A Proposal based on Frequency Response for Multi-Model Controllers

Anderson Luiz de Oliveira Cavalcanti
Federal University of Rio Grande do Norte, Natal, RN, Brazil

Keywords: MPC, Multi Model Control.

Abstract: This paper presents an alternative approach to control nonlinear plants. The nonlinear system to be controlled is decomposed into a number of operating points and a GPC controller is properly designed, based on local linear model for each point. A metric based on frequency response of each local linear model is proposed in order to consider the contribution of each local controller in the signal sent to the plant. Two applications are presented. The first application in a simulated plant consists of a continuous stirred tank reactor (CSTR) and the second consists of a coupled tanks system level control.

1 INTRODUCTION

In the industry are found processes that work in large operating ranges. Batch processes (Foss et al., 1995), as chemical reactors, are classic examples of this type of process. The main solution in these cases would be to obtain a non-linear complex model, requiring, in the case of using predictive controllers, the use of nonlinear prediction and optimization, which is not a trivial task (Camacho and Bordons, 1999).

In order to provide a simpler solution for the problems mentioned above, a known approach, in academia, as multi-models approach has been investigated (Boling et al., 2007), (Dougherty and Cooper, 2003a), (Dougherty and Cooper, 2003b), (Arslan et al., 2004) and (Normey- Bravo and Rich, 2009). This approach seeks to decompose the process range of operating at various operating points usual in the same place and obtain a valid model for each of these points. A validation function, called a metric is defined to indicate what is the most appropriate model at a given moment of sampling. Such a metric is used to calculate weighting factors, the application of which will be described below. There are basically two types of multi-model approaches, which will be detailed below also.

The first approach uses the weighting factors to compute a global model formed by the convex combination of local models obtained and a single controller is designed from said master (Foss et al., 1995) (Azimadeh et al., 1998) (Dumitrache and Constantine, 2000) (Pickhardt, 2000) and (Cavalcanti et al., 2007a).

The second type of approach designs a suitable controller for each model of each operating point. In this case, the control signal to the process is a convex combination of the computed control signals (Cavalcanti et al., 2007b), (Cavalcanti et al., 2008), (Arslan et al., 2004), (Wen et al., 2006), (Dougherty and Cooper, 2003) and (Dougherty and Cooper, 2003b).

This article is based on the second type of approach mentioned. Most existing literature are based on the metrics or statistics of the process (Foss et al., 1995) or standards (Dougherty and Cooper, 2003) (Dougherty and Cooper, 2003b), (Arslan et al., 2004) (Cavalcanti et al., 2007b) and (Cavalcanti et al., 2008). This work, in particular, consider the frequency response of each local model compared to the frequency response of each current approximate model obtained by interpolation at each sampling instant.

The choice of use of a predictive controller based on the highlight that this is gaining in terms of industrial applications (García, Prett and Morari, 1989). This highlight is observed as the same, and can be applied in a wide range of processes, including processes with long delays and non-minimum phase, you can easily incorporate the constraint treatment in the problem formulation (Camacho and Bordons 1999).

Luiz de Oliveira Cavalcanti A..
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DOI: 10.5220/0005546202230229
In *Proceedings of the 12th International Conference on Informatics in Control, Automation and Robotics* (ICINCO-2015), pages 223-229
ISBN: 978-989-758-122-9

This article is organized as follows: section 2 shows the theoretical formalization of multi-model approach used in this work; Section 3 shows the metric based on frequency response proposal; Section 4 shows the application results in a simulated continuous stirred tank reactor and in a real system level control, as well as discussions on them. Section 5 provides the conclusions.

2 THEORETICAL FORMALIZATION OF MULTI-MODEL APPROACHES

Consider a nonlinear system with multiple operating points, which is segmented into a set Φ containing NPO operating regimes. Each operating regime is defined as a subset $\Phi_i \subset \Phi$ with i = 1,..,NPO. An operating point is typically denoted by a function ϕ represented by a subset of input and output like $\phi = H(y, u)$. It is assumed that there is a metric ρ_i: $\Phi \rightarrow [0,1]$, which is designed so that its value is close to *one* for operating points where the local model *i* is a good description of the system and close to *zero* otherwise. If the system's operating range is broken down into NPO operating systems, so that $\Phi_1,\ldots,\Phi_{NPO} \subset \Phi$, then for each local model is defined a particular metric $\rho_i(\phi)$ with i = 1,…, NPO. For each operating system, a model of discrete transfer function type can be described:

$$y(z) = G_i(z)u(z) \tag{1}$$

where $G_i(z)$, $u(z)$ and $y(z)$ are the transfer function of the process valid for the operating point *i*, the deviation of the process input and the deviation of the process output. For each operating point, a controller is designed, and its output at time *k* is denoted by $u_i(k)$. The control action effectively sent to the process is given by:

$$u(k) = \sum_{i=1}^{NPO} u_i(k) w_i(\phi) \tag{2}$$

where u (k) is the control signal computed by the *i*-th controller and $w_i(\phi)$ is the weighting factor assigned to that controller given by:

$$w_i(\phi) = \frac{\rho_i(\phi)}{\sum_{j=1}^{NPO} \rho_j(\phi)} \tag{3}$$

The following property to (3) is defined to ensure the convexity:

$$\sum_{i=1}^{NPO} w_i(\phi) = 1 \tag{4}$$

This paper, specifically, the controller designed for each operating point is a Generalized Predictive Control (GPC) (Clarke et al., 1987). The considered tunning parameters are the prediction horizon (NY), the control horizon (NU) and the weighting of the control signal (λ). More details on the GPC can be obtained from (Clarke et al., 1987).

3 METRIC BASED ON FREQUENCY RESPONSE

3.1 The Proposed Metric

The methodology proposed in this work takes into account the difference in dynamics between the transfer functions, appointed by the different frequency responses. It is assumed that, for all operating points, a linear model of the process can be obtained. At each sampling time, an estimation of the parameters change, based on interpolation, is made considering the current output *y(k)* (current model). The current model is compared, in terms of frequency response, with the determined models in operating points. An index called dynamic measure (MD) sistem, is defined. The dynamic measure (MD) is given by the area under the frequency response curve corresponding to the module. Thus, for each local model of each operating point defined the MD is calculated as follow:

$$MD_i(e^{j\omega h}) = \int_0^{\omega_N} \left|G_i(e^{j\omega h})\right| d\omega,\ i = 1,\ldots,NPO \tag{5}$$

where *h* is the sampling time and ω_N is the Nyquist Frequency.

The function that describes the system's metric, according to the criteria established in section 2, is given by:

$$\rho_i(\varphi) = \frac{\max\left(MD_i(e^{j\omega h})\right) - \min\left(MD_i(e^{j\omega h})\right)}{\left|MD_i\left(e^{j\omega h}\right) - MD_a\left(e^{j\omega h}\right)\right|},\ i = 1,\ldots,NPO \tag{6}$$

where $MD_a(e^{j\omega h})$ is the dynamic measure of the system uat the current sampling time *k*. Thus, considering the metric, the proposed control algorithm is described as follows:

Step 1: Read the output of the process at time k, *y(k)*;

Step 2: Calculate the estimated $G_a(z)$ by interpolation based on y (k);
Step 3: Compute the value ($MD_a(e^{j\omega h})$) based on $G_a(z)$;
Step 4: Compute the values of $\rho_i(\phi)$, i = 1, …, NPO;
Step 5: Compute the values of the weights of the local controllers $w_i(\phi)$, i = 1, …, NPO;
Step 6: Compute the control signal by convex combining of each local controller signal - see equation (2);
Step 7: Apply the control signal in the plant;
Step 8: Wait for the next sampling period *h* and return to Step 1.

Note that the calculation of integral in (9) is computed numerically.

4 RESULTS AND DISCUSSIONS

4.1 Application Example 1: Simulated Continuous Stirred Tank Reactor (CSTR)

The plant to be controlled, in this simulated case, consists of a continuous stirring tank reactor which is a process widely found in Chemical and Biochemical Engineering. Normally, to irreversible exothermic reactions, adiabatic mode reagents leads to large production rates, so in order to maintain the temperature within a certain range, the generated heat has to be removed by cooling. The vessel is assumed to be perfectly mixed, and a single irreversible reaction exothermic first order occurs. Thus, the variable to be controlled is the temperature of the reactor and the manipulated variable is the cooling temperature.

Figure 1 shows an illustration of this reactor:

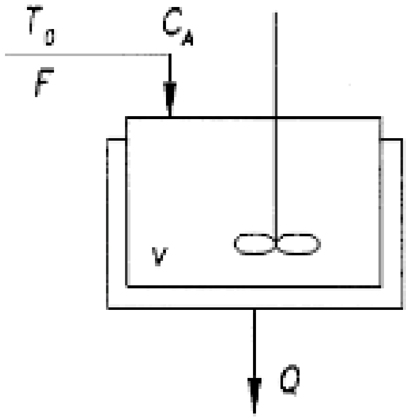

Figure 1: Continuous Stirred Tank Reactor.

A nonlinear space state model to the reactor is found in Arslan et al. (2004) and Uppal, et al. (1976). The model is based on mass and energy balance of the tank as shown below:

$$\frac{dC_A}{dt} = -C_A + D_a(1-C_A)\exp(T/(1+T/\gamma)) - (C_f - 1) \quad (7)$$

$$\frac{dT}{dt} = T + B \cdot D_a(1-C_A)\exp(T/(1+T/\gamma)) - \beta(T_R - T) \quad (8)$$

where C_A is the concentration of product of the reactor given in gmol/L, T is the temperature of the reactor (controlled variable) given in Kelvin, T_R is the cooling temperature (manipulated variable) given in Kelvin and C_f is the reactor feed concentration given in gmol/L. Typical values of the constants shown in (7) and (8) are shown in Table 1.

Table 1: Parameters of the CSTR nonlinear model.

Constant	Value
Da	0.072
γ	20
B	8
β	0.3
Cf	1

Figure 2 shows the curve of the steady state system. Here, we consider three operating points, as shown in curve for T_R = 0.

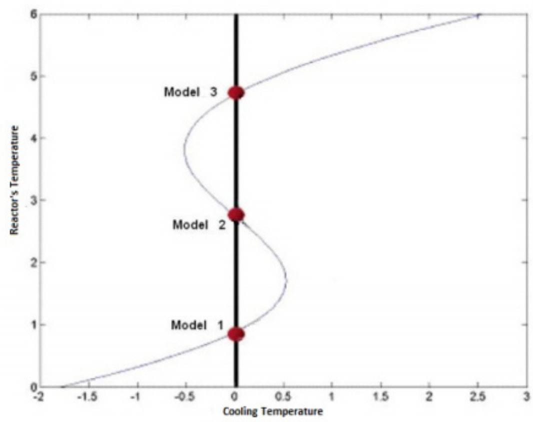

Figure 2: Steady State curve to Continuous Stirred Tank Reactor model.

For T_R = 0, we see, in Figure 4, three equilibrium points, which will be considered as the operating points in this work. Linearizing the model (7) and (8) around the equilibrium points and discretizing them to h = 0.1 minutes, we obtain the following models:

$$G_1(z) = \frac{0.02964z - 0.02638}{z^2 - 1.865z + 0.8694} \quad (9)$$

$$G_2(z) = \frac{0.03211z - 0.0269}{z^2 - 1.969z + 0.9646} \quad (10)$$

$$G_3(z) = \frac{0.03362z - 0.02199}{z^2 - 1.837z + 0.8521} \quad (11)$$

For each of these models, a GPC was tuned, to the given operating point, considering a maximum settling time of 5 minutes. The tuning parameters of each local controller are shown in Table 2. For comparison, the proposed method is compared with a single-model GPC using the model 3 shown in (11) and the method proposed in Arslan et al. (2004).

Table 2: Tuning parameters of the local GPC controller in CSTR example.

Operating Point	λ	NY=NU
Point 1	3	15
Point 2	2	15
Point 3	1	20

The objective of controllers, in this example, is to track the references as shown in Figure 3. The references considered is the operating point.

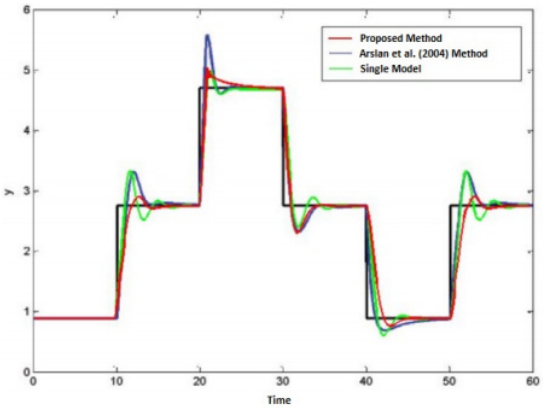

Figure 3: Process output (Cooling temperature).

Analysing the Figure 3, we observe a better performance of the multi-modelo controller, compared with the other controllers, in terms of less overshoot and shorter stabilization, particularly when the process is far from the third operating point. When the process is close to the third operating point, this improvement is not as significant, since the multi-model is replaced as the dominant controller the controller designed for that. Figure 4 shows the control signal for this simulation.

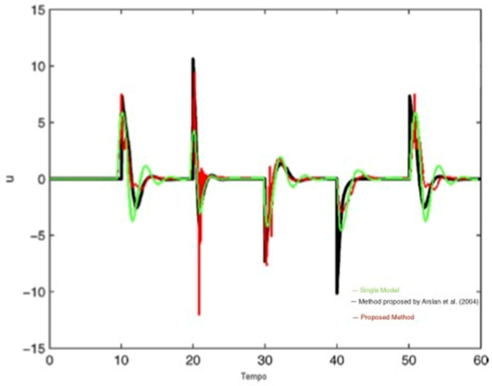

Figure 4: Control Signal.

The authors of Arslan et al. (2004) consider a metric known as Gap Metric that is based in H_∞ norm. In Arslan et al. (2004), the designed controllers are Proportional-Integral (PI). However, this comparison is possible because the design criteria used for each controller in this work are the same, that is, each controller is designed so as to obtain maximum settling time of 5 minutes.

The control effort generated by the multi-model scheme is shown in Figure 6, most of the time, less than the control effort single-model schema.

For a better analysis of the performance of controllers, some classic indices of control literature are compared. In (Goodhart, et al., 1994) the authors present some indices that assess the controller performance. The first index is given by:

$$\varepsilon_1 = \sum |u(k)| / N \quad (12)$$

where N is the number of control signals applied to the process so that it achieves the desired response. The content presented in (12) represents the energy used by the process to achieve the desired response. The variance of the actuators is given by:

$$\varepsilon_2 = \sum (u(k) - \varepsilon_1)^2 / N \quad (13)$$

The process output deviation in terms of the integral of absolute error (IAE) is:

$$\varepsilon_3 = \sum |r(k) - y(k)| / N \quad (14)$$

Table 3 shows the indexes considered for the three simulations.

Table 3: Performance indexes for the CSTR example.

Controller	ε1	ε2	ε3
Single model	0.678	2.523	0.164
Proposed Method	0.506	1.931	0.152
Method proposed in Arslan et al. (2004)	0.613	0.613	0.613

Quantitative analysis shows, according to the figures, that proposal in this paper provides less energy spent by the process, the lower variance of actuators and lower tracking error when compared to the other controllers.

The variations in the weights of the three controllers as function of time are shown in Figure 5.

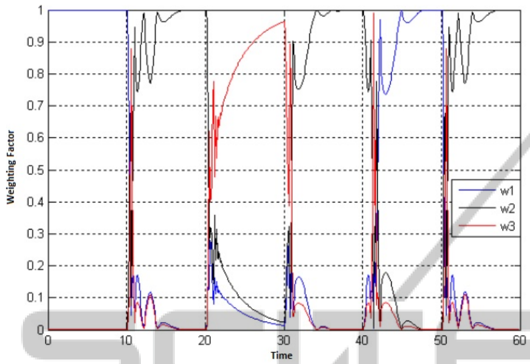

Figure 5: Weights of the controllers.

4.2 Application Example 2: Level Control in Coupled Tank

The second application of this work is on a real system of coupled tanks (Figure 6).

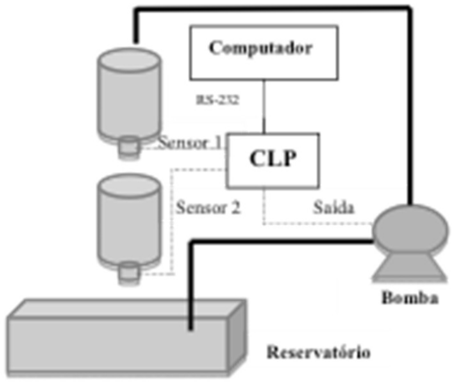

Figure 6: Coupled tanks system.

Considering L_1, L_2 and V_p as the top tank level in cm, the bottom tank level in cm and the pump voltage in volts, respectively, the following equations describe the states of the system:

$$\frac{dL_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gL_1} + \frac{K_m}{A_1}V_p \qquad (15)$$

$$\frac{dL_2}{dt} = \frac{a_1}{A_1}\sqrt{2gL_1} - \frac{a_2}{A_2}\sqrt{2gL_2} \qquad (16)$$

where a_1 and a_2 are the areas of the outlet holes of lower tank and the upper tanks respectively, A_1 and A_2 are the areas of the cross sections of the upper and lower tanks respectively, K_m is the constant of the pump, and $g = 981$ cm/s^2 is the gravitational acceleration. The values of the constants used for testing in this study are described in Table 4.

Table 4: Parameters of the coupled tanks model.

Parameters	Values
A_1 and A_2	15.518 cm^2
a_1 and a_2	0.1781 cm^2
K_m	4.6 (cm^3/s).V

In this application the controlled variable is the level of the bottom tank and the manipulated variable is the voltage in the pump. In this case are also considered three operating points. Linearizing the model for 10cm, 15cm and 20cm, we obtain the following discrete models for h = 0.5s:

$$G_1(z) = \frac{0.004057z - 0.003906}{z^2 - 1.889z + 0.8925} \qquad (17)$$

$$G_2(z) = \frac{0.00238z - 0.002328}{z^2 - 1.935z + 0.9365} \qquad (18)$$

$$G_3(z) = \frac{0.001852z - 0.001821}{z^2 - 1.95z + 0.9504} \qquad (19)$$

The tuning parameters in this case, shown in Table 5, were obtained trying to get a settling time of less than 50s.

Table 5: Tuning parameters of the local GPC controller in coupled tanks example.

Operating Point	λ	NY=NU
Point 1	85	15
Point 2	75	15
Point 3	65	15

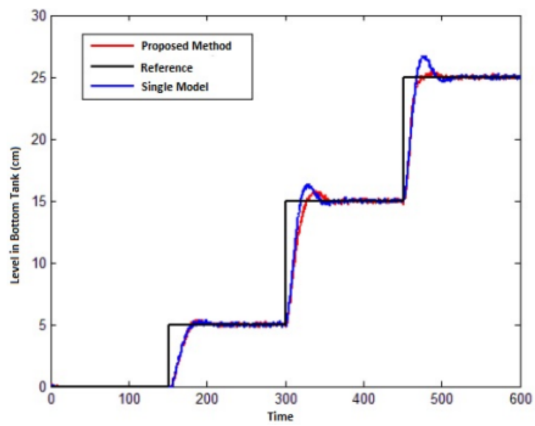

Figure 7: Controlled level in bottom tank (in cm).

In this application, we compare the proposed controller with the single-model controller using the model in operating point 1. The results are shown in Figure 7. In the experiment, the system starts with empty tanks. In a given time (time = 150 s.) Reference is changed to the first operating point. Then (time = 300 s), the reference to the controllers is switched to the second operating point, and just after (time = 450 s.) for the third operating point.

Similarly to the reactor of example, when the system diverges from the first operating point, the multi-controller model has better performance. Figure 8 shows the control signals applied to the pump, in volts, for the two trials. Note that in neither case the input constraints (maximum voltage of 22V pump) were violated because the system did not operate near its constraint region. As the designed GPCs controllers are subject to constraint on plant input, the convex combination of the output of each controller also produce a control signal within the constraint limits.

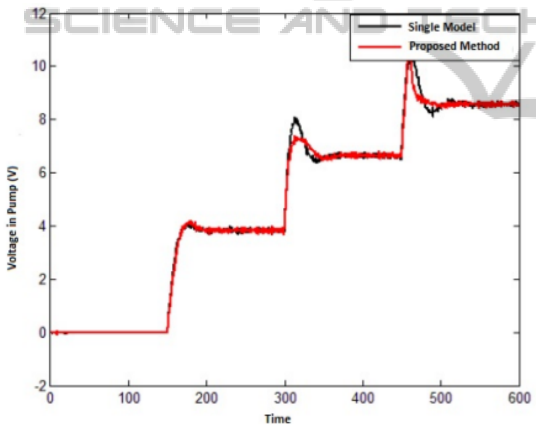

Figure 8: Control signal – voltage in the pump (in volt).

Figure 9 shows the variation in weights of the controllers as function of time.

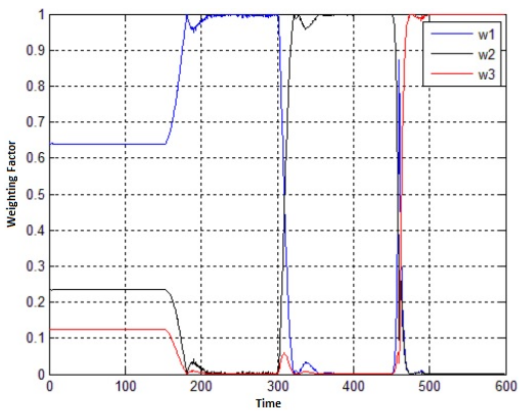

Figure 9: Weights of the controllers.

Also for this application example, to quantitatively evaluate the comparison made, the indexes showed in (12), (13) and (14) are considered. Table 6 shows the indexes to the present case.

Table 6: Performance indexes for the CSTR example.

Controller	ε_1	ε_2	ε_3
Single model	4.78	11.15	6.45
Proposed Method	4.46	11.01	6.34

We can see, for the evaluation of Table 6, the proposed controller compared to the single model controller has better performance.

5 CONCLUSIONS

Was presented in this paper an alternative approach to nonlinear predictive control. The multi-model approach has become an attractive alternative proposal, because it uses the whole theoretical framework of linear controller, which is quite consolidated in academia, to control with results, in many cases, better, as demonstrated this work and the works here referenced.

The metric proposed here reflects well the process change the operating point, pondering consistently the isolated controllers designed.

The results shown in this work, both in simulation in a physical plant, revealed an improvement, properly evaluated by indexes, compared to controllers with single model.

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