The Optimal Control Problems of Nonlinear Systems

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Abstract: This article discusses the optimal control problem of nonlinear systems, which are described by ordinary differential equations, their right parts are periodic in the angular coordinate. The particularity of the considered in the given work nonlinear control problems is that they take into account the fact that on unfairly long interval of time, preservation of a deviation of any subsystem of controlled system from nominal operating conditions conducts to danger of destruction and unbalance of other subsystems and even of all the system as a whole. Consideration was given to a numerical example of the optimal motion control of two-machine power system.

1 INTRODUCTION

The mathematical model of the modern electric power complex, consisting of turbine generators and complex multiply-connected energy blocks, is a system of nonlinear ordinary differential equations. The optimization problem, and the creation of algorithms for constructing of controls by the principle of feedback for such systems is actual and still attracts the attention of many researchers.

In this work while solving the problem of control synthesis for the considered electric power system, the constructions of the method of Bellman-Krotov function in the form of necessary and sufficient optimality conditions were used (V. F. Krotov, 1996)–(V. I. Gurman, 1997).

2 STATEMENT OF THE PROBLEM

It is required to minimize the functional

$$J(u) = 0.5\sum_{i=1}^{l}\int_0^T \left(k_i y_i^2 + r_i u_i^2\right)\exp\{\gamma_i t\}dt + \\ +\Lambda(x(T), y(T)), \tag{1}$$

under the conditions:

$$\frac{dx_i}{dt} = y_i,\ \frac{dy_i}{dt} = -\lambda_i y_i - f_i(x) + b_i u_i,$$
$$x_i(t_0) = x_{i0},\ \ y_i(t_0) = y_{i0},\quad i = \overline{1,l},\ t \in (t_0, T) \tag{2}$$
$$x(t),\ \ y(t) : (t_0, T) \to R^l,$$

where $u_i \in R$ is scalar control; $f_i(x)$ is a continuously differentiable scalar function satisfying the integrability condition:

$$\frac{\partial f_i(x)}{\partial x_k} = \frac{\partial f_k(x)}{\partial x_i},\ \forall i \neq k; \tag{3}$$

moments of time t_0, T are assumed to be given; r_i, γ_i, λ_i are positive constants, $k_i(t)$ are positive functions and the terminal value of $x(T), y(T)$ are unknown beforehand.

It should be noted that if inequalities took place $\gamma_i < 0,\ \ i = 1,2,\ldots.l,$ then these coefficients would reflect the fact of discounting (playing the important role in economical problems). In our case, these coefficients are positive,that, naturally, imposes in control problem(1)–(3) additional requirements on the condition function $x_i(t_0), y_i(t_0),\ \ i = 1,2,\ldots,l,$ and control $u_i(t),\ \ i = 1,2,\ldots,l,$ so that they in view of weight coefficients $k_i(t),\ r_i,\ i = 1,2,\ldots,l,$ decreased faster than the exponent $\exp\{\gamma_i t\},\ \ i = 1,2\ldots,l,$ and also provided definiteness of integral (1). It becomes essentially important obviously when time T is great enough.

As it is known, in many, including complex, technical devices, ”danger” of deviations of controlled system from a normal natural operating regime in time does not decrease, and can only grow. The offered quality functional (1) allows, first, to struggle with the specified ”danger” promptly and efficiently. Second, after return of the system to the normal operating regime of work it provides disappearance of control influences as soon as possible.

The problem of synthesis for the Cauchy problem (1)–(3) is very important for the problems of electric

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power systems work optimization.

2.1 The Main Lemma

Following to the Bellman-Krotov formalism (V. F. Krotov, 1996)–(V. I. Gurman, 1997), we will show the correctness of the following lemma.

Lemma. In order to provide the optimal control of $u_i^0(y_i) = -\frac{b_i}{r_i}\exp\{-\gamma_i t\}\, y_i,\ \ i = \overline{1,l}$ and the corresponding solution of the system (2)–(3) $\{x(t), y(t)\}$, it is necessary and sufficient that

$$K(x(T),\, y(T)) = \Lambda(x(T), y(T)),$$
$$k_i(t) = 2\lambda_i \exp\{-\gamma_i t\} + \frac{b_i^2}{r_i}\exp\{-2\gamma_i t\} > 0, \quad i \in \overline{1,l}, \tag{4}$$

$$K(x,y) = 0.5\sum_{i=1}^{l} y_i^2 - \sum_{\substack{i=1,\\ x_j=0,\\ j>i}}^{l} \int_0^{x_i} f_i(x_1,\ldots,x_{i-1},\xi_i,x_{i+1},..,x_l)\, d\xi_i$$

is the Bellman-Krotov function and besides,

$$J(u^0) = \min_u J(u) = K(x(t_0), y(t_0)).$$

2.2 Proof

For continuously differentiable function $K(x,y)$ functional (1) has the view:

$$J(u) = J(x(t), y(t), u(t)) = \int_{t_0}^{T} R[x(t), y(t), u(t)]\, dt + m_1(x(T), y(T)) + m_0(x(t_0),\, y(t_0)), \tag{5}$$

where

$$R[x,y,u] = \sum_{i=1}^{l} K_{x_i} y_i + K_{y_i}(-\lambda_i y_i - f_i(x) + b_i u_i) + \frac{1}{2}\left(k_i y_i^2 + r_i u_i^2\right)\exp\{\gamma_i t\}, \tag{6}$$

$$\begin{aligned} m_1(x,y) &= -K(x,y) + \Lambda(x,y),\\ m_0(x,y) &= K(x,y), \end{aligned} \tag{7}$$

$$K_{x_i} = \frac{\partial K}{\partial x_i}, \quad K_{y_i} = \frac{\partial K}{\partial y_i}, \ \ i = \overline{1,l}.$$

To find the Bellman-Krotov function $K(x,y)$ we use the Cauchy problem:

$$\begin{cases} \inf_u R[x,y,u] = 0, \quad t \in (t_0, T), \\ K(x(T),\, y(T)) = \Lambda(x(T), y(T)). \end{cases} \tag{8}$$

Assuming that

$$\begin{aligned} K_{x_i} y_i = K_{y_i} f_i(x), \quad K_{y_i} = y_i,\\ K_{x_i} = f_i(x), \quad i = \overline{1,l}. \end{aligned} \tag{9}$$

Taking into account (9), from (8) we obtain:

$$u_i(y_i) = -\frac{b_i}{r_i}\exp\{-\gamma_i t\}\, y_i,$$
$$k_i(t) = 2\lambda_i exp\{-\gamma_i t\} + \frac{b_i^2}{r_i}\exp\{-2\gamma_i t\} > 0, \quad r_i > 0, \quad i \in \overline{1,l}. \tag{10}$$

By virtue of integrability conditions of (3) and the assumptions (9) we get the representation of the Bellman-Krotov function (4).

From (8) it also follows that the value of the terminal term of the functional (1) is defined by:

$$\Lambda(x(T), y(T)) = K(x(T),\, y(T)).$$

By virtue of the representations (6)–(7) along the optimal control (10) the value of the functional $J(u)$ (1) will be equal:

$$J(u^0) = \min_u J(u) = K(x(t_0), y(t_0)).$$

To complete the proof of lemma, it remains only to note that the existence of Bellman-Krotov function in the form (4) provides not only the sufficiency of the optimality conditions (8), but also their necessity.

3 THE OPTIMUM CONTROL PROBLEM OF PHASE SYSTEM

Let us consider the problem of functional minimization:

$$J(\nu) = J(\nu_1 \ldots, \nu_l) = 0.5\sum_{i=1}^{l}\int_0^T \left(w_{s_i} S_i^2 + w_{\nu_i}\nu_i^2\right) * \exp\{\gamma_i t\}\, dt + \Lambda(\delta(T), S(T)), \tag{11}$$

under the conditions:

$$\begin{aligned} &\frac{d\delta_i}{dt} = S_i,\\ &H_i\frac{dS_i}{dt} = -D_i S_i - f_i(\delta_i) - N_i(\delta_i) + \nu_i,\\ &\delta = (\delta_1,\ldots,\delta_l), \quad S = (S_1,\ldots,S_l), \end{aligned} \tag{12}$$

where w_{s_i}, w_{ν_i} are weight coefficients, correspondingly positive functions and constants; H_i is an inertial constants; $f_i(\delta_i)$ are 2π-periodical continuously differentiated functions; $N_i(\delta)$ 2π-periodical continuous differentiated function srelative to $\delta_1,\ldots,\delta_l$;

for summands $N_i(\delta)$– the conditions of integrability are carried out (3); T is duration of transient process which is considered as specified.

The system of the equations (12) are supplemented with initial conditions

$$\delta_i(0) = \delta_{i0}, \quad S_i(0) = S_{i0}, \ i = 1, \dots, l. \tag{13}$$

Terminal values $\delta(T)$, $S(T)$ are beforehand unknown, so they also should be determined.

The following theorem is valid.

Theorem 1. For optimization of controls

$$\nu_i^0(S_i,t) = -[w_{\nu_i}]^{-1} exp\{-\gamma_i t\} S_i, \quad i = 1, \dots, l,$$

and their corresponding solution $\{\delta^0(t), S^0(t)\}$ of system (12)–(13), it is necessary and sufficient, that

$$\Lambda(\delta(T), S(T)) = K(\delta(T), S(T)),$$

$$w_{s_i}(t) = 2D_i exp\{-\gamma_i t\} + [w_{\nu_i}]^{-1} \ exp\{-2\gamma_i t\} > 0, \quad i = 1, \dots, l,$$

where

$$K(\delta,S) = 0.5 \sum_{i=1}^{l} \left[H_i S_i^2 + \int_0^{\delta_i} f_i(\delta_i)\, d\delta_i \right] + + \sum_{\substack{i=1,\\ \delta_j=0,\\ j>i}}^{l} \int_0^{\delta_i} N_i(\delta_1, \dots, \delta_{i-l}, \ \delta_{i+1}, \dots, \delta_l) d\xi_i, \tag{14}$$

Bellman-Krotov function and besides,

$$J(v^0) = \min_v J(v) = K(\delta^0, S^0).$$

The proof of the theorem 1 is received, applying the procedure of construction of Bellman-Krotov function suggested in 2 (see proof of lemma).

4 THE OPTIMAL CONTROL PROBLEM OF POWER OF STEAM TURBINES

One of the models describing the transients in the electrical system is the following system of differential equations (I. I. Blekhman, 1997)–(A. A. Gorev, 2004):

$$\frac{d\delta_i}{dt} = S_i, \quad H_i \frac{dS_i}{dt} = -D_i S_i - E_i^2 Y_{ii} \sin\alpha_{ii} - -P_i \sin(\delta_i - \alpha_i) - \sum_{j=1, j\neq i}^{l} P_{ij} \sin(\delta_{ij} - \alpha_{ij}) + u_i, \quad i \in \overline{1,l}, \quad t \in (0,T), \tag{15}$$

$$\delta_{ij} = \delta_i - \delta_j, \quad P_i = E_i U Y_{i,n+1}, \ \ P_{ij} = E_i E_j Y_{ij},$$

where δ_i is an angle of rotor turn of i-th generator with respect to some synchronous rotational axis (the axis of rotation of bus-bares of constant voltage, which rotates at the speed of 50 r./sec.; S_i is sliding of i-th generator; H_i is an inertial constant of the i-th machine; u_i is mechanical power, which are led to the generator; E_i – EMF of the i-th machine; Y_{ij}– mutual conductivity of i-th and j-th branches of the system; $U = const$ is bus-bar voltage of constant voltage; $Y_{i,n+1}$–defines the bond (conductivity) of i-th generator with bus-bar of constant voltage; $D_i = const \geq 0$ is mechanical damping; α_{ij}, α_i are constant values which take into account the influence of active resistance in the stator chains of generators.

The complexity of the analysis of the model (15) is taking into account of α_{ij}, $\alpha_{ij} = \alpha_{ji}$. Since in this case $\delta_{ij} = -\delta_{ji}$, then the model (15) is not conservative; one fails to build the Lyapunov's function in the form of the first integral for it. The system is called positional model.

Let the state variables and control variables in the steady after-emergency regime be equal to:

$$\overline{S_i} = 0, \ \overline{\delta_i} = \delta_i^F, \quad \overline{u_i} = u_i^F, \ \ i = \overline{1,l}. \tag{16}$$

To obtain the system of the disturbed movement we should proceed to the equations of deviations assuming:

$$S_i = \Delta S_i, \quad \delta_i = \delta_i^F + \Delta\delta_i, \quad u_i = u_i^F + \Delta u_i, \quad i = \overline{1,l}. \tag{17}$$

Further, for the convenience we denote again the variables Δu_i, $\Delta\delta_i$, ΔS_i by u_i, δ_i, S_i and from (15),(16) we obtain:

$$\frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = \frac{1}{H_i}\left[-D_i S_i - f_i(\delta_i) - N_i(\delta) + M_i(\delta) + u_i\right], \quad i = \overline{1,l}, \quad t \in (0,T), \tag{18}$$

where

$$f_i(\delta_i) = P_i \left[\sin\left(\delta_i + \delta_i^F - \alpha_i\right) - \sin\left(\delta_i^F - \alpha_i\right)\right],$$

$$N_i(\delta) = \sum_{j=1, j\neq i}^{l} \overline{N}_{ij}(\delta_1, \dots, \delta_l) = = \sum_{j=1, j\neq i}^{l} \Gamma_{ij}^1 \left[\sin\left(\delta_{ij} + \delta_{ij}^F\right) - \sin\delta_{ij}^F\right],$$

$$M_i(\delta) = \sum_{j=1, j\neq i}^{l} \overline{M}_{ij}(\delta_1, \dots, \delta_l) =$$

$$= \Gamma_{ij}^2\left[\cos\left(\delta_{ij}+\delta_{ij}^F\right)-\cos\delta_{ij}^F\right],$$
$$\Gamma_{ij}^1 = P_{ij}\cos\alpha_{ij}, \quad \Gamma_{ij}^2 = P_{ij}\sin\alpha_{ij}, \quad P_{ij}=P_{ji},$$
$$\Gamma_{ij}^k=\Gamma_{ji}^k, \quad k=1,2.$$

The control will be searched in the form of:

$$u_i = \nu_i - M_i(\delta), \quad i=\overline{1,l}, \tag{19}$$

where the function ν_i should be determined.

It is required to minimize the functional

$$\begin{gathered} J(\nu) = J(\nu_1,\ldots,\nu_l) = \\ = 0.5\sum_{i=1}^{l}\int_0^T\left(w_{s_i}S_i^2 + w_{\nu_i}\nu_i^2\right) * \\ *\exp\{\gamma_i t\}\,dt + \Lambda(\delta(T),S(T)), \end{gathered} \tag{20}$$

under the conditions (18)–(19), where w_{s_i}, w_{ν_i} are weight coefficients, correspondingly positive functions and constants; $f_i(\delta_i)$ is 2π-periodic continuously differentiable function; $N_i(\delta)$ 2π-periodic continuously differentiable function with respect to δ_{ij}; for $N_i(\delta)$ the integrability condition of the type (3) is realized; T is duration of the transition process which is considered to be given. In addition, the initial conditions are given:

$$\delta_i(0)=\delta_{i0}, \quad S_i(0)=S_{i0}, \quad i=\overline{1,l}, \tag{21}$$

and the values $\delta_i(T)$, $S_i(T)$ are unknown.

On the basis of the lemma from 2 we obtain

Theorem 2. In order to optimal control

$$\nu_i^0(S_i,t) = -\left[w_{\nu_i}\right]^{-1}\exp\{-\gamma_i t\}S_i, \quad i=\overline{1,l},$$

and the corresponding solution $\left\{\delta^0(t),\ S^0(t)\right\}$ of system (18), (20) it is necessary and sufficient that

$$\Lambda(\delta(T),S(T)) = K(\delta(T),S(T)),$$
$$w_{s_i}(t) = 2D_i\exp\{-\gamma_i t\} + \left[w_{\nu_i}\right]^{-1}\exp\{-2\gamma_i t\} > 0,$$
$$i=\overline{1,l},$$
$$K(\delta,S) = 0.5\sum_{i=1}^{l}\left[H_iS_i^2 + \int_0^{\delta_i} f_i(\delta_i)\,d\delta_i\right] +$$
$$\sum_{\substack{i=1,\\ \delta_j=0,\\ j>i}}^{l}\int_0^{\delta_i} N_i(\delta_1,\ldots,\delta_{i-l},\xi_i,\delta_{i+1},\ldots,\delta_l)\,d\xi_i$$

Bellman-Krotov function, and besides,

$$J\left(\nu^0\right) = \min_{\nu} J(\nu) = K\left(\delta^0,S^0\right). \tag{22}$$

It should be noted that in the proof of the theorem 2, assumptions (9) from the lemma take the form:

$$K_{\delta_i}S_i = \frac{K_{S_i}}{H_i}\left[f_i(\delta_i)+N_i(\delta)\right], \tag{23}$$

i.e.

$$K_{S_i} = H_iS_i, \quad K_{\delta_i} = f_i(\delta_i)+N_i(\delta), \quad i=\overline{1,l}.$$

5 THE NUMERICAL EXAMPLE. OPTIMAL CONTROL OF THE MOVEMENT OF TWO-MACHINE ELECTRIC POWER SYSTEM

In the system (15) we accept that $i=1,2$, and assume that the mechanical damping is absent, i.e., coefficients D_1, D_2 are equal to zero. According to the relations (15)–(21) the optimal control problem takes the form (I. I. Blekhman, 2004):

$$\begin{gathered} J(\nu) = J(\nu_1,\nu_2) = 0.5\sum_{i=1}^{2}\int_0^T\left(S_i^2+0.1\nu_i^2\right) * \\ *\exp\{\gamma_i t\}\,dt + 0.5\left[\delta^2(T)+S^2(T)\right], \end{gathered} \tag{24}$$

$$\begin{gathered} \frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = \frac{1}{H_i}\left[-f_i(\delta_i) - N_1(\delta) + \nu_i\right], \\ i=1,2, \end{gathered} \tag{25}$$

where

$$f_i(\delta_i) = P_i\left[\sin\left(\delta_i+\delta_i^F-\alpha_i\right) - \sin\left(\delta_i^F-\alpha_i\right)\right], i=1\,2,$$
$$N_1(\delta) = \Gamma_1\left[\sin\left(\delta_{12}+\delta_{12}^F\right) - \sin\delta_{12}^F\right],$$
$$M_1(\delta) = \Gamma_2\left[\cos\left(\delta_{12}+\delta_{12}^F\right) - \cos\delta_{12}^F\right],$$
$$\delta_{12}^F = \delta_1^F - \delta_2^F,$$
$$\Gamma_1 = P_{12}\cos\alpha_{12}, \ \ \Gamma_2 = P_{12}\sin\alpha_{12},$$
$$\delta_{12} = \delta_1-\delta_2, \ \ \delta_{21} = -\delta_{12}.$$

The numerical data of the system (25):

$$\gamma_1=\gamma_2=0.1;$$
$$\alpha_1=-0.052; \quad \alpha_2=-0.104;$$
$$H_1=2135; \quad H_2=1256; \quad P_1=0.85;$$
$$P_2=0.69; \quad P_{12}=0.9;$$
$$\delta_1^F=0.827; \quad \delta_2^F=0.828, \quad \alpha_{12}=-0.078;$$

and the initial data:

$$\begin{gathered} \delta_1(0)=0.18; \quad \delta_2(0)=0.1; \\ S_1(0)=0.001; \quad S_2(0)=0.002. \end{gathered} \tag{26}$$

According to formula (22) Bellman-Krotov function and its partial derivatives will have the form:

$$K(\delta,S) \approx 1067.5S_1^2 + 628S_2^2 + 2.004 - \cos(\delta_1+0.879) -$$
$$-0.77\delta_1 - \cos(\delta_2+0.932) - 0.83\delta_2,$$
$$K_{\delta_1}(\delta,S) = 0.425\left[\sin(\delta_1+0.879) - \sin 0.879\right] +$$
$$+0.9\sin(\delta_1-\delta_2),$$
$$K_{\delta_2}(\delta,S) = 0.345\left[\sin(\delta_2+0.932) - \sin 0.932\right] +$$
$$+0.9\sin(\delta_1-\delta_2),$$

$$K_{S_1}(\delta, S) = 2135 S_1, \quad K_{S_2}(\delta, S) = 1256 S_2.$$

Controls on the feedback principle (synthesis) are determined by the formulas:

$$\begin{aligned} \nu_1^0(\delta, S) &= -\exp\{-0.1t\} S_1, \\ \nu_2^0(\delta, S) &= -\exp\{-0.1t\} S_2, \end{aligned} \tag{27}$$

herewith the assumption was used (23). The results of numerical calculation of the optimal pair of vector-functions state-control $\{ \delta^0(t),\ S^0(t);\ \nu^0(t) = \nu^0\left(\delta^0(t),\ S^0(t)\right\}$ the obtained from (25)–(27), as shown on the Figures 1 and 2.

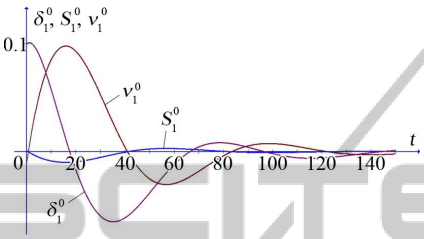

Figure 1.

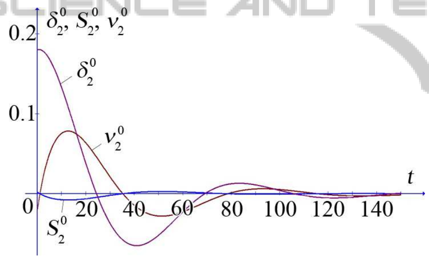

Figure 2.

6 CONCLUSIONS

On the basis of the theoretical results, the computational experiments were conducted, which showed the sufficient efficacy of the proposed procedure for constructing the function of Bellman-Krotov and the synthesising optimal control for the given power system.

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