Sign Subband Adaptive Filter with Selection of Number of Subbands

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Keywords: Adaptive Filter, Impulsive Noise, Sign Algorithm, Mean-Square Deviation.

Abstract: The sign subband adaptive filter (SSAF) algorithm is introduced to reduce performance degradation of least-mean-square-type algorithms due to a correlated input signal or an impulsive noise environments. However, this algorithmh has huge computational complexity when the length of the unknown system is large. In this paper, we focus on reduce computational complexity of the conventional SSAF algorithm and propose an SSAF algorithm which selects number of subbands according to convergence state. The specific bands which contributes to decrease the mean-square deviation are used to update the adaptive filter. Thus, the proposed algorithm reduces the computational complexity compared to the conventional SSAF algorithm. The selection mehtod is derived by analysing the mean-square deviation. Through the computer simulation, simulation results are presented that demonstrate the fast convergence rate of the proposed algorithm and save the computational cost.

1 INTRODUCTION

Adaptive filter algorithm has many applications such as channel equalization, echo cancellation, and system identification (Sayed, 2003; Lee et al., 2009). The least-mean-square (LMS) and normalized least-mean-square algorithm (NLMS), which are derived by minimizing the $\mathcal{L}_2$-norm of the error function, are widely used in this area because of its simplicity and robustness against background noise. However, these algorithms exhibit slow convergence rate when an input signal is correlated or a measured signal contains impulsive noise.

To overcome each problem, two categorizations are presented. First, the normalized subband adaptive filter (NSAF) algorithm was developed to improve the performance of the NLMS algorithm for highly correlated input signals (Lee and Gan, 2004). By taking a pre-whitening operation on the input signal, the NSAF algorithm achieves fast convergence rate. Second, the sign algorithm was developed to improve the performance of the NLMS algorithm for impulsive noise environments, because it is obtained by minimizing the $\mathcal{L}_1$-norm of the error function (Mathews and Cho, 1987).

Combining the advantages of these two techniques, in (Ni and Li, 2010), the sign subband adaptive filter (SSAF) algorithm was introduced, i.e., the SSAF algorithm is derived by taking the pre-whitening process and minimizing the $\mathcal{L}_1$-norm. Therefore, the SSAF algorithm can have good performance in correlated input signal and impulsive noise environments.

For subband-type algorithms, the correlated input signal is close to the white signal in each band when the number of subbands is high (Lee et al., 2009). However, this leads to a huge computational complexity when the length of the unknown system is long (Kim et al., 2010). Therefore, the SSAF algorithm also has the complexity problem when the algorithm is applied the long-tap unknown system.

In this paper, we focus on reducing computational complexity of the conventional SSAF algorithm and propose an SSAF algorithm with selection of number of subbands. For every iteration, the only specific bands which contributes to decrease the mean-square deviation (MSD) are used to update the adaptive filter coefficient. That is, the proposed algorithm implies smaller number of subbands, so it reduces computational complexity of the conventional SSAF algorithm. In addition, the proposed algorithm achieves a fast convergence rate than the conventional SSAF algorithm in impulsive-noise environments. Through the computer simulation, simulation results are pre-

Jeong J., Kim S., Koo G. and Kim S..
Sign Subband Adaptive Filter with Selection of Number of Subbands.
DOI: 10.5220/0005537604070411
In *Proceedings of the 12th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2015), pages 407-411
ISBN: 978-989-758-122-9

sented that demonstrate the fast convergence rate of the proposed algorithm.

This paper is organized as follows. In Section 2, the SSAF algorithm is reviewed. Section 3 presents the proposed algorithm. Section 4 deals with the simulation results which compare the proposed algorithm with the SSAF algorithm for system identification. Finally, conclusions are given in Section 5.

2 SIGN SUBBAND ADAPTIVE FILTER

The output signal $d(n)$ of the system is obtained as

$$d(n) = \mathbf{w}_{\text{opt}}^T \mathbf{u}(n) + \eta(n), \tag{1}$$

where $\mathbf{w}_{\text{opt}} = [w_0, w_1, \ldots, w_{M-1}]^T$ denotes the optimal weight vector, which is predicted by an adaptive filter; superscript T is the vector transpose; $\mathbf{u}(n) = [u(n), u(n-1), \ldots, u(n-M+1)]^T$ denotes the input signal vector; $\eta(n)$ is a noise that consists of a background and an impulsive noise; n is time index; and M is the length of the optimal weight vector. The probability density function of the noise is expressed as (Rey Vega et al., 2008)

$$p_{\eta(n)}(\eta(n)) = p\mathcal{N}(0, (K+1)\sigma_b^2) + (1-p)\mathcal{N}(0, \sigma_b^2), \tag{2}$$

where p is the probability of impulsive noise, K is the magnitude of impulsive noise, and σ_b^2 is the noise variance without the impulsive noise.

The structure of the SSAF is shown in Figure 1. The subband signals $d_i(n)$ and $\mathbf{u}_i(n)$ are obtained by filtering $d(n)$, and $\mathbf{u}(n)$ via analysis filters $H_i(z)$ for $i = 0, 1, \ldots, N-1$, respectively, where $\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \ldots, u_i(kN-M+1)]^T$ and N is the number of subbands. The decimated desired signal $d_{i,\text{D}}(k)$ and output signal $y_{i,\text{D}}(k)$ are obtained by critically decimating $d_i(n)$ and $y_i(n)$, respectively, where $y_{i,\text{D}}(k) = \mathbf{u}_i^T(k)\mathbf{w}(k)$ and subscript D means the decimated signal. n is index of original sequences and k is index of decimated sequences. $G_i(z)$ is synthesis filter for $i = 0, 1, \ldots, N-1$.

The weight update equation for the conventional SSAF algorithm is (Ni and Li, 2010)

$$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \mu \frac{\mathbf{U}(k)\text{sign}(\mathbf{e}_\text{D}(k))}{\sqrt{\sum_{i=0}^{N-1} \|\mathbf{u}_i(k)\|^2}}, \tag{3}$$

where μ is a step size, $\|\cdot\|$ denotes the $\mathcal{L}_2$-norm, sign$(\cdot)$ denotes the sign function,

$$\mathbf{U}(k) = [\mathbf{u}_0(k), \mathbf{u}_1(k), \ldots, \mathbf{u}_{N-1}(k)], \tag{4}$$

$$\mathbf{e}_\text{D}(k) = [e_{0,\text{D}}(k), e_{1,\text{D}}(k), \ldots, e_{N-1,\text{D}}(k)]^T, \tag{5}$$

$$e_{i,\text{D}} = d_{i,\text{D}}(k) - y_{i,\text{D}}(k), \tag{6}$$

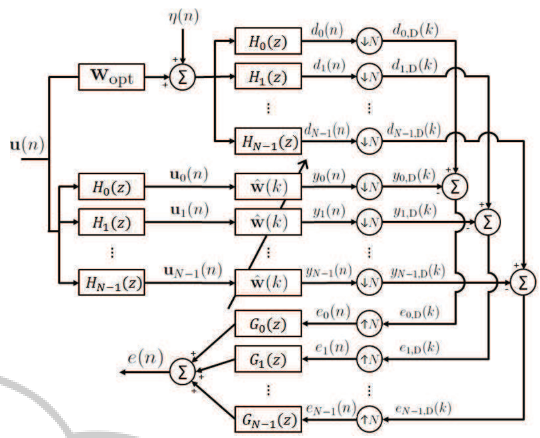

Figure 1: Structure of the SSAF.

and $d_{i,\text{D}}(k) = d_i(kN)$ denotes the decimated desired signal.

3 PROPOSED ALGORITHM

3.1 Proposed Algorithm

The proposed algorithm is determined by maximizing the decrease in the MSD. The MSD is defined as $\text{MSD}(k) \triangleq E\{\tilde{\mathbf{w}}^T(k)\tilde{\mathbf{w}}(k)\}$, where $\tilde{\mathbf{w}}(k) = \mathbf{w}_{\text{opt}}(k) - \hat{\mathbf{w}}(k)$ is the weight error vector, and $E\{\cdot\}$ is the expectation of random variables. By subtracting (3) from $\mathbf{w}_{\text{opt}}(k)$, the equation is expressed in terms of $\tilde{\mathbf{w}}(k)$ as follows:

$$\tilde{\mathbf{w}}(k+1) = \tilde{\mathbf{w}}(k) - \mu \frac{\mathbf{U}(k)\text{sign}(\mathbf{e}_\text{D}(k))}{\sqrt{\sum_{i=0}^{N-1} \|\mathbf{u}_i(k)\|^2}}. \tag{7}$$

By taking squared $\mathcal{L}_2$-norm and expectation, the MSD can be obtained as

$$\text{MSD}(k+1) = \text{MSD}(k) - 2\mu E\left\{ \frac{\tilde{\mathbf{w}}^T(k)\mathbf{U}(k)\text{sign}(\mathbf{e}_\text{D}(k))}{\sqrt{\sum_{i=0}^{N-1} \|\mathbf{u}_i(k)\|^2}} \right\} + \mu^2. \tag{8}$$

For a sufficiently long length of the weight vector (Rey Vega et al., 2008; Bershad et al., 2014), we obtain

$$E\left\{ \frac{\tilde{\mathbf{w}}^T(k)\mathbf{U}(k)\text{sign}(\mathbf{e}_\text{D}(k))}{\sqrt{\sum_{i=0}^{N-1} \|\mathbf{u}_i(k)\|^2}} \right\} \approx \gamma E\{\tilde{\mathbf{w}}^T(k)\mathbf{U}(k)\text{sign}(\mathbf{e}_\text{D}(k))\}, \tag{9}$$

where

$$\gamma \triangleq E\left\{ \frac{1}{\sqrt{\sum_{i=0}^{N-1} \|\mathbf{u}_i(k)\|^2}} \right\}. \tag{10}$$

Substituting (9) into (8) yields

$$\mathrm{MSD}(k+1) = \mathrm{MSD}(k) - 2\mu\gamma \sum_{i=0}^{N-1} E\{e_{i,a}(k)\mathrm{sign}(e_{i,\mathrm{D}}(k))\} + \mu^2, \quad (11)$$

where $e_{i,a}(k) = \mathbf{u}_i^T(k)\tilde{\mathbf{w}}(k)$. The second term on the right-hand side of (11) is calculated by assuming that $e_{i,\mathrm{D}}(k)$ and $e_{i,a}(k)$ are jointly Gaussian and have zero mean (Sayed, 2003). By using Price's theorem (Sayed, 2003), we get

$$E\{e_{i,a}(k)\mathrm{sign}(e_{i,\mathrm{D}}(k))\} = a_i \sigma^2_{e_{i,a}}(k), \quad (12)$$

where $\sigma^2_{e_{i,a}}(k) \triangleq E\{e^2_{i,a}(k)\}$ is the undisturbed error variance of the ith subband, and

$$a_i = \sqrt{\frac{2}{\pi}} \left\{ \frac{1-p}{\sqrt{\sigma^2_{e_{i,a}}(k) + \sigma^2_{b_{i,\mathrm{D}}}}} + \frac{p}{\sqrt{\sigma^2_{e_{i,a}}(k) + (K+1)\sigma^2_{b_{i,\mathrm{D}}}}} \right\}. \quad (13)$$

By assuming that the input signal and noise are mutually independent (Shin and Sayed, 2004), the $i-$th subband error variance is written as

$$\sigma^2_{e_{i,\mathrm{D}}}(k) = \sigma^2_{e_{i,a}}(k) + (1-p)\sigma^2_{b_{i,\mathrm{D}}} + p(K+1)\sigma^2_{b_{i,\mathrm{D}}}. \quad (14)$$

Combining (12) and (14) into (11), the resulting equation is expressed in terms of $\sigma^2_{e_{i,\mathrm{D}}}(k)$ as follows:

$$\mathrm{MSD}(k+1) = \mathrm{MSD}(k) - 2\mu\gamma \sum_{i=0}^{N-1} a_i \left(\sigma^2_{e_{i,\mathrm{D}}}(k) - \alpha\sigma^2_{b_{i,\mathrm{D}}} \right) + \mu^2, \quad (15)$$

where $\alpha = (1+pK)$.

In (15), the MSD tends to decrease when $\sigma^2_{e_{i,\mathrm{D}}}(k)$ is larger than $\alpha\sigma^2_{b_{i,\mathrm{D}}}$. On the other hand, the MSD increases when $\sigma^2_{e_{i,\mathrm{D}}}(k)$ is smaller than $\alpha\sigma^2_{b_{i,\mathrm{D}}}$. The proposed algorithm selects subbands satisfying $\sigma^2_{e_{i,\mathrm{D}}}(k) > \alpha\sigma^2_{b_{i,\mathrm{D}}}$ at every iteration for the largest decrease in the MSD. Consequently, the number of selected subbands, which is to update the weight vector, is less than or equal to that of the conventional SSAF algorithm.

3.2 Practical Consideration and Computational Computation

In practical application, we can not obtain the exact expected values, so we assume that the expected value is approximately the same as an instantaneous value.

$$E\{e^2_{i,\mathrm{D}}(k)\} \approx e^2_{i,\mathrm{D}}(k). \quad (16)$$

Table 1: Proposed Algorithm Summary.

Initialization : $\hat{\mathbf{w}}(0) = [0, 0, \ldots, 0]^T$

Parameters : $\alpha \geq 1$

Update :

for $i = 0, 1, \ldots, N-1$ **do**

  $S_{L(k)} = [\,]$

  If $|e_{i,\mathrm{D}}(k)| > \sqrt{\alpha}\sigma_{b_{i,\mathrm{D}}}$

    s_l is selected, $S_{L(k)} = [S_{L(k)}, s_l]$

  end

  If $L(k) \neq 0$

    $\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \mu \sum_{l=0}^{L(k)} \frac{\mathbf{u}_{s_l}(k)\mathrm{sign}(e_{s_l,\mathrm{D}}(k))}{\sqrt{\sum_{l=0}^{L(k)} \|\mathbf{u}_{s_l}(k)\|^2}}$

  end

end for

The noise variance σ^2_b can be easily estimated during silences (Yousef and Sayed, 2001; Benesty et al., 2006).

Let $S_{L(k)} = [s_1, s_2, \cdots, s_{L(k)}]$ means a subset with $L(k)$ members of the set $0, 1, \cdots, N-1$, where s_l denotes the index of the chose subbands, and $L(k)$ means the number of selected subbands at iteration k. Finally, the update equation of the proposed algorithm is expressed as

$$\hat{\mathbf{w}}(k+1) = \begin{cases} \hat{\mathbf{w}}(k) + \mu \sum_{l=0}^{L(k)} \frac{\mathbf{u}_{s_l}(k)\mathrm{sign}(e_{s_l,\mathrm{D}}(k))}{\sqrt{\sum_{l=0}^{L(k)} \|\mathbf{u}_{s_l}(k)\|^2}} & L(k) \neq 0 \\ \hat{\mathbf{w}}(k) & L(k) = 0 \end{cases}, \quad (17)$$

where $|e_{s_l,\mathrm{D}}(k)| > \sqrt{\alpha}\sigma_{b_{s_l,\mathrm{D}}}$ $(l = 1, 2, \ldots, L(k))$. The proposed algorithm is summarized in Table 1. Table 2 shows the computational cost of the conventional SSAF and the proposed algorithm.

4 SIMULATION RESULTS

The performance of the proposed algorithm is compared to the conventional SSAF algorithm via com-

Table 2: Computational Complexity.

	SSAF	Proposed algorithm
Multiplication	$3M+3NL$	$(1+2N(k))M+3NL$
Division	1	$N(k)$
Comparison	-	1

$(1/T_S,\ N(k) = L(k)/N)$

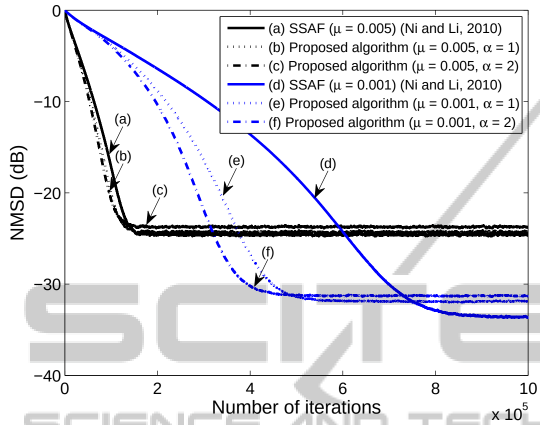

Figure 2: NMSD learning curve for the conventional SSAF (Ni and Li, 2010) and proposed algorithm with various step sizes.

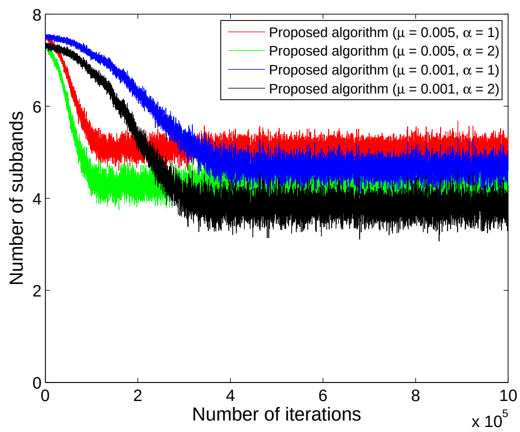

Figure 3: Average number of selected subbands in the proposed algorithm.

puter simulation. The adaptive filter has the same length as the optimal weight vector with 512 or 1024 taps. The input signal is generated by passing a zero-mean white Gaussian random sequence through

$$G(z) = \frac{1}{1-0.9z^{-1}}. \tag{18}$$

The background noise is added to the system output with a signal-to-noise ratio (SNR) $= 30$ dB. Furthermore, an impulsive noise is also added to the system output with $p = 0.01$ and $K = 10^5$. In order to compare the performance, we use the normalized MSD (NMSD), which is defined as $\|\mathbf{w}_{\text{opt}} - \hat{\mathbf{w}}(k)\|^2/\|\mathbf{w}_{\text{opt}}\|^2$ and calculated by ensemble averaging over 50 independent trials. We assume that the background noise variance σ_b^2 is known (Yousef and Sayed, 2001; Benesty et al., 2006), so $i-$th subband noise variance is obtained as $\sigma^2_{b_{i,\text{D}}} = \sigma_b^2/N$ (Yin and Mehr, 2011). In the simulations, the number of subbands $(N = 8)$ are used. The length of the prototype filter is 64.

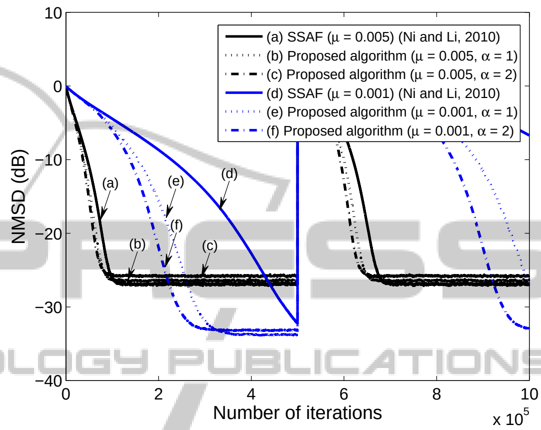

Figure 4: NMSD learning curve for the conventional SSAF (Ni and Li, 2010) and proposed algorithm with various step sizes.

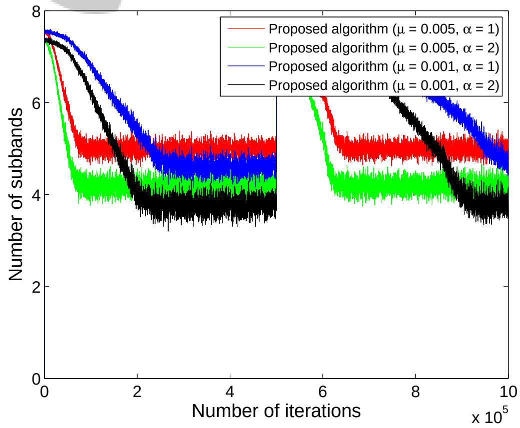

Figure 5: Average number of selected subbands in the proposed algorithm.

Figure 2 shows the normalized MSD learning curve for the conventional SSAF (Ni and Li, 2010) and the proposed algorithm for $M = 1024$, various step sizes ($\mu = 0.005$ and $\mu = 0.001$), and values of α ($\alpha = 1$ and $\alpha = 2$). As can be seen, the proposed algorithm leads to a fast convergence rate when step size is small. The proposed algorithm has a fast convergence rate but high steady-state MSD if α is large because the number of subbands quickly decreases. Figure 3 shows the average number of selected subbands. In this result, the proposed algorithm has a lower computation complexity than the conventional SSAF algorithm, because the number of used subbbands is decreased.

Tracking performance is an important issue in adaptive filter (Benesty et al., 2006). The unknown system is changed to $-\mathbf{w}_{\text{opt}}$ at 5×10^5 to evaluate its tacking performance (Zou et al., 2000). Figure 4 shows the NMSD learning curve of the conventional SSAF (Ni and Li, 2010) and proposed algorithm for $M = 1024$, various step sizes ($\mu = 0.005$ and $\mu = 0.001$), and values of α ($\alpha = 1$ and $\alpha = 2$). As can be seen, the proposed algorithm has fast convergence rate after the system change. That is the proposed algorithm properly tracks the changed system coefficient. AS can be seen from Figure 5, the average number of selected subbands is increased when the system changed, but it is decreased again as the iteration increases. Therefore, the proposed algorithm efficiently reduces the computational cost even for system tracking scenario.

In practical application, we can not exactly know the values of p and K. Therefore, it is difficult to select α. However, the user chooses $\alpha \geq 1$, because p and K are always positive values.

5 CONCLUSIONS

In this paper, we have proposed a new SSAF algorithm with a low computational complexity. By analyzing the MSD, the proposed algorithm selects the number of subbands at each iteration. In conclusion, the proposed algorithm was derived by maximizing the decrease in the MSD at every iteration. Consequentially, the proposed algorithm reduces the computational complexity compared to the conventional SSAF algorithm. In addition, the simulation results show the proposed algorithm achieves a fast convergence rate in impulsive-noise environments.

ACKNOWLEDGEMENTS

This research was supported by the MSIP(Ministry of Science, ICT and Future Planning), Korea, under the ICT Consilience Creative Program (IITP-2015-R0346-15-1007) supervised by the IITP(Institute for Information & communications Technology Promotion) and by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2013R1A1A2058975).

REFERENCES

Benesty, J., Rey, H., Rey Vega, L., and Tressens, S. (2006). A nonparametric vss nlms algorithm. *Signal Processing Letters, IEEE*, 13(10):581–584.

Bershad, N., Eweda, E., and Bermudez, J. (2014). Stochastic analysis of the lms and nlms algorithms for cyclostationary white gaussian inputs.

Kim, S.-E., Choi, Y.-S., Song, M.-K., and Song, W.-J. (2010). A subband adaptive filtering algorithm employing dynamic selection of subband filters. *Signal Processing Letters, IEEE*, 17(3):245–248.

Lee, K. A. and Gan, W. S. (2004). Improving convergence of the nlms algorithm using constrained subband updates. *Signal Processing Letters, IEEE*, 11(9):736–739.

Lee, K.-A., Gan, W.-S., and Kuo, S. M. (2009). *Subband adaptive filtering: theory and implementation*. John Wiley & Sons.

Mathews, V. J. and Cho, S. H. (1987). Improved convergence analysis of stochastic gradient adaptive filters using the sign algorithm. *Acoustics, Speech and Signal Processing, IEEE Transactions on*, 35(4):450–454.

Ni, J. and Li, F. (2010). Variable regularisation parameter sign subband adaptive filter. *Electronics letters*, 46(24):1605–1607.

Rey Vega, L., Rey, H., Benesty, J., and Tressens, S. (2008). A new robust variable step-size nlms algorithm. *Signal Processing, IEEE Transactions on*, 56(5):1878–1893.

Sayed, A. H. (2003). *Fundamentals of adaptive filtering*. John Wiley & Sons.

Shin, H.-C. and Sayed, A. H. (2004). Mean-square performance of a family of affine projection algorithms. *Signal Processing, IEEE Transactions on*, 52(1):90–102.

Yin, W. and Mehr, A. S. (2011). Stochastic analysis of the normalized subband adaptive filter algorithm. *Circuits and Systems I: Regular Papers, IEEE Transactions on*, 58(5):1020–1033.

Yousef, N. R. and Sayed, A. H. (2001). A unified approach to the steady-state and tracking analyses of adaptive filters. *Signal Processing, IEEE Transactions on*, 49(2):314–324.

Zou, Y., Chan, S., and Ng, T. (2000). A recursive least m-estimate (rlm) adaptive filter for robust filtering in impulse noise. *Signal Processing Letters, IEEE*, 7(11):324–326.