# Synchronization of Uncertain Chaotic Systems using Generalized Predictive Control based on Fuzzy PID Controllers

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Abstract: In this paper, we investigate the synchronization of chaotic systems with unknown parameters using generalized predictive control based on fuzzy PID controllers. In order to verify the efficiency of the proposed method, fuzzy PD+I and fuzzy PI+D controllers are successively used with and without prediction terms for the synchronization of two uncertain Lorenz systems. For fuzzy PD+I controller, the prediction terms seem to be efficient for the synchronization. However, with the fuzzy PI+D controller, they make a noise and worsen the performance of the controller.

## 1 INTRODUCTION

Synchronization of chaotic systems has been widely investigated in the last decades. Due to their sensitivity to initial conditions and random behavior, they have been categorized among one of the most interesting topics of nonlinear science. Uncertainties on parameters are another problem that worsens the task of synchronization. Many classical approaches failed to reach the synchronization and some advanced control approaches and improved schemes such as fuzzy logic control(FLC) (Lam and Leung, 2006), neural network(NN) (Lam and Seneviratne, 2007), adaptive control strategy (Sun et al., 2013), are used to resolve this problem.

Model predictive control (MPC) (Dumur and Boucher, 1994) is a control approach which consists in using a model of a system to predict its output over an extended horizon. In the presence of uncertainties, self-tuning and model-reference adaptive control(MRAC) were used with MPC to solve many problems such as an open-loop unstable plant, a nonminimum-phase plant, a plant with variable or unknown dead-time and a plant with unknown order. However, there was not a general algorithm to solve all these problems at once until the establishment of a general algorithm by D.W. Clarke (Clarke et al., 1987) in 1985 called generalized predictive control(GPC).

The drawback of GPC is the number of mathematical steps the algorithm requests. In order to fix this problem, several advanced control approaches have been involved in GPC such as fuzzy-model-based approach (Lam and Leung, 2006), Neural-network (Jinquan and Lewis, 2003), and PSO-based model predictive control (Wang and Xiao, 2005). One of the most interesting approaches (Lu et al., 2001) is by involving fuzzy PID controllers to minimize the cost function and to ensure the convergence.

In this paper, we consider the performance of GPC based on fuzzy PID controllers (Lu et al., 2001) for the synchronization of uncertain chaotic systems. Fuzzy PI+D and fuzzy PD+I controllers are successively used to check the performance of the proposed control method in the presence or absence of prediction terms. For the prediction of the future variation of the master and the salve system, an ARX model is used. To verify the above proposed approach performance, we apply it for the synchronization of two uncertain Lorenz systems.

The rest of the paper is arranged as follows: Section 2 presents synchronization of chaotic systems. GPC based on fuzzy PID controllers is introduced in Section 3. A brief description of fuzzy PI+D and fuzzy PD+I controllers in Section 4. Simulation results are given in Section 5. Conclusions are given in Section 6.



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## 2 SYNCHRONIZATION OF UNCERTAIN CHAOTIC SYSTEMS

Let's consider two $n$-dimensional chaotic systems, one is designed as the master system:

$$\dot{x}_m = g_m(x,t), \quad 1 \leq m \leq n \tag{1}$$
$$x = [x_1, x_2, ..., x_n] \in \Re^n$$

and the second is the controlled slave system:

$$\dot{y}_m = f_m(y,t) + u_m(t), \quad 1 \leq m \leq n \tag{2}$$
$$y = [y_1, y_2, ..., y_n] \in \Re^n$$

$f$ and $g$ represent unknown nonlinear functions, and $u \in \Re^n$ is the control input.

The Synchronization problem can be considered as a control problem which consists in the design of an appropriate control law $u(t)$ such that:

$$\lim_{t \to \infty} \|y(t) - x(t)\| \to 0 \tag{3}$$

The error states between the two systems are given by:

$$\dot{e}_m = f_m(y,t) - g_m(x,t) + u_m(t), \qquad 1 \leq m \leq n \tag{4}$$

And the objective is how to design an efficient control law $u_m(t)$ such that the error states converge to zero when the time goes further.

## 3 GPC BASED ON FUZZY PID CONTROLLERS

GPC algorithm consists mainly in minimizing a cost function that contains the predicted values. There have been many attempts to reduce the complexity of the algorithm by involving some advanced control approaches. To avoid the tedious mathematical steps, fuzzy PID controllers can be used (Lu et al., 2001). For the synchronization, the following criterion is used:

$$J_m(k) = \textstyle\sum_{i=-1}^{N} [y_m(k-i) - x_m(k-i)]^2 + \lambda \sum_{j=0}^{N_c} [\Delta u_m(k-j)]^2$$
$$J_m(k) = \textstyle\sum_{i=-1}^{N} [e_m(k-i)]^2 + \lambda \sum_{j=0}^{N_c} [\Delta u_m(k-j)]^2 \tag{5}$$

Where $N$ is the prediction horizon, $N_c$ is the control increment horizon, $\Delta u_m$ is the incremental output of a controller, $\lambda \geq 0$ is a control increment weight. Figure 1 represents the main structure of GPC based on fuzzy PID controllers for synchronization of uncertain chaotic systems.

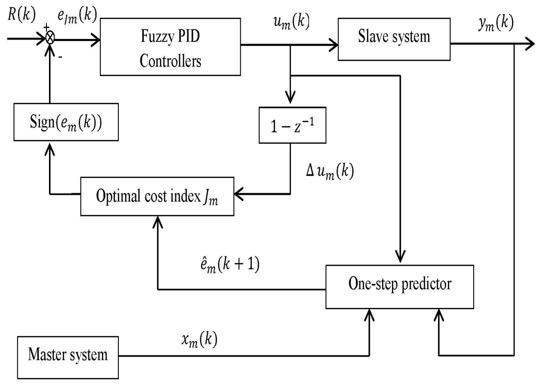


Figure 1: Block diagram of GPC based on fuzzy PID controllers.

To get the predicted values of both systems, we use ARX model. For the slave system, the model is given by:

$$\widehat{y}_m(k+1) = a_1 y_m(k) + a_2 y_m(k-1) + a_3 y_m(k-2) + a_4 y_m(k-3) + b_1 u_m(k-1) \tag{6}$$

while the model of the master is given by:

$$\widehat{x}_m(k+1) = a_1 x_m(k) + a_2 x_m(k-1) + a_3 x_m(k-2) + a_4 x_m(k-3) \tag{7}$$

Where $x_m(k)$, $y_m(k)$ and $u_m(k)$ are the output of the master system, the output of the slave system and the control input respectively; $a_1, a_2, a_3, a_4, b_1$ are constant parameters.
Thus, the one-step ahead predictor of the error states is given as:

$$\widehat{e}_m(k+1) = a_1 e_m(k) + a_2 e_m(k-1) + a_3 e_m(k-2) + a_4 e_m(k-3) + b_1 u_m(k-1) \tag{8}$$

## 4 FUZZY PID CONTROLLERS

Fuzzy PI+D and fuzzy PD+I controllers are used to perform two tasks: drive the salve system to track the output of the master system , make the cost function $J_m$ as small as possible.

The main steps to design both fuzzy PI+D and fuzzy PD+I controllers are developed as follows. The continuous form of a conventional fuzzy PI controller (Tang et al., 2001) is given by:

$$\begin{cases} u_{PI}(t) = K_p e_{Jm}(t) + K_i \int e_{Jm}(t)dt \\ e_{Jm}(t) = R(t) - J_m(t) \times Sign(e_m(t)) \end{cases} \tag{9}$$

Where $R(t)$ is the reference for the optimal cost index; $K_p$ is the constant proportional gain; $K_i$ is integral gain. $e_{Jm}(t)$ and $e_m(t)$ are the error signal from the optimal index $J_m$, the error between the master and the slave system, respectively.

Using the Laplace transform, we obtain the analog PI controller in the frequency domain

$$u_{PI}(s) \quad = (K_p + \frac{K_i}{s})E_{Jm}(s) \qquad (10)$$

By applying the bilinear transform $s = (2/T)((z+1)/(z-1))$, where $T > 0$ is the sampling period, we obtain the discrete version

$$u_{PI}(z) \quad = (K_p - \frac{K_i T}{2} + \frac{K_i T}{1-z^{-1}})E_{Jm}(z) \qquad (11)$$

By taking

$$K_P = K_p - \frac{K_i T}{2} \ , \ K_I = K_i T$$

and then using the inverse z-transform, we obtain

$$u_{PI}(k) \quad = u_{PI}(k-1) + T\Delta u_{PI}(k) \qquad (12)$$

Where

$$\Delta u_{PI}(k) = K_P \dot{e}_{Jm}(k) + K_I e_{Jm}(k) \qquad (13)$$

In equation (12), $T\Delta u_{PI}(k)$ represents the incremental output of a conventional PI controller.

By replacing the term $T\Delta u_{PI}(k)$ with a fuzzy control action $K_{uPI}\Delta u_{PI}(k)$ , we obtain the equation of the fuzzy PI controller

$$u_{PI}(k) = u_{PI}(k-1) + K_{uPI}\Delta u_{PI}(k) \qquad (14)$$

where $K_{uPI}$,$K_P$ and$K_I$ are constant control gains.
By applying the same steps, we obtain the fuzzy D control law given by

$$u_D(k) = -u_D(k-1) + K_{uD}\Delta u_D(k) \qquad (15)$$

The overall fuzzy PI+D control law can be obtained by algebraically summing the fuzzy PI control law ( 14) and the fuzzy D law ( 15):

$$\begin{aligned} u_{PID}(k) = u_{PI}(k-1) + K_{uPI}\Delta u_{PI}(k) \\ +u_D(k-1) - K_{uD}\Delta u_D(k) \end{aligned} \qquad (16)$$

Using the same steps of fuzzy PI+D (Lu et al., 2001), we get the equations of fuzzy PD+I controller:

$$u_{pdi}(k) = u_{pd}(k) + u_i(k) \qquad (17)$$

Where

$$u_{pd}(k) = -u_{pd}(k-1) + K_{u_{pd}}\Delta u_{pd}(k)$$

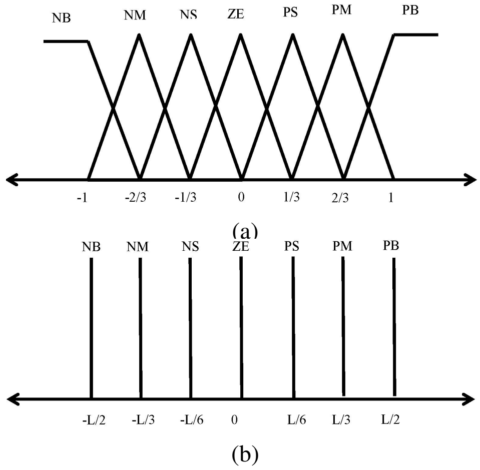


Figure 2: Input-output membership functions for fuzzy PI+D and fuzzy PD+I controllers. (a) Membership functions of $e_{Jm}, \dot{e}_{Jm}$. (b) Membership functions of $\Delta u_{PI}(k)$, $\Delta u_D(k)$, $\Delta u_{pd}(k)$ and $\Delta u_i(k)$.

$$u_i(k) = u_i(k-1) + K_{u_i}\Delta u_i(k)$$

For the both controllers, seven triangular membership functions are used for input linguistic variables $e_{Jm}$,$\dot{e}_{Jm}$. The same number of functions is assigned to output linguistic variables $\Delta u_{PI}(k)$, $\Delta u_D(k)$, $\Delta u_{pd}(k)$ and $\Delta u_i(k)$.

The functions are designed as negative big (NB), negative medium (NM), negative small (NS), zero (ZE), positive small (PS), positive medium (PM) and positive big (PB). The membership functions are shown in Figure 2 while the fuzzy rule table is designed in Table 1.

Table 1: Fuzzy rule of fuzzy PI+D and fuzzy PD+I controllers.

| | | $\dot{e}_{Jm}$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | **PB** | **PM** | **PS** | **ZE** | **NS** | **NM** | **NB** |
| | **PB** | NB | NB | NB | NM | NS | NS | ZE |
| | **PM** | NB | NM | NM | NM | NS | ZE | PS |
| | **PS** | NB | NM | NS | NS | ZE | PS | PM |
| $e_{Jm}$ | **ZE** | NB | NM | NS | ZE | PS | PM | PB |
| | **NS** | NM | NS | ZE | PS | PS | PM | PB |
| | **NM** | NS | ZE | PS | PM | PM | PM | PB |
| | **NB** | ZE | PS | PS | PM | PB | PB | PB |

# 5 SIMULATION RESULTS

The performance of the proposed algorithm is tested for the synchronization of two uncertain Lorenz systems.

The master is defined by:

$$\begin{cases} \dot{x_1} &= \alpha_1(x_2 - x_1) \\ \dot{x_2} &= (-x_1x_3 + \rho_1x_1 - x_2) \\ \dot{x_3} &= x_1x_2 - \beta_1x_3 \end{cases} \qquad (18)$$

Where $x_1, x_2, x_3$ are the state variables and $\alpha_1, \rho_1, \beta_1$ are positive uncertain parameters of the system.

And the slave by:

$$\begin{cases} \dot{y_1} &= \alpha_2(y_2 - y_1) + u_1 \\ \dot{y_2} &= (-y_1 y_3 + \rho_2 y_1 - y_2) + u_2 \\ \dot{y_3} &= y_1 y_2 - \beta_2 y_3 + u_3 \end{cases} \tag{19}$$

Where $y_1, y_2, y_3$ are the state variables , $\alpha_2, \rho_2, \beta_2$ are positive uncertain parameters and $u_1, u_2, u_3$ are the GPC based on fuzzy PI+D or fuzzy PD+I controller.

The synchronization errors are defined as:

$$\begin{cases} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{cases} \tag{20}$$

And the error states as:

$$\begin{cases} \dot{e_1} = \alpha_2(y_2 - y_1) - \alpha_1(x_2 - x_1) + u_1 \\ \dot{e_2} = \rho_2 y_1 - y_2 - y_1 y_3 - \rho_1 x_1 + x_2 + x_1 x_3 + u_2 \\ \dot{e_3} = y_1 y_2 - \beta_2 y_3 - x_1 x_2 + \beta_1 x_3 + u_3 \end{cases} \tag{21}$$

To synchronize these chaotic systems ,we chose the following optimal index:

$$J_m(k) = \sum_{i=-1}^{3} [e_m(k-i)]^2 + \lambda \sum_{j=0}^{3} [\Delta u_m(k-j)]^2 \tag{22}$$

Where $\lambda = 0.001$.

The one-step predictor of the error states is given as:

$$\begin{aligned} e_m(k+1) &= 0.9497 e_m(k) + 0.0141 e_m(k-1) \\ &+ 0.6806 e_m(k-2) + 0.6440 e_m(k-3) \\ &+ 0.051 u_m(k-1) \end{aligned} \tag{23}$$

Figures 3 and 4 show the one step predictors of the states $x_2$, $y_2$, and their prediction errors.

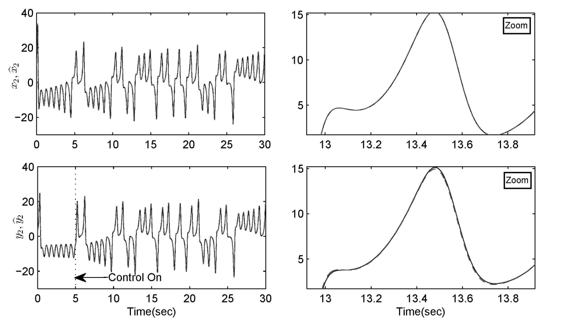


Figure 3: One step predictors of the states $x_2, y_2$ using PI+D controller.

For the numerical simulation, the parameters of the master and the slave systems are chosen respectively as:

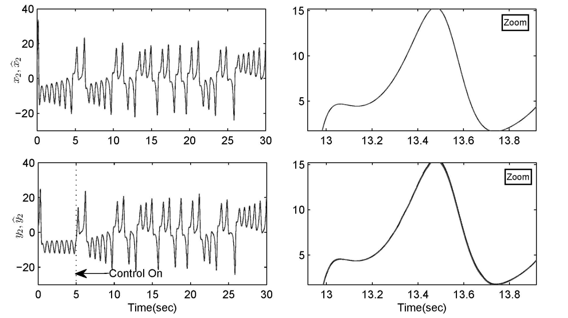


Figure 4: One step predictors of the states $x_2, y_2$ using PD+I controller.

$\alpha_1 = 10, \rho_1 = 28, \beta_1 = 8/3$,
$\alpha_2 = 10.5, \rho_2 = 25, \beta_2 = 8/3 + 0.2$

The initial conditions of the master and the slave systems are taken as :
$x_1(0) = 2, x_2(0) = 10, x_3(0) = -6$,
$y_1(0) = -2, y_2(0) = 5, y_3(0) = 1$

In the first part of the simulation, we present two results obtained using fuzzy PD+I controller. One with prediction terms and the other without. The parameters of the fuzzy PD+I are chosen as:$K_p = 1.014, K_d = 0.594, K_{upd} = 0.1, K = 1, K_i = 2.045, K_{ui} = 0.1, L = 30$.

In the second part, fuzzy PI+D controller is used instead of fuzzy PD+I controller. The parameters of the fuzzy PI+D controller are set: $L = 30, Ku_{PI} = 1, K_I = 1, K_P = 1, K_{uD} = 0.001, K_D = 1$.

Figure 5 and Figure 6 show the results of the synchronization of the two systems and the variations of the cost functions without prediction terms and with prediction terms respectively for the first case, while Figure 7 and Figure 8 give the results for the second case.

For the first case, we can notice that in the absence of prediction terms, the synchronization between the two systems is destroyed, and the cost functions take huge values. However, in the presence of the prediction terms, the synchronization is achieved and the cost functions converge to zero.

For the second case, the synchronization between the two systems is achieved with and without prediction terms.

Table 2 summarizes all the results obtained by the two controllers with and without prediction terms. In the case of fuzzy PD+I controller, one-step prediction terms ensure the synchronization between the two systems. However, with the fuzzy PI+D controller, they make a noise and reduce the performance of the controller. Moreover, the performance of fuzzy PI+D controller is better than the fuzzy PD+I controller in the two cases. The table shows also that the

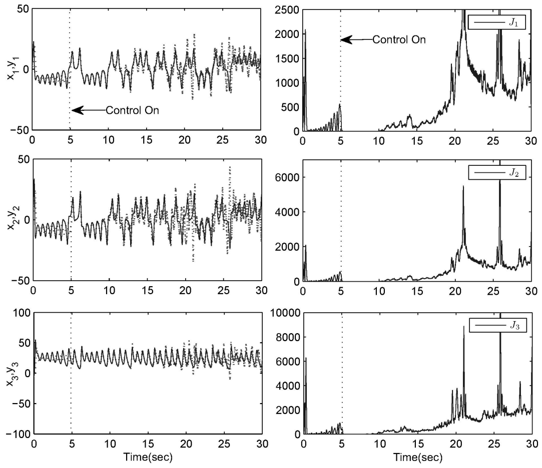


Figure 5: Synchronization of the Lorenz systems and cost functions variations without prediction terms using fuzzy PD+I controller.

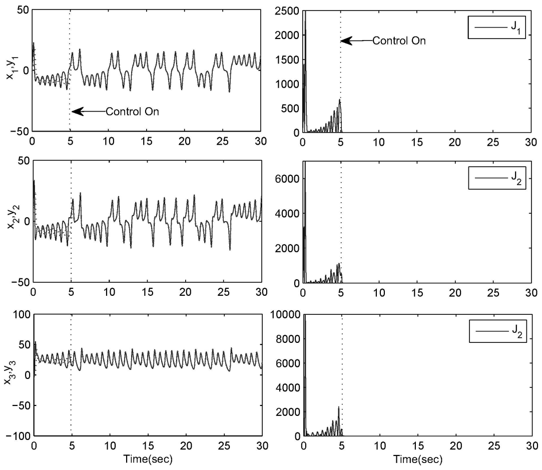


Figure 6: Synchronization of the Lorenz systems and cost functions variations with prediction terms using fuzzy PD+I controller.

prediction terms worsen the results, and this can be explained by: the modeling error which is considered as perturbation terms added to the cost functions, the unpredictability behavior of chaotic systems, or the structure of the proposed control method which may need improvements. Although the structure of the algorithm is simpler than many others (Yan and Wang, 2012; Wang and Sun, 2010; Song et al., 2007; Causa et al., 2008; Mercieca and Fabri, 2011), the role of the prediction is still questionable.

Table 2: Synchronization and cost function errors.

| | Without prediction terms | | With prediction terms | |
|---|---|---|---|---|
| | PD+I | PI+D | PD+I | PI+D |
| $\sum_{m=1}^{3} e_m$ | 1.1 10ˆ5 | 60.8 | 1092.9 | 89.09 |
| $\sum_{m=1}^{3} e_{Jm}$ | 2.15 10ˆ7 | 285.11 | 6486.7 | 3699.6 |

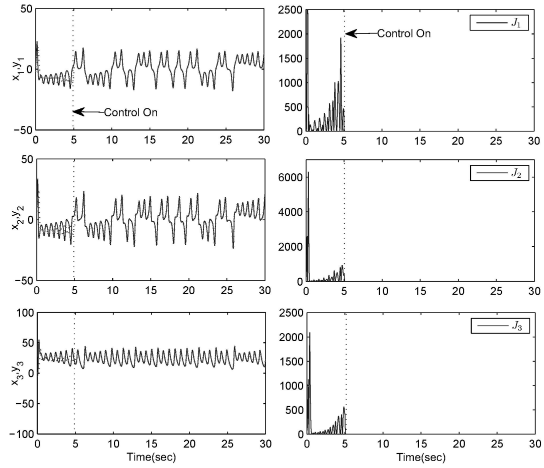


Figure 7: Synchronization of the Lorenz systems and cost functions variations without prediction terms using fuzzy PI+D controller.

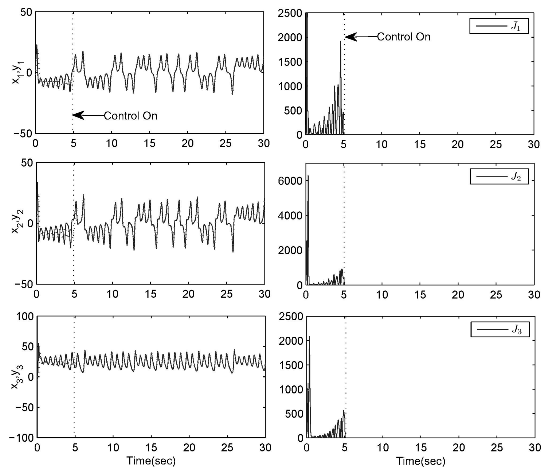


Figure 8: Synchronization of the Lorenz systems and cost functions variations with prediction terms using fuzzy PI+D controller.

# 6 CONCLUSION

In this paper, we use generalized predictive control based on fuzzy PID controllers for synchronization of uncertain chaotic systems. The synchronization, between the two systems, is achieved without mathematical complexities and by using fuzzy PID controllers, which serve as an optimizer. generalized predictive control based on fuzzy PID controllers has a negative and a positive aspect. The positive aspect, which concerns fuzzy PD+I controller, appears when the controller cannot ensure the synchronization. In this case, one step ahead prediction term helps to ensure asymptotic stability, which obviously leads to reduce the cost function value. However, when a stronger controller is used instead, as an example fuzzy PI+D controller. The prediction term causes a negative action and enlarges the synchronization and the cost function error.

## REFERENCES

Causa, J., Karer, G., Núnez, A., Sáez, D., Škrjanc, I., and Zupančič, B. (2008). Hybrid fuzzy predictive control based on genetic algorithms for the temperature control of a batch reactor. *Computers & chemical engineering*, 32(12):3254–3263.

Clarke, D. W., Mohtadi, C., and Tuffs, P. (1987). Generalized predictive controlpart i. the basic algorithm. *Automatica*, 23(2):137–148.

Dumur, D. and Boucher, P. (1994). Predictive control application in the machine tool field. *Advances in Model-Based Predictive Control, Oxford University Press, Oxford*, pages 471–482.

Jin-quan, H. and Lewis, F. L. (2003). Neural-network predictive control for nonlinear dynamic systems with time-delay. *Neural Networks, IEEE Transactions on*, 14(2):377–389.

Lam, H. and Leung, F. F. (2006). Synchronization of uncertain chaotic systems based on the fuzzy-model-based approach. *International Journal of Bifurcation and Chaos*, 16(05):1435–1444.

Lam, H.-K. and Seneviratne, L. D. (2007). Synchronization of chaotic systems using neural-network-based controller. *International Journal of Bifurcation and Chaos*, 17(06):2117–2125.

Lu, J., Chen, G., and Ying, H. (2001). Predictive fuzzy pid control: theory, design and simulation. *Information Sciences*, 137(1):157–187.

Mercieca, J. and Fabri, S. (2011). Particle swarm optimization for nonlinear model predictive control. In *ADVCOMP 2011, The Fifth International Conference on Advanced Engineering Computing and Applications in Sciences*, pages 88–93.

Song, Y., Chen, Z., and Yuan, Z. (2007). New chaotic pso-based neural network predictive control for nonlinear process. *Neural Networks, IEEE Transactions on*, 18(2):595–601.

Sun, Z., Zhu, W., Si, G., Ge, Y., and Zhang, Y. (2013). Adaptive synchronization design for uncertain chaotic systems in the presence of unknown system parameters: a revisit. *Nonlinear Dynamics*, 72(4):729–749.

Tang, K., Man, K. F., Chen, G., and Kwong, S. (2001). An optimal fuzzy pid controller. *Industrial Electronics, IEEE Transactions on*, 48(4):757–765.

Wang, X. and Xiao, J. (2005). Pso-based model predictive control for nonlinear processes. In *Advances in Natural Computation*, pages 196–203. Springer.

Wang, Z. and Sun, Y. (2010). Generalized predictive control based on particle swarm optimization for linear/nonlinear process with constraints. In *Computational Intelligence and Natural Computing Proceedings (CINC), 2010 Second International Conference on*, volume 1, pages 303–306. IEEE.

Yan, Z. and Wang, J. (2012). Model predictive control of nonlinear systems with unmodeled dynamics based on feedforward and recurrent neural networks. *Industrial Informatics, IEEE Transactions on*, 8(4):746–756.