Control Algorithm for a Cooperative Robotic System in Fault Conditions

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Abstract: This paper expounds a control procedure and a control algorithm with two levels to solve the control problem of a cooperating multi-arm robotic system. This system is composed of a structure like a gripper with n fingers manipulating a usual object. The control system is a hierarchical system. The problems of the inter-coordination and the force distribution are decided by the top tier coordinator which brings together all the appropriate information. This information is directed towards the n inferior level subsystems. The local control is solved by assigning the local controllers based on the inverse model method. The robotic structure is either in a correct position when possible, or by minimising the movements and using the adequate commands to the functional joints, in an acceptable proximity position of the desired co-ordinates. It is also proposed a synthesis of the commands. The paper presents a workspace analysis and an algorithm for the actuators in the terms of a good working for finding the optimal motions by blocking or unblocking some robotic joints.

1 INTRODUCTION

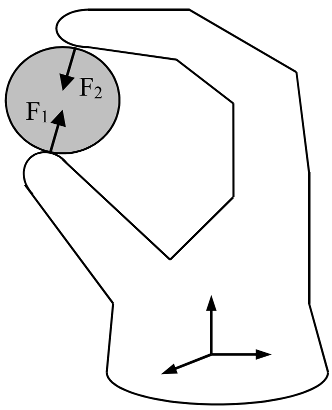

Figure 1: A cooperative robotic system.

There are an important number of aspects in control of the robotic systems with cooperative tasks in real time (Figure 1), as dispatching of mobile robot legs, mechanical hand fingers, dispatching two robotic arms in co-operant tasks, etc. The two aspects of control system are as follows: the first one is the general coordination that presumes dispatching of a couple of robotic elements to assure a required trajectory of the tip and the second one is the local control problem which delivers the control of the individual components of the arms (fingers, legs) to generate the appropriate position and orientation. The force allocation must be determined, mentioning that the motion is completely specified and the internal forces/torques which generate this motion must be found. A two-level hierarchical control system (Cheng and Orin, 1991a), (Cheng and Orin, 1991b), (Cheng, 1995), (Zheng and Luh, 1998), (Khatib, 1996), (Wang, 1996) is used to determine the solution for this control problem. All the appropriate information is gathered by the top-tier system and is determined by the inter-chain dispatching problem, the force allocation problem. The problem is divided into a lot of inferior-level problems, one for every element of the robotic system. An algorithm for establish the inputs of the control system (joint variables) on lower-level in fault conditions and without, is also expounds.

Analysing the work-envelope geometry it is essential the locus of points in $\boldsymbol{R}^3$ that could be reachable by the tool tip. If the tool tip is considered a reference point, it must include the effects of both the major axes used to position the wrist and the minor axes used to orient the tool.

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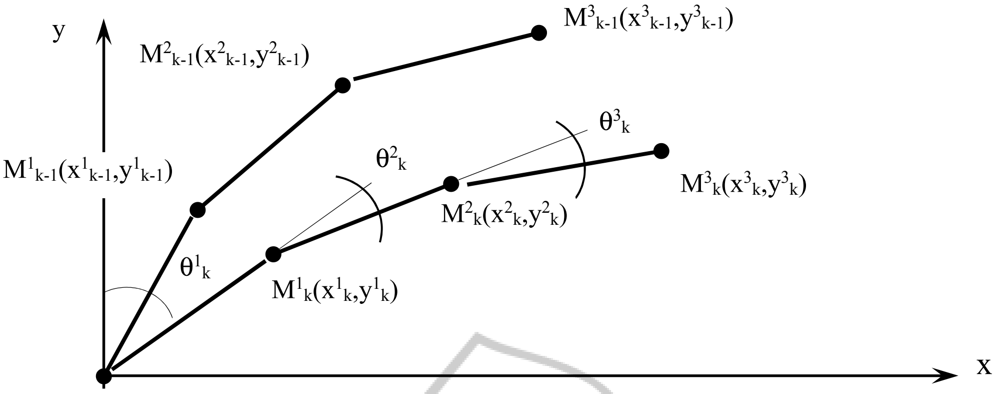

Figure 2: The finger.

Considering the shape or geometry of the work envelope as a subset of $\boldsymbol{R^3}$, although this work envelope varies from robot to robot, it can be viewed as well within the framework of joint space $\boldsymbol{R^n}$.

The work envelope is typically characterised by bounds on linear combinations of joint variables related to joint space. The constraints of this nature generate a convex polyhedron in $\boldsymbol{R^n}$ named the joint-space work envelope. Let $q_{\min}$ and $q_{\max}$ be the joint limits vectors in $\boldsymbol{R^n}$ and let A be an $m \times n$ joint coupling matrix. The set of all values of the joint variables q is called the joint-space work envelope. It is denoted Q and is of the form:

$$Q = \left\{ q \in R^n : q^{\min} \leq Aq \leq q^{\max} \right\} \qquad (1)$$

The relation A = I represents no inter axis coupling. The joint-space work envelope Q is the locus of points in $\boldsymbol{R^3}$ that can be reached by the tool tip.

The locus of the points reachable from at least one tool orientation is referred to as the total work envelope, or simply the work envelope and the locus of points reachable from an arbitrary tool orientation is called the dextrous work envelope (Shilling, 1993), (Beni and Hackwood, 1985), (Craig, 1990).

Let's consider the RRR planar robotic structure as it is shown in Figure 2. The structure presented in Figure 2 is a non-redundant structure because the joints variable number $\left(\theta^1_k, \theta^2_k, \theta^3_k\right)$ as well as the operational co-ordinate number (x, y and θ_z) are equal to 3. The structure can be a finger, a leg, a robotic arm, etc. The point $M^3_k\left(x^3_k, y^3_k\right)$ is belonging to a specified trajectory and their values are known. The index k represents the actual step in the evolution on the trajectory. So:

$$q_k = \left[q^1_k, q^2_k, q^3_k\right]^T = \left[\theta^1_k, \theta^2_k, \theta^3_k\right]^T \qquad (2)$$

For simplicity we consider that the length of the 3 arm elements is the same: $l_1 = l_2 = l_3 = l$. In this paper is proposed to establish the values of the angles $\theta^1_k, \theta^2_k, \theta^3_k$ as well as the differences $\Delta\theta^1_k, \Delta\theta^2_k, \Delta\theta^3_k$ which are the base for generating the commands to the actuators in the terms of a good working (finding the optimal motions) and in terms of the blocking of some robotic segments.

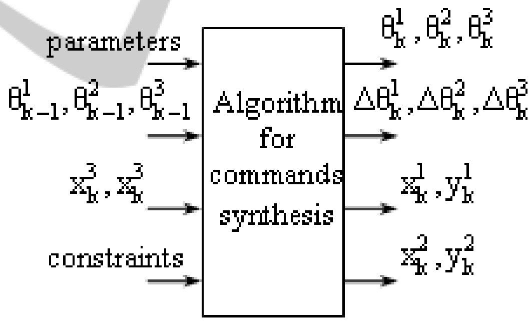

Figure 3: The inputs and outputs of the algorithm.

Practically is an inverse kinematics problem. The input and output variables of the proposed algorithm are shown in Figure 3. The angles $\theta^1_k, \theta^2_k, \theta^3_k$, the displacements $\Delta\theta^1_k, \Delta\theta^2_k, \Delta\theta^3_k$ and the co-ordinates of the points M_1 and M_2 (which are necessary in workspace analysis for avoiding some existing obstacles) are determined on the base of the angles $\theta^1_{k-1}, \theta^2_{k-1}, \theta^3_{k-1}$ from the previous step, on the base of the desired co-ordinates x^3_k, y^3_k of the arm tip and on the base of some information related of the physical structure (segments length, maximal and minimal limits of the angle displacement and the blocking status of some segments). The algorithm proposed by the authors allows, if the blocking exists, either a correct positioning by other displacements of the unblocked segments (if it is possible) or a positioning in an acceptable proximity of the desired coordinates by minimising of optimal criteria (Iancu et al., 1999), (Vinatoru et al., 1998).

2 ALGORITHM FOR UNBLOCKED JOINTS

Let's consider the robotic arm as shown in Figure 1. We wish the positioning of the arm tip in the point $M_k^3\left(x_k^3, y_k^3\right)$ without any specification of the orientation. The co-ordinates of the point M_k^3 are:

$$x_k^3 = l\sin\theta_k^1 + l\sin\left(\theta_k^1 + \theta_k^2\right) + l\sin\left(\theta_k^1 + \theta_k^2 + \theta_k^3\right)$$
$$y_k^3 = l\cos\theta_k^1 + l\cos\left(\theta_k^1 + \theta_k^2\right) + l\cos\left(\theta_k^1 + \theta_k^2 + \theta_k^3\right) \quad (3)$$

Because the inverse kinematics problem has infinity of solutions, let's consider some supplementary conditions imposed by an optimal working, the avoiding of blocking, a. s. o. (e.g.:

$\theta_k^1 + \theta_k^2 + \theta_k^3 = \theta_k^*$ =constant; $\theta_k^1 = \theta_k^2 = \theta_k^3$;

$\theta_k^3 = \theta_k^*$ - imposed, a. s. o.). (4)

In some situations, when the passing from the point M_{k-1}^3 to the next point M_k^3 because of its advantageous position is made, it is not necessary the movement of all the 3 elements, being possible an energetic consumption economy.

If we note L_1, L_2, and L_3 the elements having the length l, A -"Active" status and B –"Blocked" status, 1 logic - the movement status (operative) of the active element and 0 logic - the non-operative status of the active element, all the possible situations for unblocked joints above mentioned are:

(L_1 L_2 L_3) $\rightarrow$ (0 0 0), (0 0 1), (0 1 0), (1 0 0), (0 1 1), (1 0 1), (1 1 0), (1 1 1).

The proposed algorithm is presented in the following step sequence:

STEP 1: The robot parameter set is read:

l, θ_0^i, $\theta_{\min}^i$, $\theta_{\max}^i$, i = 1, 2, 3.

STEP 2: k = 1

STEP 3: x_k^3, y_k^3, θ_k^* are specified.

STEP 4: If (L_1 L_2 L_3) = (A A A)
then Jump to STEP 5 else Jump to STEP 13

STEP 5: If $\left(x_k^3\right)^2 + \left(y_k^3\right)^2 \le 9l^2$
then Jump to STEP 7, else Jump to STEP 6

STEP 6: "IMPOSSIBLE TO REACH THE POINT (operational space too small)" is displayed. The information is transferred to upper level controller. Jump to STEP 3.

STEP 7: If $\left(x_k^3 - x_{k-1}^2\right)^2 + \left(y_k^3 - y_{k-1}^2\right) = l^2$
then Jump to Ad001, else Jump to STEP 8

STEP 8:
If $\left(x_k^3 - x_{k-1}^1\right)^2 + \left(y_k^3 - y_{k-1}^1\right)^2 = \left(2l\cos\left(\theta_{k-1}^3/2\right)\right)^2$
then Jump to Ad010, else Jump to STEP 9

STEP 9: $\alpha = \operatorname{atan}\dfrac{\sin\theta_{k-1}^2 - \sin\theta_{k-1}^3}{1+\cos\theta_{k-1}^2 + \cos\theta_{k-1}^3}$

$l_k^{123} = l\left[\cos\left(\theta_{k-1}^2 - \alpha\right) + \cos\alpha + \cos\left(\theta_{k-1}^3 + \alpha\right)\right]$

If $\left(x_k^3\right)^2 + \left(y_k^3\right)^2 = \left(l_k^{123}\right)^2$
then Jump to Ad100, else Jump to STEP 10

STEP 10: If $\left(x_k^3 - x_{k-1}^1\right)^2 + \left(y_k^3 - y_{k-1}^1\right)^2 \le 4l^2$
then Jump to Ad011, else Jump to STEP 11

STEP 11: $l_k^{12} = 2l\cos\left(\theta_{k-1}^2/2\right)$

If $\left(l_k^{12} - l\right)^2 \le \left(x_k^3\right) + \left(y_k^3\right) \le \left(l_k^{12} + l\right)^2$
then Jump to Ad101, else Jump to STEP 12

STEP 12: $l_k^{23} = 2l\cos\left(\theta_{k-1}^3/2\right)$

If $\left(l_k^{23} - l\right)^2 \le \left(x_k^3\right) + \left(y_k^3\right) \le \left(l_k^{23} + l\right)$
then Jump to Ad110, else Jump to Ad111

3 ALGORITHM FOR BLOCKED JOINTS

If during the movement process on the trajectory, one or more joint are blocked (the information is given by the transducers), then the control system try to control the arm to continue on the trajectory by the rest of the joints. The possible situations are:

(L_1 L_2 L_3) $\rightarrow$ (0 0 B), (0 1 B), (1 0 B), (1 1 B), (0 B 0), (0 B 1), (1 B 0), (1 B 1), (B 0 0), (B 0 1), (B 1 0), (B 1 1), (0 B B), (1 B B), (B 0 B), (B 1 B), (B B 0), (B B 1), (B B B).

The step sequence of the algorithm is:

STEP 13: If (L_1 L_2 L_3) = (A A B)
then Jump to STEP 14, else Jump to STEP 17

STEP 14: If
$$\left(x_k^3 - x_{k-1}^1\right)^2 + \left(y_k^3 - y_{k-1}^1\right)^2 = \left(2l\cos\left(\theta_{k-1}^3/2\right)\right)^2$$
then Jump to Ad010 (for 01B status)

else Jump to STEP 15

STEP 15: $\alpha = \text{atan}\dfrac{\sin\theta_{k-1}^2 - \sin\theta_{k-1}^3}{1+\cos\theta_{k-1}^2+\cos\theta_{k-1}^3}$

$$l_k^{123} = l\left[\cos\left(\theta_{k-1}^2-\alpha\right)+\cos\alpha+\cos\left(\theta_{k-1}^3+\alpha\right)\right]$$

If $\left(x_k^3\right)^2+\left(y_k^3\right)^2=\left(l_k^{123}\right)^2$

then Jump to Ad100 (for 10B status)

else Jump to STEP 16

STEP 16: $l_k^{23} = 2l\cos\left(\theta_{k-1}^3/2\right)$

If $\left(l_k^{23}-l\right)^2 \le \left(x_k^3\right)+\left(y_k^3\right) \le \left(l_k^{23}+l\right)$

then Jump to Ad110 (for 11B status)

else Jump to STEP 31

STEP 17: If (L_1 L_2 L_3) = (A B A)

then Jump to STEP 18

else Jump to STEP 21

STEP 18: If $\left(x_k^3-x_{k-1}^2\right)^2+\left(y_k^3-y_{k-1}^2\right)=l^2$

then Jump to Ad001 (for 0B1 status)

else Jump to STEP 19

STEP 19: $\alpha = \text{atan}\dfrac{\sin\theta_{k-1}^2 - \sin\theta_{k-1}^3}{1+\cos\theta_{k-1}^2+\cos\theta_{k-1}^3}$

$$l_k^{123} = l\left[\cos\left(\theta_{k-1}^2-\alpha\right)+\cos\alpha+\cos\left(\theta_{k-1}^3+\alpha\right)\right]$$

If $\left(x_k^3\right)^2+\left(y_k^3\right)^2=\left(l_k^{123}\right)^2$

then Jump to Ad100 (for 1B0 status)

else Jump to STEP 20

STEP 20: $l_k^{12} = 2l\cos\left(\theta_{k-1}^2/2\right)$

If $\left(l_k^{12}-l\right)^2 \le \left(x_k^3\right)+\left(y_k^3\right) \le \left(l_k^{12}+l\right)^2$

then Jump to Ad101 (for 1B1 status)

else Jump to STEP 31

STEP 21: If (L_1 L_2 L_3) = (B A A)

then Jump to STEP 22, else Jump to STEP 25

STEP 22: If $\left(x_k^3-x_{k-1}^2\right)^2+\left(y_k^3-y_{k-1}^2\right)=l^2$

then Jump to Ad001 (for B01 status)

else Jump to STEP 23

STEP 23: If

$$\left(x_k^3-x_{k-1}^1\right)^2+\left(y_k^3-y_{k-1}^1\right)^2=\left(2l\cos\left(\theta_{k-1}^3/2\right)\right)^2$$

then Jump to Ad010 (for B10 status)

else Jump to STEP 24

STEP 24: If $\left(x_k^3-x_{k-1}^1\right)^2+\left(y_k^3-y_{k-1}^1\right)^2 \le 4l^2$

then Jump to Ad011 (for B11 status)

else Jump to 31

STEP 25: If (L_1 L_2 L_3) = (A B B)

then Jump to STEP 26, else Jump to STEP 27

STEP 26: $\alpha = \text{atan}\dfrac{\sin\theta_{k-1}^2 - \sin\theta_{k-1}^3}{1+\cos\theta_{k-1}^2+\cos\theta_{k-1}^3}$

$$l_k^{123} = l\left[\cos\left(\theta_{k-1}^2-\alpha\right)+\cos\alpha+\cos\left(\theta_{k-1}^3+\alpha\right)\right]$$

If $\left(x_k^3\right)^2+\left(y_k^3\right)^2=\left(l_k^{123}\right)^2$

then Jump to Ad100 (for 1BB status)

else Jump to STEP 31

STEP 27: If (L_1 L_2 L_3) = (B A B)

then Jump to STEP 28, else Jump to STEP 29

STEP 28: If

$$\left(x_k^3-x_{k-1}^1\right)^2+\left(y_k^3-y_{k-1}^1\right)^2=\left(2l\cos\left(\theta_{k-1}^3/2\right)\right)^2$$

then Jump to Ad010 (for B1B status)

else Jump to STEP 31

STEP 29: If (L_1 L_2 L_3) = (B B A)

then Jump to STEP 30, else Jump to STEP 31

STEP 30: If $\left(x_k^3-x_{k-1}^2\right)^2+\left(y_k^3-y_{k-1}^2\right)=l^2$

then Jump to Ad001 (for BB1 status)

else Jump to 31

STEP 31: "IMPOSIBLE ACTION (because of blocking)" is displayed. The information is transferred to upper level controller. STOP.

4 VERIFICATION OF THE CONSTRANTS

Different constraints can exist and these must be certified before generating the outputs of the algorithm. The step sequence for that is:

STEP 32: If

$$\left(\theta_{\min}^1,\theta_{\min}^2,\theta_{\min}^3\right) \le \left(\theta_k^1,\theta_k^2,\theta_k^3\right) \le \left(\theta_{\max}^1,\theta_{\max}^2,\theta_{\max}^3\right)$$

then Jump to STEP 33, else Jump to STEP 36

STEP 33: $\Delta\theta_k^i = \theta_k^i - \theta_{k-1}^i$; i = 1, 2, 3.

$$x_k^1 = l\sin\theta_k^1;\; y_k^1 = l\cos\theta_k^1$$

$$x_k^2 = l\sin\theta_k^1 + l\sin\left(\theta_k^1+\theta_k^2\right);$$

$$y_k^2 = l\cos\theta_k^1 + l\cos\left(\theta_k^1+\theta_k^2\right)$$

STEP 34: k = k+1

STEP 35: Jump to STEP 3

STEP 36: "IMPOSIBLE DEPLACEMENT FOR L_i (because of constraints)" is displayed. The information is transferred to upper level controller.

Jump to STEP 3

5 DETERMINATION OF THE INTERNAL VARIABLES

To different addresses there are statements of program which determine the expressions of the internal variables for the situations (with blocked and unblocked joints) above mentioned:

Ad001: $\theta_k^1 = \theta_{k-1}^1$; $\theta_\kappa^2 = \theta_{\kappa-1}^2$;

$$\theta_k^3 = \operatorname{atan}\frac{x_k^3 - x_{k-1}^2}{y_k^3 - y_{k-1}^2} - \left(\theta_k^1 + \theta_k^2\right)$$

Jump to STEP 32

Ad010: $\theta_k^1 = \theta_{k-1}^1$; $\theta_k^3 = \theta_{k-1}^3$;

$$\theta_k^2 = \theta_{k-1}^2 + \left(\operatorname{atan}\frac{x_k^3 - x_{k-1}^1}{y_k^3 - y_{k-1}^1} - \operatorname{atan}\frac{x_{k-1}^3 - x_{k-1}^1}{y_{k-1}^3 - y_{k-1}^1}\right)$$

Jump to STEP 32

Ad100: $\theta_k^2 = \theta_{k-1}^2$; $\theta_k^3 = \theta_{k-1}^3$;

$$\theta_k^1 = \theta_{k-1}^1 + \left(\operatorname{atan}\frac{x_k^3}{y_k^3} - \operatorname{atan}\frac{x_{k-1}^3}{y_{k-1}^3}\right)$$

Jump to STEP 32

Ad011: $\theta_k^1 = \theta_{k-1}^1$

$$c_k^3 = \frac{\left(x_k^3 - x_{k-1}^1\right)^2 + \left(y_k^3 - y_{k-1}^1\right)^2 - 2l^2}{2l^2}$$

$$\theta_k^3 = \operatorname{atan}\frac{\sqrt{1 - \left(c_k^3\right)^2}}{c_k^3}$$

$$s_k^{21} = \frac{x_k^3 - x_{k-1}^1}{2l} - \frac{\sin\theta_k^3\left(y_k^3 - y_{k-1}^1\right)}{2l\left(1 + \cos\theta_k^3\right)}$$

$$\theta_k^2 = \operatorname{atan}\frac{s_k^{21}}{\sqrt{1 - \left(s_k^{21}\right)^2}} - \theta_{k-1}^1$$

Jump to STEP 32

Ad101: $\theta_k^2 = \theta_{k-1}^2$

$$c_k^{3*} = \frac{\left(x_k^3\right)^2 + \left(y_k^3\right)^2 - \left(l_k^{12}\right)^2 - l^2}{2ll_k^{12}}$$

$$\theta_k^{3*} = \operatorname{atan}\frac{\sqrt{1 - \left(c_k^{3*}\right)^2}}{c_k^{3*}} \text{ ; } \theta_k^3 = \theta_k^{3*} - \theta_{k-1}^2 / 2$$

$$b = 2\left(l_k^{12} + l\cos\theta_k^{3*}\right) \text{ ; } c = l\sin\theta_k^{3*}$$

$$\theta_k^1 = 2\operatorname{atan}\frac{b \pm \sqrt{b^2 - 4\left[\left(x_k^3\right)^2 - c^2\right]}}{2\left(x_k^3 + c\right)} - \frac{\theta_{k-1}^2}{2}$$

Jump to STEP 32

Ad110: $\theta_k^3 = \theta_{k-1}^3$

$$c_k^{23} = \frac{\left(x_k^3\right)^2 + \left(y_k^3\right)^2 - \left(l_k^{23}\right)^2 - l^2}{2ll_k^{23}} \text{ ;}$$

$$\theta_k^2 = \operatorname{atan}\frac{\sqrt{1 - \left(c_k^{23}\right)^2}}{c_k^{23}} - \frac{\theta_{k-1}^3}{2}$$

$$b = \frac{\left(x_k^3\right)^2 + \left(y_k^3\right)^2 - \left(l_k^{23}\right)^2 + l^2}{2l} \text{ ;}$$

$$\theta_k^1 = 2\operatorname{atan}\frac{x_k^3 \pm \sqrt{\left(x_k^3\right)^2 + \left(y_k^3\right)^2 - b^2}}{b + y_k^3}$$

Jump to STEP 32

Ad111: $r_k^1 = x_k^3 - l\sin\theta_k^*$; $r_k^2 = y_k^3 - l\cos\theta_k^*$

$$r_k^3 = \frac{\left(r_k^1\right)^2 + \left(r_k^2\right)^2}{2l} \text{ ; } c_k^2 = \frac{r_k^3}{l} \text{ ;}$$

$$\theta_k^2 = \operatorname{atan}\frac{\sqrt{1 - \left(c_k^2\right)^2}}{c_k^2}$$

$$\theta_k^1 = \operatorname{atan}\left(2\frac{r_k^1 \pm \sqrt{\left(r_k^1\right)^2 + \left(r_k^2\right)^2 - \left(r_k^3\right)^2}}{r_k^2 + r_k^3}\right)$$

$\theta_k^3 = \theta_k^* - \left(\theta_k^1 + \theta_k^2\right)$; Jump to STEP 32

6 A MODEL FOR COOPERATIVE ROBOTIC SYSTEM

An example of multiple-chain robotic system is depicted in Figure 1. The robotic system forms closed-kinematics loops. The individual chains are closely coupled with one another through the load (Iancu and Vinatoru, 1999). The dynamic relations for each chain of the system (finger) are:

$$\mathrm{M}^{\mathrm{i}}\ddot{\mathrm{q}}^{\mathrm{i}} + \mathrm{C}^{\mathrm{i}}\dot{\mathrm{q}}^{\mathrm{i}} + \mathrm{D}^{\mathrm{i}}(\mathrm{q}^{\mathrm{i}})\,\mathrm{F}^{\mathrm{i}} = \mathrm{T}^{\mathrm{i}} \tag{5}$$

where M^i, C^i are (n^i x n^i) contact diagonal matrixes, D is (n^i x 2) non-linear matrix.

$$F^i = \mathrm{col}\left(F_x^i, F_Y^i\right)$$
$$q^i = \mathrm{col}\left(q_1^i \ldots q_{n^i}^i\right) \quad (6)$$
$$T = \mathrm{col}\left(T_1^i \ldots T_{n^i}^i\right)$$

In the relation (5), F^i assures the object motion on the established trajectory. The uncertainty of the object specificates an uncertainty of the force F^i. F^{Mi} is an estimation of the force upper bound. We assume:

$$\left|F^{Mi} - F^i\right|_j \le \rho_j ; j = 1,2\ldots \quad (7)$$

$$\sum_{i=1}^{n} \tau^i = \sum_{i=1}^{n} F_x^i \left(-l_i \mathrm{sin} q^i\right) + \sum_{i=1}^{n} F_y^i \left(l_i \mathrm{cos} q^i\right) \; i=1, 2.. \quad (8)$$

We employ the symbols: q^j - inner generalized coordinate of finger i, $t \in [0, t_f]$, τ^i = the moment vector which establishes the required trajectory of the object. All these variables are related to the coordinate frame of the finger i. All the relations are closely coupled through the terms τ^i, $F_X{}^i$, $F_Y{}^i$ where all of these terms define the required comportment. We use a hierarchical control scheme with two-level (Cheng, 1995) for this robotic system. The control strategy is to decouple the control system into k control sub-systems that are controlled by the upper level control system. The task of the top tier coordinator is to collect all the appropriate information to establish the force distribution and then to decide this constrained, optimization problem. The optimal solutions for the contact forces F^i are established. The optimal contact forces became the inputs for the second level subsystems. We use the notations F^0- the resultant force vector which acts to load related to the inertial coordinate frame (R_0), ${}^0H_i$ - the partial spatial transform from the frame of the finger i to the frame (R_0). We consider a hard point contact with friction and that the force balance relation on the load is:

$$F^0 = \sum {}^0 H_i F^i \quad (9)$$

The load dynamic relations have the form

$$M_0 \ddot{r} = GF^0 \quad (10)$$

where M_0 is inertial matrix of the load and **r** is the load coordinate vector

$$\mathbf{r} = (x, y, \theta)^T \quad (11)$$

and r(t) depicts the required trajectory. The inequality constraints which define the friction constraints and the maximum force constraints may be adjoint to (9):

$$\sum P^i F^i \le Q \quad (12)$$

where P^i is a coefficient matrix of inequality constraints and Q is a boundary-value vector of inequality constraints (Mason, 1981). The problem of the contact forces F^i can be considered as an optimal control problem if an optimal index is associated to the equations (9) - (12)

$$\Psi = \sum A^i F^i \quad (13)$$

This situation is answered in the papers: (Cheng, 1995), (Zheng and Luh, 1988), (Khatib, 1996), (Wang, 1996) by the general procedures of the optimization or by the specific methods (Cheng and Orin, 1991a), (Cheng and Orin, 1991b). When all of the contact forces F^j are established, the dynamical relations of each finger i are decoupled. The equations (5), (8) become decoupled and τ^i act on the tip of the finger i.

7 CONTROL SYSTEM

The control system needs determining the torques (control variable) $T_j{}^i$ such that the motion of the overall system (object and fingers) will generate the desired trajectory. The inverse model of the robot will be used here to acquire the control law for a desired motion. The used closed-loop control is presented in Figure 4. Let $q_d^i, \dot{q}_d^i, \ddot{q}_d^i$ be prescribed parameters of the motion, $F_d{}^i$ the prescribed force applied at the i - contact point of the object, and $q^i, \dot{q}^i, \ddot{q}^i$, F^i - the same variables measured on the real or estimated system. The error and its derivatives of the feedback system are: $\Delta q^i = q_d^i - q^i$; $\Delta \dot{q}^i = \dot{q}_d^i - \dot{q}^i$; $\Delta \ddot{q}^i = \ddot{q}_d^i - \ddot{q}^i$; ΔF^i = $F_d{}^i$ - F^i. The controller represents a trajectory perturbation controller which generates the new variations δq^i, $\delta \dot{q}^i$, $\delta \ddot{q}^i$, δF^i. It assures the performances of the motion for the overall system on the trajectory. We propose the control law (Ivanescu and Stoian, 1998):

$$\begin{aligned}
\delta q^i &= K_{11}^i \Delta q^i + K_{12}^i \Delta \dot{q}^i + K_{13}^i \Delta \ddot{q}^i \\
\delta \dot{q}^i &= K_{21}^i \Delta q^i + K_{22}^i \Delta \dot{q}^i + K_{23}^i \Delta \ddot{q}^i \\
\delta \ddot{q}^i &= K_{31}^i \Delta q^i + K_{32}^i \Delta \dot{q}^i + K_{33}^i \Delta \ddot{q}^i \qquad (14) \\
\delta F_X^i &= K_{f_{1X}}^i \Delta F_X^i + K_{f_{2X}}^i \Delta \dot{F}_X^i + K_{f_{3X}}^i \Delta \ddot{F}_X^i \\
\delta F_Y^i &= K_{f_{1Y}}^i \Delta F_Y^i + K_{f_{2Y}}^i \Delta \dot{F}_Y^i + K_{f_{3Y}}^i \Delta \ddot{F}_Y^i
\end{aligned}$$

From (14) and error definitions result:

$$q^i = q_d^i - \Delta q^i, \dot{q}^i = \dot{q}_d^i - \Delta \dot{q}^i, \ddot{q}^i = \ddot{q}_d^i - \Delta \ddot{q}^i \quad (15)$$

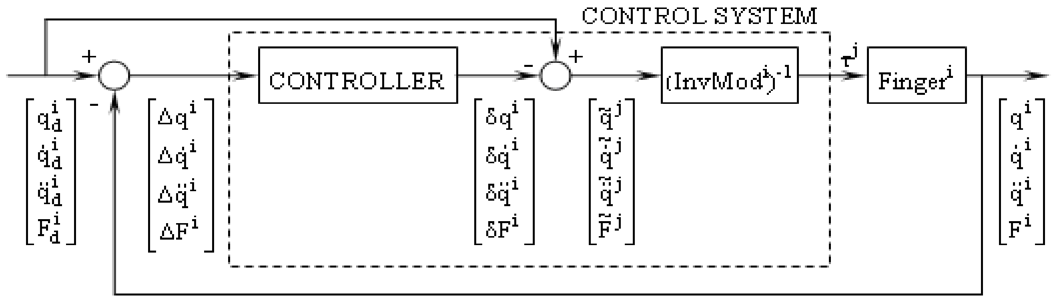

Figure 4: The control system.

$$\tilde{q}^i = \tilde{q}^i_d - \delta q^i,\ \dot{\tilde{q}}^i = \dot{\tilde{q}}^i_d - \delta\dot{q}^i,\ \ddot{\tilde{q}}^i = \ddot{\tilde{q}}^i_d - \delta\ddot{q}^i \quad (16)$$

$$\ddot{\tilde{q}}^i + f(\tilde{q}^i) + F^i_x a(\tilde{q}^i) + F^i_y c(\tilde{q}^i) = BT^i \quad (17)$$

Assuming that

$$\delta q^i << q^i_d,\ \delta\dot{q}^i << \dot{q}^i_d,\ \delta\ddot{q}^i << \ddot{q}^i_d$$

$$\Delta q^i << q^i_d,\ \Delta\dot{q}^i << \dot{q}^i_d,\ \Delta\ddot{q}^i << \ddot{q}^i_d \quad (18)$$

Using Taylor-series expansion and neglecting the high-order terms from (17) it results:

$$\left(\Delta\ddot{q}^i - \delta\ddot{q}^i\right) + d\left(q^i_d, F^i_d\right)\left(\Delta q^i - \delta q^i\right)$$

$$+ a\left(q^i_d\right)\left(\Delta F^i_x - \delta F^i_x\right) + c\left(q^i_d\right)\left(\Delta F^i_y - \delta F^i_y\right) = 0 \quad (19)$$

"d" is a $[n^i \times n^i]$ matrix and

$$d\left(q^i_d, F^i_d\right) = \left(\frac{\delta f}{\delta q}\right)_{q^i_d} + F^i_{xd}\left(\frac{\delta a}{\delta q}\right)_{q^i_d} + F^i_{yd}\left(\frac{\delta c}{\delta q}\right)_{q^i_d} \quad (20)$$

From (14) and (19) it results:

$$\left(I - K^i_{33} - d\cdot K^j_{13}\right)\Delta\ddot{q}^i - \left(K^i_{32} - d\cdot K^j_{12}\right)\Delta\dot{q}^i$$

$$+ \left[d\cdot\left(I - K^i_{11}\right) - K^i_{31}\right]\Delta q^i = 0 \quad (21)$$

$$K^i_{f_{3x}}\Delta\ddot{F}^i_x + K^i_{f_{2x}}\Delta\dot{F}^i_x + \left(1 - K^i_{f_{1x}}\right)\Delta F^i_x = 0 \quad (22)$$

$$K^i_{f_{3y}}\Delta\ddot{F}^i_y + K^i_{f_{2y}}\Delta\dot{F}^i_y + \left(1 - K^i_{f_{1y}}\right)\Delta F^i_y = 0 \quad (23)$$

For the nesingular matrix $\left(I - K^i_{33} - d\cdot K^i_{13}\right)$, these equations of the motion can be written as:

$$\Delta\ddot{q}^i - \left(V^i\right)^{-1} W^i\Delta\dot{q}^i - \left(V^i\right)^{-1} R^i\Delta q^i = 0 \quad (24)$$

$$K^i_{f_3}\Delta\ddot{F}^i + K^i_{f_2}\Delta\dot{F}^i + \left(1 - K^i_{f_1}\right)\Delta F^i = 0 \quad (25)$$

The control laws for the motion (i) and for the force (j) ask to be stable the matrix:

$$E = \begin{bmatrix} 0 & I \\ -V^{-1}R & -V^{-1}W \end{bmatrix} \quad (26)$$

and to be right (Ivanescu and Stoian, 1998):

$$K^{i2}_{f2} \le 4K^i_{f3}\left(1 - K^i_{f1}\right) \quad (27)$$

The relations use the notations:

$$V^i = I - K^i_{33} - d\,K^i_{13}$$

$$W^i = K^i_{32} + d\,K^i_{12} \quad (28)$$

$$R^i = d\,(I - K^i_{11}) - K^i_{31}$$

where we consider

$$K^i_{f1x} = K^i_{f1z} = K^i_{f1}; K^i_{f2x} = K^i_{f2z} = K^i_{f2};$$

$$K^i_{f3x} = K^i_{f3z} = K^i_{f3} \quad (29)$$

The relations (26), (27) specificate the principal conditions required to the control system to assure the global stability for the motion and for the force F^j_d at the terminal point of the arm. The condition from relation (27) is easy to apply but the stability established by the matrix (26) is more complicated to determine. We can obtain a simplified procedure if we choose appropriate matrices $K^j_{m,n}$ (m, n =1, 2, 3) in the control law from relations (14):

$$I - K^i_{33} - d\,K^i_{13} = \alpha I$$

$$K^i_{32} + d\,K^i_{12} = 2\,\Xi^i \quad (30)$$

$$d\,(I - K^i_{11}) - K^i_{31} = \Omega^i$$

where

$$\Xi^i = \mathrm{diag}\left(\xi^i_1, \xi^i_2 \ldots \xi^i_n\right)$$

$$\Omega^i = \mathrm{diag}\left(\omega^{i2}_1, \omega^{i2}_2 \ldots\ldots \omega^{i2}_n\right) \quad (31)$$

Now, the relations (g) become

$$\alpha\cdot\Delta\ddot{q}^i_j - 2\xi^i_j\cdot\Delta\dot{q}^i_j + \omega^{i2}_j\cdot\Delta q^i_j = 0 \quad (32)$$

The relations for the control of the finger parameters (32) and for the control of the force are adequate for a Direct Sliding Mod Control which presumes two phases. In first phase the system motion develops towards the switching line:

$$S_q:\ \Delta\dot{q}^i_j + p^i_j\Delta q^i_j = 0$$

$$S_F:\ \Delta\dot{F}^i + p^i_{jF}\Delta F^i = 0 \quad (33)$$

On this trajectory segment:

$$\xi_j^i < \min\left[\alpha\omega_j^{i2}(s)\right]^{1/2}$$

$$K_{f_2}^i < 2\left[K_{f_3}^i\left(1-K_{f_1}^i\right)\right]^{1/2} \qquad (34)$$

When the trajectory penetrates S_q (or S_F), the damping coefficients $\xi_j^i, K_{f_2}^i$ are increased (Shilling, 1993), (Ivanescu and Stoian, 1998):

$$\xi_j^i > \max\left[\alpha\omega_j^{i2}(s)\right]^{1/2}$$

$$K_{f_2}^i > 2\left[K_{f_3}^i\left(1-K_{f_1}^i\right)\right]^{1/2} \qquad (35)$$

In the second phase, on the last trajectory segment, the system develops towards the origin, directly, on the switching line S_q (or S_F).

8 CONCLUSIONS

This paper presents a control procedure and a control algorithm with two levels to solve the control problem of a cooperating multi-arm robotic system like a gripper with n fingers manipulating a usual object. The control system is a hierarchical system. The problems of the inter-coordination and the force distribution are decided by the upper-level coordinator which brings together all the appropriate information. This information is directed towards the n lower-level subsystems. The local control is solved by assigning the local controllers based on the inverse model method.

A control algorithm is also presented. This allows for the robotic structure, under the terms of the actuator blocking occurrence during the working, either a correct positioning (if it is possible) or a positioning in an acceptable proximity of the desired co-ordinates by minimising the movements (by the adequate commands to the functional elements).

A synthesis of the commands is proposed. First, a workspace analysis is made and then an algorithm for the actuators in the terms of a good working (finding the optimal motions) is presented in terms of the blocking or unblocking of some robotic segments.

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