

# Application of Sliding Mode Control to the Ball and Plate Problem

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**Abstract:** This paper proposes and investigates the application of sliding mode control to the ball and plate problem. The nonlinear properties of the ball and plate control system are first presented. Then the experimental setup designed and built specifically for the purpose of this research is discussed. The paper then focuses on the implementation and thorough evaluation of the experimental results obtained with two different control schemes: the linear full-state feedback controller and the sliding mode controller. The latter control strategy was selected for its robust and order reduction properties. Finally the control performance of the two controllers is analysed. The sliding controller manages to obtain a faster and more accurate operation for continuously changing reference inputs. The robustness of the proposed control scheme is also verified, since the system's performance is shown to be insensitive to parameter variations.

## 1 INTRODUCTION

The ball and plate system, depicted in Figure 1, is one of the most popular educational models developed by control engineers to teach and validate various control strategies. The control objective of the ball and plate problem is to balance a ball, or to make it track a desired trajectory, on a flat plate, solely by tilting the plate relative to the horizontal plane. This system is of particular interest to the control community because it allows the user to study and validate a wide class of both linear and nonlinear control schemes, before applying them to real-life applications that exhibit similar dynamics.

This control challenge, which reportedly originated in the mid 1990s from Rockwell laboratory of Czechoslovakia University, is an extension of the traditional ball and beam system (Moarref et al., 2008), (Wang et al., 2007), (Liu and Liang, 2010). The ball and beam problem is a two degrees of freedom (DOF) system whose objective is to stabilize a rolling ball on a rigid beam. In contrast the ball and plate system exhibits four DOF, namely the two independent motions of the free rolling ball about the plate's plane and the two independent and orthogonal inclinations of the plate which indirectly control the ball's motion. Since the system exhibits less actuators than DOF, then it is clearly underactuated. Another property of this setup is that it is a multiple-input

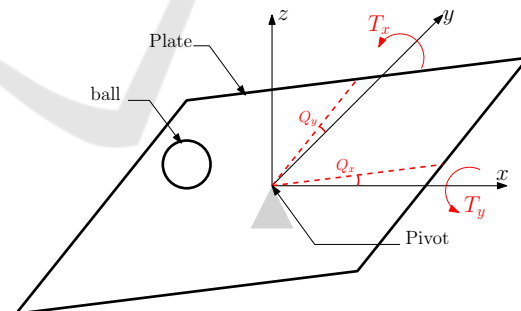


Figure 1: The ball and plate system.

multiple-output (MIMO) system, creating an interesting situation where engineers can study and observe the effects of cross-coupling between different inputs and outputs. In addition the ball and plate setup is also nonlinear and open-loop unstable. All these properties lead to several control challenges that are still being addressed by current research.

Throughout the years a variety of control topologies have been applied to the ball and plate problem. Awatar and Craig, in (Awatar et al., 2002), applied a two-loop cascaded control strategy where the inner loop controlled the plate's actuation mechanism and the outer loop controlled the ball dynamics of the system. The response of the inner loop, for such topologies, needs to appear instantaneous with respect to the outer loop. The inner loop controller consisted of a Proportional-Integral-Derivative (PID)

servo controller while the ball dynamics were controlled through a lead compensator. Similar linear cascaded topologies like the linear full-state feedback controller can also be used where the inner loop usually consists of a digital torque controller while standard LQR and pole-placement design procedures are applied to the outer loop. Such linear topologies manage to achieve a limited performance. This is due to the unaccounted nonlinear dynamics that become more dominant the further the states are from the system's equilibrium point. On the other hand in (Moarref et al., 2008), (Wang et al., 2007) and (Yuan and De-hu, 2009) the authors used a fuzzy estimator in the outer loop to include some "intelligence" in the control action used to regulate the ball position.

In (Ho et al., 2013) a standard linear full-state feedback controller is used with the feedback linearisation topology to control the ball and plate system. Feedback linearisation techniques are subject to limitations such as the undefined relative degree of the linearised ball and plate system at certain locations. The authors managed to apply an approximative input-output feedback linearisation technique, where higher terms of the output could be ignored. A similar control topology that tries to force a nonlinear system to behave like a linear system, is the recursive backstepping method (Khalil, 2002). This method has been successfully applied to regulate the ball's position of two different ball and plate setups in (Ker et al., 2007) and (Hongrui et al., 2008). On the other hand papers (Liu and Liang, 2010) and (Liu et al., 2009) manage to successfully simulate different sliding mode topologies on the ball and plate model.

Sliding mode control is a robust control strategy that guarantees a suitable response even in the face of model imprecisions and external disturbances. Robustness is achieved by employing a discontinuous control action. This paper focuses on the study, comparison and experimental evaluation of this robust control technique. In addition it presents the design and construction of the physical ball and plate experimental test-bed, designed and build specifically for the purpose of such research.

Despite its interesting nonlinear properties, literature indicates that the ball and plate system has not attracted as much attention as other setups like the ball and beam and the inverted pendulum experiments. Recent works, like (Liu and Liang, 2010) and (Liu et al., 2009), present the ball and plate results within a simulation environment. According to Moarref (Moarref et al., 2008), this is due to the structural

complexities involved in the ball and plate setup. Papers (Moarref et al., 2008), (Awatar et al., 2002), (Ker et al., 2007) present different mechanisms that can be used to implement the actual ball and plate system. The L-shaped mechanism presented in (Awatar et al., 2002), is the most popular mechanical structure, to transmit the necessary torques. In fact this is the standard actuation mechanism used in educational setups and research papers (Jadlovská et al., 2009), (Wang et al., 2012). Variations of the same mechanisms are presented in (Yuan and De-hu, 2009), (Ker et al., 2007) and (Yuan and Zhang, 2010) where different types of actuators (pneumatic cylinders and magnetic levitation) are used. The main limitation of such structures is the resulting small angle plate deflections allowed by the actuating mechanism. Moarref (Moarref et al., 2008) presented a different actuation mechanism where two stepper motors are located at the sides of the plate. One of the motors is directly coupled to a metal frame supporting the rotating plate. The other motor is actuating the plate's motion through a mechanical linkage. This structure allows a larger range of motion, but it is subject to limitations when the two motors are operating simultaneously. Like the previous mechanical structures, the encoders do not provide direct feedback of the plate's angular movements. Inaccurate feedback will deteriorate the overall performance of the closed-loop system. Such limitations are not desirable for a setup intended to test nonlinear topologies. Section 3 discusses the ball and plate hardware that we designed and constructed to overcome the limitations imposed by the already available structures.

The rest of the paper is organised as follows. Section 2 introduces the mathematical properties of the ball and plate system. This is followed by a detailed explanation of the constructed experimental setup, which is one of the contributions of this paper. Section 4 focuses on the design of the standard linear topology that was implemented on the ball and plate system. The following section describes the sliding mode scheme and how this was adapted to tackle a multi-variable control problem like the ball and plate system. Section 6 evaluates the resulting robust response that was obtained when the sliding controller was implemented on the experimental setup. The proposed robust control scheme and its experimental comparison and evaluation constitute the other two contributions of this work.

## 2 MATHEMATICAL MODEL

The full nonlinear model of the ball and plate system was derived by using the Euler-Lagrange method and König's theorem. Table 1 lists the parameters used in the mathematical derivation of the model.

$$\ddot{x} = \frac{m_b}{\left(m_b + \frac{J_b}{r_b^2}\right)} \left(-g \sin \theta_x + (x\dot{\theta}_x + y\dot{\theta}_y)\dot{\theta}_x\right) \quad (1)$$

$$\ddot{y} = \frac{m_b}{\left(m_b + \frac{J_b}{r_b^2}\right)} \left(-g \sin \theta_y + (y\dot{\theta}_y^2 + x\dot{\theta}_y\dot{\theta}_x)\right) \quad (2)$$

$$T_x = (J_{px} + J_b + m_b x^2)\ddot{\theta}_x + 2m_b x \dot{x}\dot{\theta}_x + m_b x y \ddot{\theta}_y + m_b \dot{x}y\dot{\theta}_y + m_b x \dot{y}\dot{\theta}_y + m_b g x \cos(\theta_x) \quad (3)$$

$$T_y = (J_{py} + J_b + m_b y^2)\ddot{\theta}_y + 2m_b y \dot{y}\dot{\theta}_y + m_b y x \ddot{\theta}_x + m_b \dot{y}x\dot{\theta}_x + m_b x \dot{y}\dot{\theta}_x + m_b g y \cos(\theta_y) \quad (4)$$

Equations (1) and (2) represent the ball dynamics, while (3) and (4) represent the plate dynamics. These mathematical expressions show how the state variables of the ball and plate setup are interrelated together in a complicated manner making the system much harder to control. The effects of these nonlinear terms become more dominant for faster ball and plate movements, larger plate angles displacements and the displacement of ball from plate's centre. A linear model of the system was derived as shown in (Awtar et al., 2002) and (Ker et al., 2007). Naturally this linear model approximates the nonlinear model only for a limited range of operation around an equilibrium point. If the operating conditions vary widely, such a linear model becomes inadequate to represent of the actual system behaviour. Section 4 explores how these approximations limit the performance that can be reached by standard linear regulators since their design procedures are based on such linear models. The implementation of more complex control topologies, like sliding mode control, should lead to a faster and more accurate performance as well as a larger range of operation, since they are not based on linear models.

## 3 THE EXPERIMENTAL SETUP

From the mathematical analysis presented in Section 2, it can be concluded that the physical ball and plate setup should ideally allow large angular displacements. Recall that the nonlinear terms are more dominant when the system is operated with faster and larger plate angles. The mechanical linkage

Table 1: Parameters of ball and plate mathematical model.

Symbol	Units	Description
$m_b$	kg	Mass of the ball
$r_b$	m	Radius of the ball
$J_b$	kgm <sup>2</sup>	Moment of inertia of the Ball
$x$	m	Ball's position along the x-axis
$y$	m	Ball's position along the y-axis
$\theta_x$	rad	Plate tilt angle in the x-axis
$\theta_y$	rad	Plate tilt angle in the y-axis
$L_x$	m	Plate's length along the x-axis
$L_y$	m	Plate's length along the y-axis
$J_{px}$	kgm <sup>2</sup>	Plate's inertia in the x-axis
$J_{py}$	kgm <sup>2</sup>	Plate's inertia in the y-axis
$T_x$	Nm	Torque applied in the x-axis
$T_y$	Nm	Torque applied in the y-axis

structures discussed in Section 1, (Moarref et al., 2008; Awtar et al., 2002), suffer from a very limited range of inclination angles.

Ideally both actuators should be directly coupled to the plate's axis of rotation. One possible way to achieve this is through a gimbal structure, where the plate is mounted on a metal frame. We have designed and constructed such a setup which is depicted in Figures 2 and 3. The stationary motor is responsible to turn the whole gimbal structure. Hence its rotation is directly coupled to the plate's and frame's axis of rotation, similar to the structure presented in (Moarref et al., 2008). The second actuator provides the second degree of motion by rotating the plate within the frame structure. This creates a non-symmetric setup because the stationary motor needs to move the entire gimbal structure, which includes the second motor. To balance the greater load experienced by the stationary motor a smaller secondary actuator is chosen and a counter-weight is added to balance the whole structure, when stationary.

Another challenge of the ball and plate problem is the sensing method used to accurately track the ball's position on the plate. Awtar in (Awtar et al., 2002), presented a number of different sensors that could be used to track the ball's position. In this work the visual tracking method was chosen since the plate provides a plain black background that contrasts with the coloured ball. Hence simpler colour tracking algorithms could be used. Still a personal computer is not the best machine to handle the computational demands required by real-time image processing algorithms. Hence an intelligent visual sensor was used, namely the CMUcam4. This visual module does all the required image processing algorithms on board,

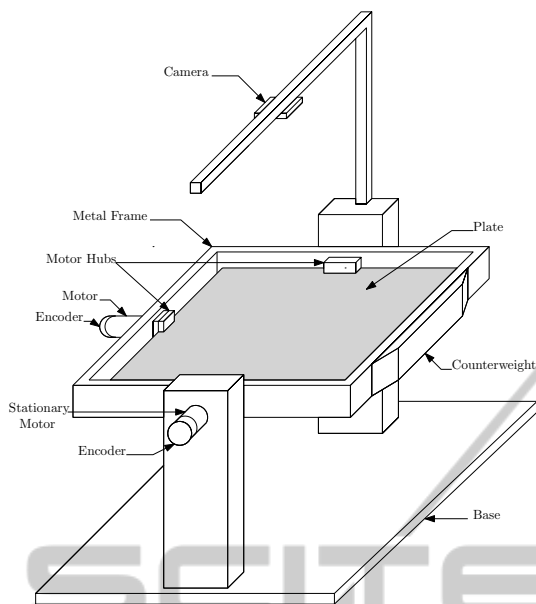


Figure 2: The mechanical design of the proposed ball and plate system.



Figure 3: The constructed ball and plate system.

by dividing the computations between eight processors working simultaneously. Hence it is able to track and provide the ball's location, to the main processing unit, at a 30Hz rate. This tracking update rate was verified that is high enough to capture the desired closed-loop dynamics.

## 4 LINEAR CONTROL TOPOLOGIES

Classical control is a mature field of study offering a set of powerful tools for the analysis and design of linear time-invariant systems. Section 1 showed how standard classical topologies, like the state-feedback controller, have successfully been applied to the ball and plate problem. These methods are based on the linearised state-space models of the nonlinear ball and plate dynamics shown in (1) - (4). The pole-placement design procedure entails the selection of appropriate state-feedback gains, that enforce the desired closed-loop eigenvalues. The full-state control feedback law,  $u$ , is given by:

$$u = -\mathbf{K}^T \mathbf{x} + gr \quad (5)$$

where:

$r$  is the desired reference input.

$\mathbf{K}$  is a gain matrix that ensures that the system achieves the desired closed-loop eigenvalues.

$\mathbf{x}$  is the state vector of the system.

$g$  is the feedforward gain that ensures zero steady-state error between the system's output and its reference input.

Figure 4 shows how this topology is applied to the ball and plate problem. Note that the block diagram does not show the inner-most loop which controls the armature currents requested by the outer state-feedback regulators. In this case the current response is assumed to be instantaneous with respect to the state-feedback regulator. Another observation, from Figure 4, is that two state-feedback regulators are required to control the two axes of the system. This is due to the linearisation process presented in Section 2 which decouples the two axes of the ball and plate system from each other. Hence, when using classical control design procedures, like the state-feedback linear regulators, the effects introduced by cross-coupling terms are ignored. Recall from the nonlinear model, shown in (1) - (4), that these terms become more dominant for larger non-equilibrium conditions and thus limit the type and range of achievable performance. This is further discussed in Section 6.

The state-feedback regulator, shown in Figure 4, is unsuitable for continuously changing inputs or in cases where the system is subjected to external disturbances. Hence an extra integral action is added to the feed forward path of each regulator. This modification adds another state variable to both axes while leaving the actual design process, for the calculation



of value  $\mathbf{K}$ , unchanged. The results shown in Section 6 were obtained with this modified state-feedback topology, which is usually referred to as the state-feedback tracker.

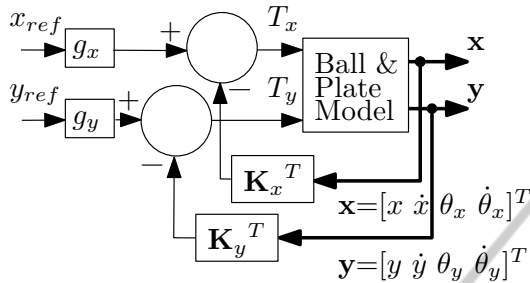


Figure 4: Linear regulator topology.

## 5 SLIDING MODE CONTROL

Sliding mode control aims to enforce the system trajectories into a desired manifold in order to obtain the desired closed-loop dynamics. This is achieved by using a discontinuous control action (Slotine and Li, 1991). This topology can guarantee a suitable system response even in the face of model uncertainties and external disturbances. Moarref (Moarref et al., 2008) shows only one set of results from the experimental implementation of the sliding mode controller, where the ball had to balance at the centre of the plate. The plot indicates that the ball took approximately nine seconds to stabilise at the origin. In (Liu and Liang, 2010) and (Liu et al., 2009) the authors manage to simulate this nonlinear control topology on the ball and plate model. Liu and Liang, specify that they used a double feedback structure to control the full ball and plate dynamics (Liu and Liang, 2010). On the other hand (Liu et al., 2009) assumes that the plate dynamics do not affect the ball's position. In both cases, only the ball dynamic equations are used, and an ideal servo response is assumed for the plate dynamics. This assumption was made due to the limitations of standard sliding mode control design topologies, which are based on nonlinear models that have a scalar output and are affine in the control input. Hence the simulations, in both papers ignore the effects introduced by the plate dynamics.

Figure 5 shows the full sliding mode control topology that we are proposing and that we have implemented on the constructed ball and plate experimental testbed, presented in Section 3. To control the plate dynamics a PID controller was designed, leading to a total of three cascaded loops. Figure 5, like Figure 4, does not

show the inner-most current loop since it is assumed that this exhibits an instantaneous response with respect to the control actions requested by the PID regulators. Ideally the PID controllers should also appear instantaneous with respect to the switching control action that is requested by the outer sliding mode controllers. The high control activity requested by the sliding controller is very difficult to achieve with any mechanical servo mechanism, resulting in a degradation of the system's output response. Recall that the simulation results shown in (Liu and Liang, 2010) and (Liu et al., 2009) did not take into consideration the plate dynamics and their effects. But such matters cannot be ignored when the proposed scheme is to be implemented and validated on the the constructed ball and plate hardware. This is further discussed in Section 6.

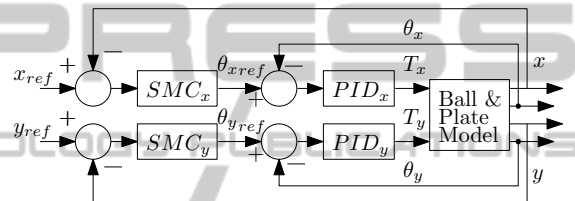


Figure 5: The sliding mode control topology.

The works in (Liu and Liang, 2010) and (Liu et al., 2009) differ in the way they tackle the nonlinearities present in the ball dynamic equations. Liu and Lang remove the cross-coupling terms leading to a decoupling of the two axes In (Liu et al., 2009) the authors assume a plate deflection of  $\pm 5^\circ$ , leading to the removal of trigonometric terms from the mentioned equations. Unlike (Liu and Liang, 2010), the authors of (Liu et al., 2009) retain the cross-coupling terms which are estimated through an uncertain item observer. More accurate system dynamics would reduce the need for large switching functions to ensure system robustness. For the scope of this research the effects introduced by the cross-coupling terms were not considered during the design procedure. Equation (1) describes the ball dynamics, in the x-direction, of the ball and plate system. Removing the cross-coupling terms from (1) results in the standard affine nonlinear second order model:

$$\ddot{x} = \frac{1}{\left(m_b + \frac{J_b}{r_b^2}\right)} (-m_b g \sin \theta_x) = bu(\theta_x) \quad (6)$$

where:

$$u(\theta_x) = \sin \theta_x \quad \text{and} \quad b = \frac{-m_b g}{\left(m_b + \frac{J_b}{r_b^2}\right)}$$

The primary aim of the discontinuous switching controller is to impose the desired dynamics on the sys-

tem being controlled. For a second order system, the resulting surface  $s$  would have the following form:

$$s = \dot{\tilde{x}} + \lambda\tilde{x} \quad (7)$$

where:

$\tilde{x} = x - x_d$  is the tracking error between the system's output and the desired reference input,  $x_d$ .

$\lambda$  is a strictly positive constant that sets the desired dynamics of the sliding surface.

The switching action ensures that the state trajectory of the system reaches and remains on the sliding surface. Once the state trajectory reaches the sliding surface, the dynamics defined by  $\lambda$  will be imposed on the system's output response. Another action that can be added in conjunction with the switching function is the equivalent control term,  $u_{eq}$ . To obtain the  $u_{eq}$  expression,  $\dot{s}$  is assumed to be equal to zero and  $\ddot{x}$  terms are substituted with (6). Theoretically the derived expression would result in a continuous control law that can maintain the trajectory on the desired sliding surface provided that the exact model of the system is known. In this case equivalent control could theoretically replace the discontinuous control function. Practically this is never the case due to model uncertainties and external disturbances present in the actual system. Hence the following control action is used:

$$u = \arcsin\left(\frac{1}{\hat{b}}(\ddot{x}_d - \lambda\dot{\tilde{x}} - \beta\text{sgn}(s))\right) \quad (8)$$

where:

$\hat{b}$  is an estimate of the nonlinear model coefficient  $b$ , shown in (6).

$\beta\text{sgn}(s(t))$  is the bang-bang action, multiplied by a gain  $\beta$ , which ensures the system's robustness.

One of the greatest advantages of this topology is the intuitive tuning of control parameters  $\lambda$  and  $\beta$ , due to the sliding mode's order reduction property. Control parameter  $\lambda$  has a direct influence on the control bandwidth of the whole system. Two factors that limit the selection of  $\lambda$ , hence limiting the performance that can be achieved by the system, are neglected time delays and the control loop's available sampling rate. Switching gain  $\beta$  determines how fast the trajectory is moving towards the surface and the resulting switching that slides the trajectory along the selected sliding surface. When selecting parameter  $\beta$  a compromise between a faster response and smaller chattering effects must be found. Proper selection of these control variables should lead to an asymptotically stable system as discussed in (Liu and Liang, 2010).

Each paper contributed different terms to the standard sliding mode control equation, shown in (8), to improve the overall performance of the system. In (Liu and Liang, 2010), the authors add the proportional term  $\alpha s(t)$  which reduces the time the trajectory takes to reach the sliding surface. On the other hand, in (Moarref et al., 2008) the authors tried to eliminate the chattering effects by replacing the discontinuous signum function with a continuous saturating function. Another modification is to add an integral term to the sliding surface which will improve the controller's performance to continuously changing reference inputs. When adding the integrator to the sliding mode controller, (8) changes to:

$$u = \arcsin\left(\frac{1}{\hat{b}}(\ddot{x}_d - 2\lambda\dot{\tilde{x}} - \lambda^2\tilde{x} - \beta\text{sgn}(s))\right) \quad (9)$$

## 6 RESULTS AND EVALUATION

This section will focus on the results obtained when the presented control topologies were applied to the constructed hardware. Figure 6 shows the results obtained when the linear full-state tracker and the sliding mode tracker were implemented on the constructed ball and plate system. Recall from Section 4 that linear control topologies do not take into consideration nonlinear and cross-coupling terms. These terms become more dominant with faster responses and larger ranges of operation. Faster specifications always result in smaller ranges of operation and stability. Hence the performance requirements had to be limited to ensure that the ball reached the desired trajectory even if its initial conditions are not on the desired trajectory. For successful operation the tracker specifications had to be decreased to a 0.6s rise time and a 15% peak overshoot. Faster requirements are possible, but would not guarantee stability if the initial position of the ball is not on the desired trajectory. For the set specifications the following tracker parameters were derived:

$$\begin{aligned} \mathbf{K}_x^T &= [-18.43 \quad -7.023 \quad 10.41 \quad 1.049] \\ \mathbf{K}_y^T &= [-5.412 \quad -1.783 \quad 2.643 \quad 0.2689] \\ g_x &= -23.86 \quad g_y = -6.059 \end{aligned}$$

The linear tracker results, in Figure 6, show how the system behaves when subjected to a sinusoidal reference input with a  $\pm 0.12\text{m}$  magnitude and frequency of 0.1Hz. The tracker's response resulted in an attenuated output that lagged behind the desired reference input. The resulting root mean square (RMS) tracking errors, for the  $x$  and  $y$  axes, were equal to 0.0545m and 0.0497m respectively. The

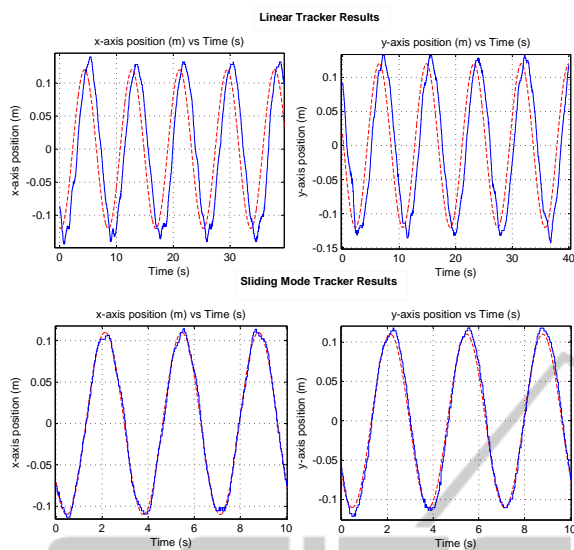


Figure 6: Results obtained when Linear and Sliding Mode trackers were applied to the ball and plate setup.

observed output delay and attenuation continued to increase when faster reference inputs were requested. The resulting tracking error, when the reference input was increased to 0.3Hz, increased to 0.1217m and 0.113m for the  $x$  and  $y$  axes respectively. These large delays and errors are due to the system's closed-loop bandwidth which is unable to handle such high-speed dynamics. On the other hand faster closed-loop bandwidths are not possible due to the limitations introduced by the unaccounted nonlinear terms.

The sliding mode control topology presented in Section 5 does not take into consideration the plate dynamics of the ball and plate system. Hence a PID control loop was added between the outer sliding mode controller and the inner torque controller to control the plate dynamics. Recall that the high control activity requested by the sliding controller is very difficult to achieve with any mechanical servo mechanism. The best performance that was achieved by the PID controller was insufficient to keep up with the discontinuous switching control action that was being updated every 0.033s. This would result in an imperfect and delayed sliding control action which would result in a degradation of the system's output response and an increase in the chattering problem. The effects introduced by the plate dynamics are not shown in (Liu and Liang, 2010) and (Liu et al., 2009) since both papers assumed an ideal servo response in their simulations. Despite these limitations, when applying the switching control action shown in (9), we still managed to obtain a very accurate tracking response. The sliding mode control parameters  $\beta$  and  $\lambda$  were

heuristically tuned according to the selection criteria discussed at the end of Section 5:

$$\beta = 0.35 \quad \lambda = 2.5$$

The sliding mode tracker results, in Figure 6, show the system's output response when it was subjected to a sinusoidal signal with a magnitude of  $\pm 0.11$ m and a frequency equal to 0.3Hz. The resulting RMS tracking errors for the  $x$  and  $y$  axes were equal to 0.0055m and 0.0096m respectively. Unlike the linear tracker scenario, the trajectory response of the sliding controller was not limited to  $\pm 0.11$ m but during operation the magnitude of the sinusoidal reference input was successfully increased to  $\pm 0.14$ m without deteriorating the overall response of the system. Hence the sliding mode controller managed to get a faster and more precise response for bigger ranges of operation. This is due to the controller's robust nature which makes the output response more insensitive to the increasing effects introduced by the nonlinear dynamics present in the system. Moreover, in contrast to the linear controller, the sliding-mode scheme managed to cope very well even in cases when the plant dynamics were modified deliberately, by changing the ball with one that is forty times heavier (a metal ball bearing instead of a tennis ball). In the case of the sliding-mode controller there was no degradation in performance, while the linear controller simply went unstable. This verifies experimentally that proposed controller is very robust to high variations in the model parameters.

It is interesting to note that the sliding mode controller does not perform well when subjected to constant reference inputs, even when the plate dynamics were not being considered in the simulation environment. In this case the sliding mode response of the system was never able to reach the speed and range that was achieved with the linear state-space regulator. This is due to the equivalent control action which is working against the switching function during the initial phase when the system's trajectory is reaching the sliding surface. This scenario is made worse by the imperfect switching introduced by the slow plate dynamics and the large  $\beta$  values due to the motors' dead-zones. The latter effects were reduced by switching the signum function with a saturation function, at the expense of reducing the controller's robustness.

## 7 CONCLUSION

In this paper a mechatronic design of the ball and plate system was presented. Two different control schemes were presented and their respective performance, was experimentally validated on a physical

testbed that was designed and constructed for the purpose of this research. Linear state-feedback controller was discussed in detail. This was followed by an analysis of the results obtained by the sliding mode controller. When compared to the linear tracker results, the sliding controller managed to obtain a more precise response at much higher speeds.

The imperfect switching, introduced by the servo-loop which could not handle the high control activity requested by the sliding mode controller, was one of the main factors which limited the performance of this robust control strategy. Future research should focus on designing a switching function that takes into consideration all the states of the system. Hence effectively designing a multidimensional sliding mode controller that would consider both the ball and plate dynamics.

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## REFERENCES

- Awtar, S., Bernard, C., Boklund, N., Master, A., Ueda, D., and Craig, K. (2002). Mechatronic design of a ball-on-plate balancing system. *Mechatronics*, 12(2):217–228.
- Ho, M.-T., Rizal, Y., and Chu, L.-M. (2013). Visual servoing tracking control of a ball and plate system: Design, implementation and experimental validation. *International Journal of Advanced Robotic Systems*, 10.
- Hongrui, W., Yantao, T., Siyan, F., and Zhen, S. (2008). Nonlinear control for output regulation of ball and plate system. In *Control Conference, 2008. CCC 2008. 27th Chinese*, pages 382–387. IEEE.
- Jadlovská, A., Jajcisi, S., and Lonscak, R. (2009). Modelling and pid control design of nonlinear educational model ball and plate. In *Proceedings of the 17th International Conference on Process Control 2009*, pages 475–483. Slovak University of Technology in Bratislava.
- Ker, C. C., Lin, C. E., and Wang, R. T. (2007). Tracking and balance control of ball and plate system. *Journal of the Chinese Institute of Engineers*, 30(3):459–470.
- Khalil, H. K. (2002). *Nonlinear systems*, volume 3. Prentice Hall Upper Saddle River.
- Liu, D., Tian, Y., and Duan, H. (2009). Ball and plate control system based on sliding mode control with uncertain items observe compensation. In *Intelligent Computing and Intelligent Systems, 2009. ICIS 2009. IEEE International Conference on*, volume 2, pages 216–221. IEEE.
- Liu, H. and Liang, Y. (2010). Trajectory tracking sliding mode control of ball and plate system. In *Informatics in Control, Automation and Robotics (CAR), 2010 2nd International Asia Conference on*, volume 3, pages 142–145. IEEE.
- Moarref, M., Saadat, M., and Vossoughi, G. (2008). Mechatronic design and position control of a novel ball and plate system. In *Control and Automation, 2008 16th Mediterranean Conference on*, pages 1071–1076. IEEE.
- Slotine, J.-J. E. and Li (1991). *Applied nonlinear control*, volume 199. Prentice hall New Jersey.
- Wang, H., Tian, Y., Sui, Z., Zhang, X., and Ding, C. (2007). Tracking control of ball and plate system with a double feedback loop structure. In *Mechatronics and Automation, 2007. ICMA 2007. International Conference on*, pages 1114–1119. IEEE.
- Wang, Y., Li, X., Li, Y., and Zhao, B. (2012). Identification of ball and plate system using multiple neural network models. In *System Science and Engineering (ICSSE), 2012 International Conference on*, pages 229–233. IEEE.
- Yuan and De-hu (2009). Pneumatic servo ball and plate system based on touch screen and oscillating cylinder. In *Intelligent Systems and Applications, 2009. ISA 2009. International Workshop on*, pages 1–4. IEEE.
- Yuan, D. and Zhang, Z. (2010). Modelling and control scheme of the ball-plate trajectory-tracking pneumatic system with a touch screen and a rotary cylinder. *IET Control Theory & Applications*, 4(4):573–589.