sin x_2 - \frac{f_p x_4}{(m_c + m_p) l_d}] \right\} \\
&/[\frac{J_p}{(m_c + m_p) l_d^2} - \cos^2 x_2],
\end{aligned} \tag{21}$$

parameter values used in the experimental setup are (Turnau et al., 2008; Precup et al., 2014)

$$\begin{aligned}
&u_{\max} = 0.5, m_c = 0.76 \text{ kg}, m_p = 0.052 \text{ kg}, \\
&l_d = 0.011 \text{ m}, J_p = 0.00292 \text{ kg} \cdot \text{m}^2, \\
&p_1 = 9.4 \text{ N}, p_2 = -0.548 \text{ N s/m}, \\
&f_c = 0.5 \text{ N s/m}, f_p = 6.65 \cdot 10^{-5} \text{ N m s/rad}.
\end{aligned} \tag{22}$$

The OIA presented in the previous sections has been applied in order to obtain the evolving TSK fuzzy models of the pendulum dynamics, i.e. $y = x_2$. This section gives a part of the results. The OIA has been coded as an extension of the implementation in terms of eFS Lab (Ramos and Dourado, 2004; Aires et al., 2009) of the OIAs given in (Angelov and Filev, 2004; Precup et al., 2014).

Setting the sampling period to 0.01 s, the control signal u has been generated as two weighted sums of pseudo-random binary signals according to Figure 1 that covers different ranges of magnitudes. As shown in (Precup et al., 2012c, 2014), this process input has been applied to the laboratory setup to generate the input-output data points $(\mathbf{z}_k, y_k)$, $k = 1...D$. Figure 1 leads to a total number of 6000 data points separated in training data and validation data. The first $D = 2500$ data points (the time frame from 0 s to 25 s) in Figure 1 belong to the validation data, the rest of $D = 3500$ data points (the time frame from 25 s to 60 s) in Figure 1 belong to the testing (validation) data, and the process output y will be illustrated as follows.

The input selection algorithm included in the step 1 of the OIA has been applied for three values of the importance threshold, namely $\lambda = 0.4$, $\lambda = 0.3$ and $\lambda = 0.2$, and one value of the significance threshold, $\tau = 0.5$. This leads to three TSK fuzzy models with the following inputs: the TSK fuzzy model 1, with the input u_{k-1}, the TSK fuzzy model 2 with the inputs u_{k-1} and y_{k-1}, and the TSK fuzzy model 3 with the inputs u_{k-1}, y_{k-1} and y_{k-2}. The output of these three TSK fuzzy models is y_k. The inputs of the fuzzy models have been obtained from delayed system inputs and/or outputs extracted from the training and validation data sets. The value of the parameter Ω in the step 1 of the OIA has been set to $\Omega = 10000$.

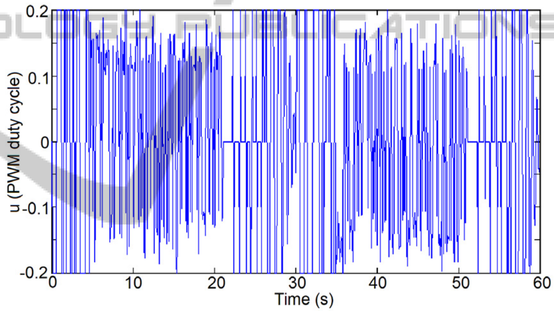

Figure 1: Control signal versus time: training data and testing data.

The TSK fuzzy model 1 has evolved to $n_R = 2$ rules. The parameter values of the TSK fuzzy model 1, computed by the OIA for $n = 1$, are presented in Table 1.

The evolutions of the system output (i.e., the pendulum angle) y versus time of the TSK fuzzy model 1 and of the real-world process (the laboratory setup) are presented in Figure 2. Figure 2 gives the responses of the TSK fuzzy model 1 and of the process for the validation data and shows the poor behaviour of this model. The system output for the validation data is not illustrated as follows.

Table 1: Parameter values of TSK fuzzy model 1.

Rule number i	z_{i1}^*	r_s	a_{i0}	a_{i1}
1	0	0.0424	5.3009	–1.6296
2	0.1156	0.0424	5.3689	0.3121

Table 2: Parameter values of TSK fuzzy model 2.

Rule number i	z_{i1}^*	z_{i2}^*	r_{s1}	r_{s2}	a_{i0}	a_{i1}	a_{i2}
1	0	0	0.0424	1.2502	0.6903	1.0025	0.4226
2	–0.1167	3.1861	0.0424	1.2502	0.0639	–0.5125	0.9956
3	–0.1167	2.3332	0.0424	1.2502	0.6877	–1.1108	0.6857
4	0	6.2186	0.0424	1.2502	–0.4733	0.0435	1.0744
5	–0.1167	6.2186	0.0424	1.2502	–0.6956	–0.0229	1.1099
6	0	6.1161	0.0424	1.2502	–0.4597	–0.1376	1.0472
7	0	5.9841	0.0424	1.2502	–0.5298	0.7453	1.0385

Table 3: Parameter values of TSK fuzzy model 3.

Rule number i	z_{i1}^*	z_{i2}^*	z_{i3}^*	r_{s1}	r_{s2}	r_{s3}	a_{i0}	a_{i1}	a_{i2}	a_{i3}
1	0	0	0	0.0424	1.2502	1.2502	0.1161	0.2383	2.1021	–0.9404
2	–0.1175	3.1861	0	0.0424	1.2502	1.2502	0.4574	–0.0915	0.8872	0.0096
3	–0.1175	3.5558	3.4423	0.0424	1.2502	1.2502	–0.2529	–0.0368	1.8198	–0.7841
4	0.1293	1.9835	1.9643	0.0424	1.2502	1.2502	0.2798	0.1254	2.0211	–1.0080
5	–0.1175	2.3332	1.9643	0.0424	1.2502	1.2502	0.8986	–0.2488	1.5784	–0.8115
6	–0.1175	6.2186	6.2282	0.0424	1.2502	1.2502	0.0827	–0.0133	1.4412	–0.4550
7	0	6.2186	6.2282	0.0424	1.2502	1.2502	–0.6813	0.0502	1.6820	–0.5745
8	0	6.1161	6.2282	0.0424	1.2502	1.2502	0.2990	0.5435	0.8706	0.0532
9	0	5.9841	6.1161	0.0424	1.2502	1.2502	–1.2067	0.1298	2.1044	–0.9259

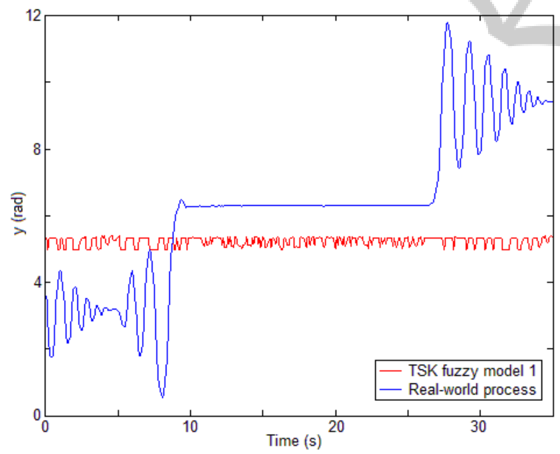

Figure 2: Pendulum angle versus time of TSK fuzzy model 1 and of real-world process for validation data.

The TSK fuzzy model 2 has evolved to $n_R = 7$ rules. The parameter values of the TSK fuzzy model 2, computed by the OIA for $n = 2$, are presented in Table 2.

The time responses of y of the TSK fuzzy model 2 and of the real-world process are presented in Figure 3. Figure 3 shows a slightly improved behaviour compared to Figure 2.

The TSK fuzzy model 3 has evolved to $n_R = 9$ rules. The parameter values of the TSK fuzzy model 3, computed by the OIA for $n = 3$, are presented in Table 3.

The time responses of y versus time of the TSK fuzzy model 3 and of the real-world process are illustrated in Figure 4. Figure 4 shows an improved behaviour with respect to Figure 3.

As pointed out in Section 1, the OIA and the TSK fuzzy model performance (as the result of the OIA) have been compared with the following three OIAs that lead to evolving TSK fuzzy models: ANFIS, DENFIS and FLEXFIS. Since Figure 2 illustrates the poor performance of the TSK fuzzy model 1, the comparison has been focused on the TSK fuzzy models 2 and 3. Two TSK fuzzy models have been obtained for each OIA. The fair comparison of all fuzzy models has been conducted in terms of using the same inputs, numbers and shapes of m.f.s as those of the TSK fuzzy models 2 and 3, and the numbers of rules n_R have been set such that to be very close.

The comparison of the models is carried out in terms of the root mean square error (RMSE) between the pendulum angles of the TSK fuzzy models and of the real-world process. The expression of this global performance index is

$$RMSE = \sqrt{(1/D)\sum_{k=1}^{D}(y_k - x_{2,k})^2}, \qquad (23)$$

where y_k is the output (the pendulum angle) of the TSK fuzzy models and $x_{2,k}$ is the output (the pendulum angle) of the laboratory setup at the discrete time moment *k*. The RMSE has been computed and measured for the training data and for the testing (validation) data.

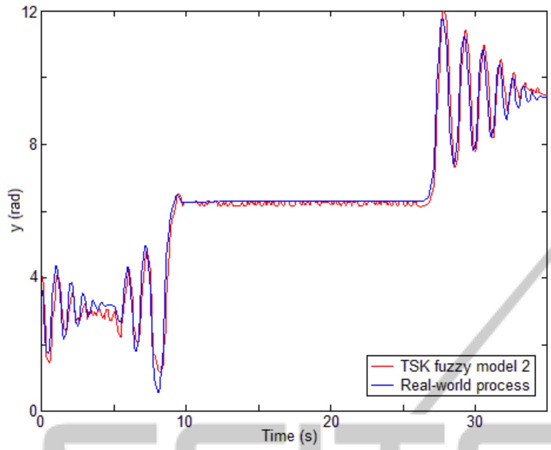

Figure 3: Pendulum angle versus time of TSK fuzzy model 2 and of real-world process for validation data.

The results obtained for the eight TSK fuzzy models on the testing data are summarized in Table 4. Table 4 includes the numbers of parameters n_P of the final evolved TSK fuzzy models.

Table 4 and Figures 2, 3 and 4 prove that the best performance on the testing data is exhibited by the TSK fuzzy model 3 obtained by the OIA presented in Section 2. Table 4 illustrates the performance improvement achieved by the evolving TSK fuzzy models obtained by proposed OIA compared to other three OIAs. In addition, the performance improvement with respect to another implementation of the OIA given in (Precup et al., 2012c) is ensured.

The results presented in Table 5 and in Figures 3 and 4 also show that the performance of the proposed TSK fuzzy models are consistent with the testing data. However, a different scaling used, for example, in Figures 3 and 4, could show in a more illustrative way the differences.

As expected, Table 4 confirms that more inputs lead to improved model performance. But the selection of the input variables is carried out systematically in the step 1 of the OIA by that input selection algorithm that guides the modelling.

The models and the performance depend on the values of the parameters λ and τ. Different models and results for these models are obtained for other values of these two parameters.

Based on these experimental results, presented only for the testing data and not for the validation data, the proposed evolving TSK fuzzy models can be accepted as very close to the real-world nonlinear process. However, different conclusions can be drawn if other nonlinear processes are considered (Precup et al., 2004; Deliparaschos et al., 2006; Gusikhin et al., 2007; Precup and Preitl, 2007; Ferreira and Ruano, 2009; Filip and Leiviskä, 2009; Bošnak et al., 2012; Precup et al., 2012b; Guerra et al., 2012; Lam and Lauber, 2013) if they are viewed such that to belong to control systems. The OIA should be reorganized such that to enable the cost-effective implementation of the control solutions (Precup et al., 2011, 2012a, 2012d;

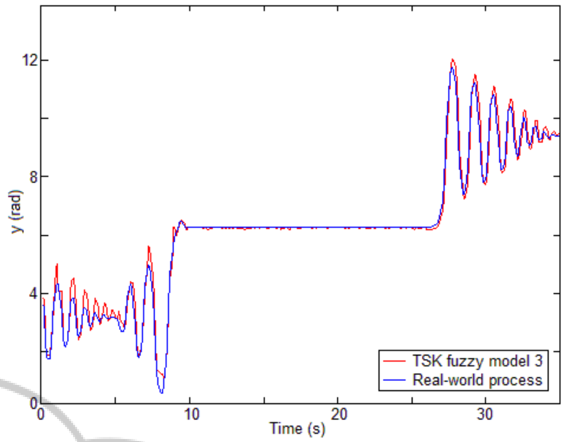

Figure 4: Pendulum angle versus time of TSK fuzzy model 3 and of real-world process for validation data.

Table 4: Results for eight TSK fuzzy models on testing data.

TSK fuzzy model	OIA	n_R	n_P	RMSE
2	Section 2	7	49	0.1672
2	ANFIS	8	56	0.2537
2	DENFIS	8	56	0.4094
2	FLEXFIS	7	49	0.3011
3	Section 2	9	90	0.1505
3	ANFIS	12	120	0.1814
3	DENFIS	10	90	0.3392
3	FLEXFIS	10	90	0.2506

4 CONCLUSIONS

This paper has given implementation details on an OIA, which continuously evolves the rule bases and the parameters of TSK fuzzy models by adding new rules with more summarization power and modifying the existing rules and parameters. The OIA consists of seven steps, and the step 1 includes an input selection algorithm that guides the modelling in terms of ranking the inputs according to their importance factors.

The main advantages of the new results given in this paper are the simplicity and transparency of the OIA, the simplicity of the evolving TSK fuzzy models and their consistency with both the testing data. These advantages have been proved by real-time experimental results related to the fuzzy modelling of a representative nonlinear process, i.e., the pendulum dynamics in the framework of pendulum-crane systems.

The OIA has been implemented by the extension of the OIAs given in (Angelov and Filev, 2004; Precup et al., 2014) using the core of eFS Lab reported in (Ramos and Dourado, 2004; Aires et al., 2009). The comparison of the experimental results shows the performance improvement exhibited by two proposed TSK fuzzy models with respect to other fuzzy models obtained by similar OIAs.

Future research will concern the further performance improvement of the TSK fuzzy models. Several optimization algorithms including nature-inspired optimization algorithms (Duleba and Sasiadek, 2003; Haber et al., 2009; Valdez et al., 2011; Johanyák and Papp, 2012; Vaščák and Paľa, 2012; David et al., 2013; El Amraoui and Mesghouni, 2014; Osaba et al., 2014; Tang et al., 2014; Savio et al., 2014; Zhang et al., 2014) will be incorporated to replace the RLS algorithm in the step 6 of the OIA. The OIA will be applied to other representative nonlinear processes as well. Since the goal of the development of these TSK fuzzy models is the model-based design of fuzzy control systems, the models will be included in such control system structures.

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