Adaptive Unscented Kalman Filter at the Presence of Non-additive Measurement Noise

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Abstract: This paper proposes an Adaptive Unscented Kalman Filter (AUKF) for nonlinear systems having non-additive measurement noise with unknown noise statistics. The proposed filter algorithm is able to estimate the nonlinear states along with the unknown measurement noise covariance (R) online with guaranteed positive definiteness. By this formulation of adaptive sigma point filter for non-additive measurement noise, the need of approximating non-additive noise as additive one (as is done in many cases) may be waived. The effectiveness of the proposed algorithm has been demonstrated by simulation studies on a nonlinear two dimensional bearing-only tracking (BOT) problem with non-additive measurement noise. Estimation performance of the proposed filter algorithm has been compared with (i) non adaptive UKF, (ii) an AUKF with additive measurement noise approximation and (iii) an Adaptive Divided Difference Filter (ADDF) applicable for non-additive noise. It has been found from 10000 Monte Carlo runs that the proposed AUKF algorithm provides (i) enhanced estimation performance in terms of RMS errors (RMSE) and convergence speed, (ii) almost 3-7 times less failure rate when prior measurement noise covariance is not accurate and (iii) relatively more robust performance with respect to the initial choice of R when compared with the other nonlinear filters involved herein.

1 INTRODUCTION

Adaptive state estimation techniques have got more attention of researchers in recent few years due to their renowned superiorities (Almagbile, 2010) over non adaptive state estimators. In this paper an Adaptive Unscented Kalman Filter (AUKF) has been proposed which is applicable for the situation where non-additive measurement noise is present.

In earlier stages adaptive filters were mostly formulated on linear Kalman Filtering framework (Mehra, 1972). However recent trends of research are directed towards adaptive nonlinear estimation techniques. The earliest adaptive nonlinear filter reported in literature is Adaptive Extended Kalman Filter (AEKF) (Busse, 2003; Meng, 2000). AEKF is found to fraught with dificulties like singularity problems, complex jacobian calculations (Wan, 2000; Fathabadi, 2009) etc. which further leads toward an alternate adaptive nonlinear estimation technique named as Adaptive Sigma Point Kalman Filter (ASPKF) (Das, 2013). Adaptive Unscented Kalman Filter (AUKF) (Das, 2014; Soken, 2011; Chai, 2012), Adaptive Central Difference Filter (ACDF) (Das, 2015) or Adaptive Divided Difference Filter (ADDF) (Dey, 2015) all belong to the class of ASPKF. The present work in this paper is formulated on Unscented Kalman Filtering (UKF) framework as it is able to exhibit better estimation performance compared to Extended Kalman Filter (EKF) and first order Central or Divided Difference Filter (CDF/ DDF) (Norgaard, 2000; Ito, 2000).

The proposed filter herein determines the covariance (R) online of the nonadditive measurement noise and therefore it fits into R-adaptive filter category. There are several existing methodologies of adapting R in literature (Mehra, 1972; Das, 2014). Direct covariance matching method depending on the innovation or residual sequence is the simplest and straight forward technique among them (Mehra, 1972; Mohamed, 1999). The rudimental idea of R adaptation in the present work is adopted from (Mohamed, 1999) where direct covariance matching method of adaptation depending on residual sequence is utilized for linear signal models.

However, adaptive formulation of sigma point filters considering non-additive measurement noise is very rare in literature. In (Dey, 2015) an adaptive Divided Difference Filter (ADDF) for nonlinear

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systems with non-additive measurement noise has been proposed. Our current work presented in this paper focuses on adaptive formulation of another sigma point filter named as Unscented Kalman Filter (UKF) considering non-additive measurement noise.

To demonstrate the performance of the proposed filtering algorithm a two dimentional bearing-only tracking (BOT) problem with non-additive measurement noise as may be found in (Sadhu, 2006; Lin, 2002) has been chosen. Although in (Sadhu, 2006; Lin, 2002) the non-additive measurement noise has been approximated as additive one, in our current work this approximation is waived to circumvent the approximation errors. It may be discerned from the simulation results presented herein that waiving this additive noise approximation may enhance the estimation performance significantly in some particular cases.

The significant contributions of this paper may be summarized as follows:

- An Adaptive Unscented Kalman Filter (AUKF) has been formulated which utilizes the augmented form of UKF and is applicable for nonlinear systems with non-additive measurement noise.
- Residual sequencees in lieu of innovation sequences have been utilized for guaranteed positive definiteness of the adapted R matrix.
- The proposed algorithm has been exemplified with a 2-D bearing-only tracking (BOT) test problem with non-additive measurement noise.

The organization of the paper is as follows. Section 2 describes the proposed Adaptive Unscented Kalman Filter (AUKF) in augmented form applicable for non-additive measurement noise. In section 3 case studies on 2-D bearing only tracking (BOT) problem have been illustarted. Discussions and conclusions are presented in section 4.

2 PROPOSED AUKF ALGORITHM

2.1 Problem Statement

Nonlinear state estimation problem with non-additive noise where the prior knowledge of measurement noise covariance is unavailable is considered here. The dynamic equations of process and measurement therefore may be represented by equations (1) and (2). Where, $x_k \in R^n$, $z_k \in R^m$ and w_k, v_k are the non-additive process and measurement noise with covariance Q_k and R_k respectively.

$$x_k = f(x_{k-1}, w_k) \tag{1}$$

$$z_k = h(x_k, v_k) \tag{2}$$

Here Q_k is assumed to be known and constant (therefore it will be represented by Q in the rest of the paper) whereas R_k is assumed to be unknown. The adaptive nonlinear filter designed for this particular problem is able to adapt the covariance R_k of the non-additive measurement noise v_k.

2.2 Filter Algorithm

As non-additive noise is considered here, the augmented form of UKF (Wan, 2000) has been utilized here and the adaptive version of augmented UKF has been formulated. The weight values (Wan, 2000) are calculated as given below:

$$\left.\begin{aligned} W_0^m &= \frac{\lambda}{n^a+\lambda};\ W_0^c = \frac{\lambda}{n^a+\lambda} + (1-\alpha^2+\beta) \\ W_i^m &= W_i^c = \frac{1}{2(n^a+\lambda)} \quad \text{for, i} \neq 0 \end{aligned}\right\} \tag{3}$$

Where, n^a is the dimension of augmented state vector, λ is a scaling factor given by $\alpha^2(n^a+\kappa)-n^a$, the parameters α and β determines respectively the spread of the sigma points and a prior knowledge about the noise distribution. The values of the tuning parameters α, β and κ are considered to be 0.5, 2 and 0 in the present work.

2.2.1 Initialization

Initialize, State error covariance (P_{k-1}), Estimated states ($\hat{x}_{k-1}$) and measurement noise covariance ($\hat{R}_{k-1}$).

2.2.2 Prediction (Time Update)

Form augmented state vector as:

$$\hat{x}_{k-1}^a = \begin{bmatrix} \hat{x}_{k-1} \\ \overline{w} \\ \overline{v} \end{bmatrix} \tag{4}$$

Where, $\overline{w}$ and $\overline{v}$ are respectively mean of process and measurement noise and $\hat{x}^a_{k-1} \in R^{n^a}$.

Form augmented state error covariance matrix as:

$$P^a_{k-1} = \begin{bmatrix} P_{k-1} & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & \hat{R}_{k-1} \end{bmatrix} \qquad (5)$$

It is assumed here that the state errors, process noise and the measurement noise are not correlated to each other. Hence the off-diagonal sub-blocks of the matrix in equation (5) are considered as zero.

Calculate the sigma points of $\hat{x}^a_{k-1}$ as:

$$\hat{\chi}^a_{k-1} = [\,\hat{x}^a_{k-1} \quad ..... \quad \hat{x}^a_{k-1}\,]_{n^a \times (2n^a+1)} + \sqrt{(n^a + \lambda)}$$
$$[\,zeros(n^a \times 1) \quad \sqrt{P^a_{k-1}} \quad -\sqrt{P^a_{k-1}}\,]$$
$$= \begin{bmatrix} \hat{\chi}^x_{k-1} \\ \hat{\chi}^w_{k-1} \\ \hat{\chi}^v_{k-1} \end{bmatrix} \qquad (6)$$

Propagate the sigma points ($\hat{\chi}^x_{k-1}$ and $\hat{\chi}^w_{k-1}$) through the function *f* as:

$$\hat{\chi}^-_k = f\,(\hat{\chi}^x_{k-1}, \hat{\chi}^w_{k-1}) \qquad (7)$$

Project the state ahead as:

$$\hat{x}^-_k = \sum_{i=0}^{2n^a} W^m_i \hat{\chi}^-_{k,i} \qquad (8)$$

Project the state error covariance ahead as:

$$P^-_k = \sum_{i=0}^{2n^a} W^c_i \{(\hat{\chi}^-_{k,i} - \hat{x}^-_k)(\hat{\chi}^-_{k,i} - \hat{x}^-_k)^T\} \qquad (9)$$

Propagate the sigma points ($\hat{\chi}^-_k$ and $\hat{\chi}^v_{k-1}$) through the function *h* as:

$$\hat{Z}^-_{k,i} = h(\hat{\chi}^-_{k,i}, \hat{\chi}^v_{k-1,i}) \qquad (10)$$

Predict the measurement as:

$$\hat{z}^-_k = \sum_{i=0}^{2n^a} W^m_i \hat{Z}^-_{k,i} \qquad (11)$$

2.2.3 Correction (Measurement Update)

Calculate the innovation covariance as:

$$P_{zz} = \sum_{i=0}^{2n^a} W^c_i \{(\hat{Z}^-_{k,i} - \hat{z}^-_k)(\hat{Z}^-_{k,i} - \hat{z}^-_k)^T\} \qquad (12)$$

Calculate the cross covariance as:

$$P_{xz} = \sum_{i=0}^{2n^a} W^c_i \{(\hat{\chi}^-_{k,i} - \hat{x}^-_k)(\hat{Z}^-_{k,i} - \hat{z}^-_k)^T\} \qquad (13)$$

Calculate the Kalman gain as:

$$K_k = P_{xz} P^{-1}_{zz} \qquad (14)$$

Estimate the state as:

$$\hat{x}_k = \hat{x}^-_k + K_k(z_k - \hat{z}^-_k) \qquad (15)$$

Calculate the estimated state error covariance as:

$$P_k = P^-_k - K_k P_{zz} K^T_k \qquad (16)$$

2.2.4 Adaptation of 'R'

Form augmented state vector $\hat{x}^a_k$ by augmenting the estimated state $\hat{x}_k$ with mean of process and measurement noise as is done in equation (4).

Form augmented state error covariance matrix $P1^a_k$ and $P2^a_k$ as given below:

$$P1^a_k = \begin{bmatrix} P_k & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & I_{m \times m} \end{bmatrix} \qquad (17)$$

$$P2^a_k = \begin{bmatrix} P_k & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & \hat{R}_{k-1} \end{bmatrix} \qquad (18)$$

Calculate the sigma points:

$$\hat{\chi}1^a_k = [\,\hat{x}^a_k \quad ..... \quad \hat{x}^a_k\,]_{n^a \times (2n^a+1)} + \sqrt{(n^a + \lambda)}$$
$$[\,zeros(n^a \times 1) \quad \sqrt{P1^a_k} \quad -\sqrt{P1^a_k}\,]$$
$$= \begin{bmatrix} \hat{\chi}1^x_k \\ \hat{\chi}1^w_k \\ \hat{\chi}1^v_k \end{bmatrix} \qquad (19)$$

$$\hat{\chi}2^a_k = [\,\hat{x}^a_k \quad ..... \quad \hat{x}^a_k\,]_{n^a \times (2n^a+1)} + \sqrt{(n^a + \lambda)}$$
$$[\,zeros(n^a \times 1) \quad \sqrt{P2^a_k} \quad -\sqrt{P2^a_k}\,]$$
$$= \begin{bmatrix} \hat{\chi}2^x_k \\ \hat{\chi}2^w_k \\ \hat{\chi}2^v_k \end{bmatrix} \qquad (20)$$

Propagate the sigma points ($\hat{\chi}2_k^x$ and $\hat{\chi}2_k^v$) through the function h as:

$$\hat{Z}_{k,i} = h(\hat{\chi}2_{k,i}^x, \hat{\chi}2_{k,i}^v) \tag{21}$$

Estimate the measurement as:

$$\hat{z}_k = \sum_{i=0}^{2n^a} W_i^m \hat{Z}_{k,i} \tag{22}$$

Calculate residual as:

$$res_k = z_k - \hat{z}_k \tag{23}$$

Calculate the residual covariance with in sliding window (of size '*ws*') as:

$$P_{res} = \frac{1}{ws} \sum_{i=k-ws+1}^{k} (res_i)(res_i)^T \tag{24}$$

Now propagate the estimated state and sigma points of measurement noise ($\hat{\chi}1_k^v$) through the function h as:

$$\hat{Z}1_{k,i} = h(\hat{x}_k, \hat{\chi}1_{k,i}^v) \tag{25}$$

Similarly, propagate sigma points of estimated states ($\hat{\chi}2_k^x$) and mean of measurement noise through the function h as:

$$\hat{Z}2_{k,i} = h(\hat{\chi}2_{k,i}^x, \bar{v}) \tag{26}$$

Calculate the covariance matrix DD^T (this matrix is analogous to DRD^T of Extended Kalman Filter where R is considered as unity) as:

$$DD^T = \sum_{i=0}^{2n^a} W_i^c \{(\hat{Z}1_{k,i} - \hat{z}1_k)(\hat{Z}1_{k,i} - \hat{z}1_k)^T\} \tag{27}$$

Where, $\hat{z}1_k$ is calculated as:

$$\hat{z}1_k = \sum_{i=0}^{2n^a} W_i^m \hat{Z}1_{k,i} \tag{28}$$

Find the matrix D by Cholesky factorization of DD^T.

Calculate the covariance matrix C (which is analogous to HPH^T of Extended Kalman Filter) as:

$$C = \sum_{i=0}^{2n^a} W_i^c \{(\hat{Z}2_{k,i} - \hat{z}2_k)(\hat{Z}2_{k,i} - \hat{z}2_k)^T\} \tag{29}$$

Where, $\hat{z}2_k$ is calculated as:

$$\hat{z}2_k = \sum_{i=0}^{2n^a} W_i^m \hat{Z}2_{k,i} \tag{30}$$

Estimate 'R' as:

$$R_k = D^{-1}(P_{res} + C)(D^T)^{-1} \tag{31}$$

2.2.5 Comments on the Algorithm

Following specific comments can be made about the proposed filter algorithm.

- A sliding window has been utilized here to calculate the residual covariance as shown by equation (24). However, if the current instant (k) is less than the window size (ws) defined by the user, the residual covariance should be calculated from all the available residual sequences.
- The window size should be chosen carefully. Too small window size may lead to quick adaptation in the cost of noisy performance. Whereas choosing too large window may provide very smooth R adaptation compromising in the speed of adaptation.
- In equation (5) $\hat{R}_{k-1}$ has been utilized due to the unavailability of adapted R at k^{th} instant. Once adaptation of 'R' at k^{th} instant is completed, an iteration of the filtering steps can also be carried out by utilising $\hat{R}_k$. However, due to increased calculation burden this iterative structure is not included in the proposed algorithm.
- Due to the use of augmented state vector and additional calculation steps for R adaptation, computation burden of the proposed AUKF is more compared to normal UKF. But the effectiveness of the proposed algorithm for nonlinear state estimation at the presence of non-additive noise (with unknown noise statistics) compensates the shortcoming related to extra calculation burden.

3 CASE STUDY

3.1 2-Dimensional BOT Problem

3.1.1 Process Model

Two dimensional bearing only tracking (BOT) problem (Sadhu, 2006) consists of two components, platform kinematics and target kinematics, as shown in figure 1. The target moves along x axis and the platform accompanied with a sensor moves parallel to the target with constant velocity.

Target motion is considered here as the process model and is given by

$$x_1(k) = x_1(k-1) + Tx_2(k-1) + \frac{T^2}{2} w(k) \quad (32)$$

$$x_2(k) = x_2(k-1) + Tw(k) \quad (33)$$

Where, $x_1(k)$ is target position along x axis and $x_2(k)$ is target velocity which is also assumed to be constant. T=1 sec. is the sampling time and $w(k)$ is process noise with covaraiance Q=0.01m^2/sec^4. Values of these parameters are adopted from (Sadhu, 2006).

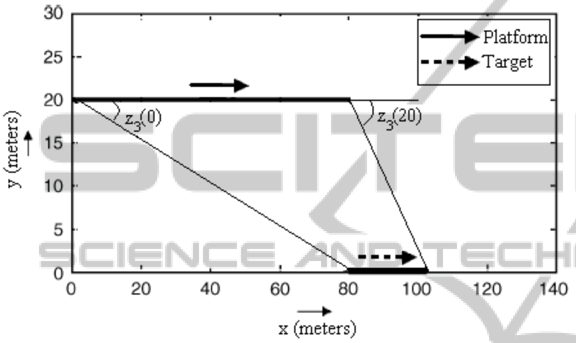

Figure 1: BOT trajectory.

3.1.2 Measurement Model

Platform position (along x and y axis) and bearing between x axis and line of sight from sensor to target are considered as measurement equations and are given by:

$$z_1(k) = y_p(k) = 20 + v_1(k) \quad (34)$$

$$z_2(k) = x_p(k) = 4kT + v_2(k) \quad (35)$$

$$z_3(k) = tan^{-1}\left(\frac{y_p(k)}{x_1(k) - x_p(k)}\right) + v_3(k)$$
$$= tan^{-1}\left(\frac{20 + v_1(k)}{x_1(k) - 4kT - v_2(k)}\right) + v_3(k) \quad (36)$$

First two measurement variables are the platform positions along y axis and x axis respectively. 'k' is the current time instant. It is evident from the measurement model that the third measurement variable is a nonlinear function of state as well as the measurement noises ($v_1(k)$ and $v_2(k)$). However, $v_3(k)$ is the additive measurement noise. Measurement noise vector v therefore may be formed as $[v_1(k); v_2(k); v_3(k)]$ with true covariance R_k. Where R_k is given by:

$$R_k = \begin{bmatrix} \sigma_{v1}^2 & 0 & 0 \\ 0 & \sigma_{v2}^2 & 0 \\ 0 & 0 & \sigma_{v3}^2 \end{bmatrix} \quad (37)$$

σ_{v1}, σ_{v2} and σ_{v3} are the standard deviation of three measurement noises with nominal values of 1 meter, 1 meter and 3° respectively. All initial values of the model and the filters are same as in (Sadhu, 2006).

3.2 Simulation Results

To assess the performance of the proposed filter, it has been compared with non adaptive UKF, AUKF with additive noise (Chai, 2012) approximation and ADDF for non-additive noise (Dey, 2015). For AUKF with additive noise (Chai, 2012), the non-additive measurement noise has been approximated to additive one in the same way as is done in (Sadhu, 2006). When the appropriate knowledge of measurement covariance (R) is available, it has been found that performances of all the considered filtering algorithms are closely comparable. Since the main aim of the paper has been to propose an adaptive algorithm when the proper knowledge of measurement noise covariance (R) is unavailable, the results presented here have considered the prior knowledge of R as wrong. Two simulation scenarios have been considered here (i) when prior knowledge of R is scaled up (true R X 100) and (ii) when prior knowledge of R is scaled down (true R / 100). Figure 2, 3 and 4 shows respectively the RMS errors of position, velocity and track losses (as defined in (Sadhu, 2006)) for 10,000 Monte Carlo runs when prior knowledge of R is (true R) X 100.

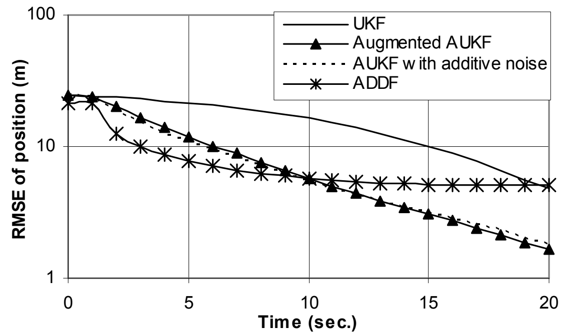

Figure 2: RMSE of position (m) when prior knowledge of R is (true R) X 100.

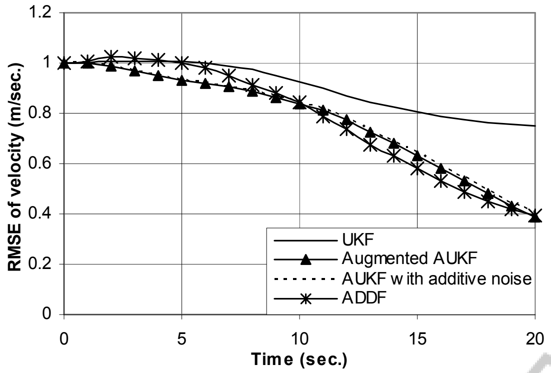

Figure 3: RMSE of velocity (m/sec.) when prior knowledge of R is (true R) X 100.

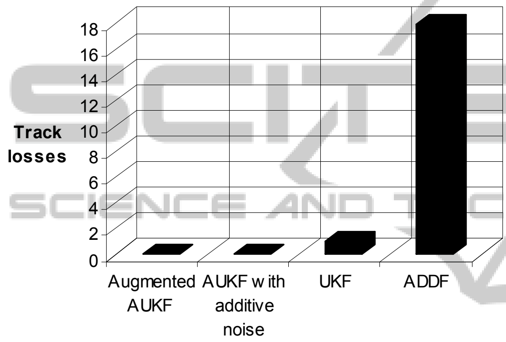

Figure 4: Number of track losses when prior knowledge of R is (true R) X 100.

Figure 4, 5 and 6 shows respectively the RMS errors of position, velocity and track losses for 10,000 Monte Carlo runs when prior knowledge of *R* is (true *R*) / 100.

It may be found from all these simulation results that the proposed adaptive filter provides less RMS errors compared to the other filtering algorithms involved in both the simulation scenarios. Track loss counts are also less in the proposed filter algorithm compared to the other filtering algorithms involved.

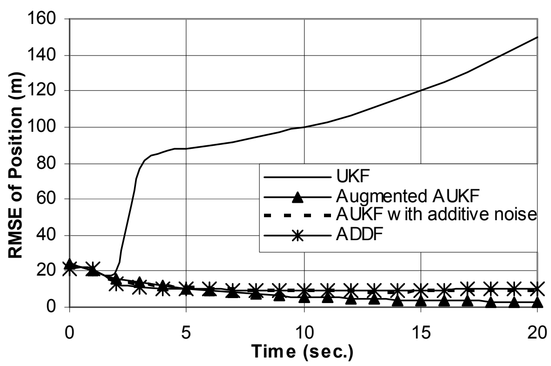

Figure 5: RMSE of position (m) when prior knowledge of R is (true R) / 100.

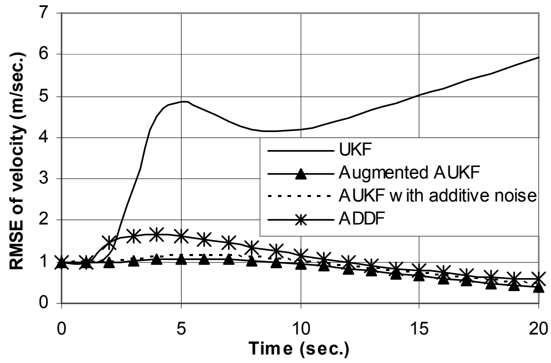

Figure 6: RMSE of velocity (m/sec.) when prior knowledge of R is (true R) / 100.

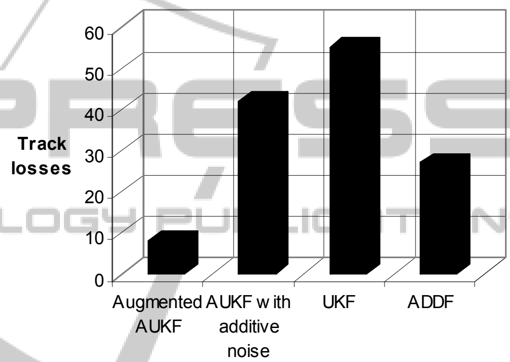

Figure 7: Number of track losses when prior knowledge of R is (true R) / 100.

The chosen 2D-BOT problem is infamous for the track loss problem associated with it. However it has been found that the track loss counts (failure rate) in the proposed Adaptive UKF is less in all simulation scenarios compared to the other nonlinear filtering algorithms considered here. The steady state values of the RMS errors provided by the proposed algorithm are also found to be less compared to the other considered adaptive and non adaptive nonlinear filtering algorithms.

4 CONCLUSIONS

The problem of nonlinear state estimation at the presence of non-additive measurement noise with unknown noise covariance has been considered here. Towards the solution of the above stated problem an Adaptive Unscented Kalman Filter (AUKF) has been proposed which utilizes the augmented form of state vector. It has been found from the total corpus of simulation results that the proposed adaptive filter provides less RMS errors and is more robust to the initial uncertainties associated with the measurement noise covariance (R) compared to few selected existing adaptive and non-adaptive filtering techniques. Probability of failure in the proposed filtering technique has also been observed to be negligible compared to the other filtering techniques involved. The results provided in this paper to demonstrate the superiority of the proposed adaptive filter are expected to encourage further studies on Adaptive Unscented Kalman Filtering techniques for non-additive noise.

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