Piecewise Chebyshev Factorization based Nearest Neighbour Classification for Time Series

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Keywords: Time Series, Piecewise Approximation, Similarity Measure.

Abstract: In the research field of time series analysis and mining, the nearest neighbour classifier (1NN) based on dynamic time warping distance (DTW) is well known for its high accuracy. However, the high computational complexity of DTW can lead to the expensive time consumption of classification. An effective solution is to compute DTW in the piecewise approximation space (PA-DTW), which transforms the raw data into the feature space based on segmentation, and extracts the discriminatory features for similarity measure. However, most of existing piecewise approximation methods need to fix the segment length, and focus on the simple statistical features, which would influence the precision of PA-DTW. To address this problem, we propose a novel piecewise factorization model for time series, which uses an adaptive segmentation method and factorizes the subsequences with the Chebyshev polynomials. The Chebyshev coefficients are extracted as features for PA-DTW measure (ChebyDTW), which are able to capture the fluctuation information of time series. The comprehensive experimental results show that ChebyDTW can support the accurate and fast 1NN classification.

1 INTRODUCTION

In the research field of time series analysis and mining, time series classification is an important task. A plethora of classifiers have been developed for this task (Esling et al, 2012; Fu, 2011), e.g., decision tree, nearest neighbor (1NN), naive Bayes, Bayesian network, random forest, support vector machine, etc. However, the recent empirical evidence (Ding et al, 2008; Hills et al, 2014; Serra et al, 2014) strongly suggests that, with the merits of robustness, high accuracy, and free parameter, the simple 1NN classifier employing generic time series similarity measure is exceptionally difficult to beat. Besides, due to the high precision of dynamic time warping distance (DTW), the 1NN classifier based on DTW has been found to outperform an exhaustive list of alternatives (Serra et al, 2014), including the decision trees, the multi-scale histograms, the multi-layer perception neural networks, the order logic rules with boosting, as well as the 1NN classifiers based on many other similarity measures. However, the computational complexity of DTW is quadratic to the time series length, i.e., $O(n^2)$, and the 1NN classifier has to search the entire dataset to classify an object. As a result, the 1NN classifier based on DTW is low efficient for the high-dimensional time series. To address this problem, researchers have proposed to compute DTW in the alternative piecewise approximation space (PA-DTW) (Keogh et al, 2001; Keogh et al, 2004; Chakrabarti et al, 2002; Gullo et al, 2009), which transforms the raw data into the feature space based on segmentation, and extracts the discriminatory and low-dimensional features for similarity measure. If the original time series with length n is segmented into N ($N \ll n$) subsequences, the computational complexity of PA-DTW will reduce to $O(N^2)$.

Many piecewise approximation methods have been proposed so far, e.g., piecewise aggregation approximation (PAA) (Keogh et al, 2001), piecewise linear approximation (PLA) (Keogh et al, 2004; Keogh et al, 1999), adaptive piecewise constant approximation (APCA) (Chakrabarti et al, 2002), derivative time series segment approximation (DSA) (Gullo et al, 2009), piecewise cloud approximation (PWCA) (Li et al, 2011), etc. The most prominent merit of piecewise approximation is the ability of capturing the local characteristics of time series. However, most of the existing piecewise approximation methods need to fix the segment

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length, which is hard to be predefined for the different kinds of time series, and focus on the simple statistical features, which only capture the aggregation characteristics of time series. For example, PAA and APCA extract the mean values, PLA extracts the linear fitting slopes, and DSA extracts the mean values of the derivative subsequences. If PA-DTW is computed on these methods, its precision would be influenced.

In this paper, we propose a novel piecewise factorization model for time series, named piecewise Chebyshev approximation (PCHA), where a novel code-based segmentation method is proposed to adaptively segment time series. Rather than focusing on the statistical features, we factorize the subsequences with Chebyshev polynomials, and employ the Chebyshev coefficients as features to approximate the raw data. Besides, the PA-DTW based on PCHA (ChebyDTW) is proposed for the 1NN classification. Since the Chebyshev polynomials with different degrees represent the fluctuation components of time series, the local fluctuation information can be captured from time series for the ChebyDTW measure. The comprehensive experimental results show that ChebyDTW can support the accurate and fast 1NN classification.

2 RELATED WORK

2.1 Data Representation

In many application fields, the high dimensionality of time series has limited the performance of a myriad of algorithms. With this problem, a great number of data approximation methods have been proposed to reduce the dimensionality of time series (Esling et al, 2012; Fu, 2011). In these methods, the piecewise approximation methods are prevalent for their simplicity and effectiveness. The first attempt is the PAA representation (Keogh et al, 2001), which segments time series into the equal-length subsequences, and extracts the mean values of the subsequences as features to approximate the raw data. However, the extracted single sort of features only indicates the height of the subsequences, which may cause the local information loss. Consecutively, an adaptive version of PAA, named piecewise constant approximation (APCA) (Chakrabarti et al, 2002), was proposed, which can segment time series into the subsequences with adaptive lengths and thus can approximate time series with less error. As well, a multi-resolution version of PAA, named MPAA (Lin et al, 2005), was proposed, which can iteratively segment time series into 2^i subsequences. However, both of the variations inherit the poor expressivity of PAA. Another pioneer piecewise representation is the PLA (Keogh et al, 2004; Keogh et al, 1999), which extracts the linear fitting slopes of the subsequences as features to approximate the raw data. However, the fitting slopes only reflect the movement trends of the subsequences. For the time series fluctuating sharply with high frequency, the effect of PLA on dimension reduction is not prominent. In addition, two novel piecewise approximation methods were proposed recently. One is the DSA representation (Gullo et al, 2009), which takes the mean values of the derivative subsequences of time series as features. However, it is sensitive to the small fluctuation caused by the noise. The other is the PWCA representation (Li et al, 2011), which employs the cloud models to fit the data distribution of the subsequences. However, the extracted features only reflect the data distribution characteristics and cannot capture the fluctuation information of time series.

2.2 Similarity Measure

DTW (Esling et al, 2012; Fu, 2011; Serra et al, 2014) is one of the most prevalent similarity measures for time series, which is computed by realigning the indices of time series. It is robust to the time warping and phase-shift, and has high measure precision. However, it is computed by the dynamic programming algorithm, and thus has the expensive $O(n^2)$ computational complexity, which largely limits its application to the high dimensional time series (Rakthanmanon et al, 2012). To overcome this shortcoming, the PA-DTW measures were proposed. The PAA representation based PDTW (Keogh et al, 2000) and the PLA representation based SDTW (Keogh et al, 1999) are the early pioneers, and the DSA representation based DSADTW (Gullo et al, 2009) is the state-of-the-art method. Rather than in the raw data space, they compute DTW in the PAA, PLA, and DSA spaces respectively. Since the segment numbers are much less than the original time series length, the PA-DTW methods can greatly decrease the computational complexity of the original DTW. Nonetheless, the precision of PA-DTWs greatly depends on the used piecewise approximation methods, where both the segmentation method and the extracted features are crucial factors. As a result, with the weakness of the existing piecewise approximation methods, the PA-DTWs cannot achieve the high precision. In our proposed ChebyDTW, a novel adaptive segmentation method and the Chebyshev factorization are used, which overcomes the drawback of the fixed segmentation, and can capture the fluctuation information of time series for similarity measure.

3 PIECEWISE FACTORIZATION

Without loss of generality, we first give the relevant definitions as follows.

Definition 1. (Time Series): The sample sequence of a variable X over n contiguous time moments is called time series, denoted as $T = \{t_1, t_2, \ldots, t_i, \ldots, t_n\}$, where $t_i \in \boldsymbol{R}$ denotes the sample value of X on the i-th moment, and n is the length of T.

Definition 2. (Subsequence): Given a time series $T = \{t_1, t_2, \ldots, t_i, \ldots, t_n\}$, the subset S of T that consists of the continuous samples $\{t_{i+1}, t_{i+2}, \ldots, t_{i+l}\}$, where $0 \leq i \leq n\text{-}l$ and $0 \leq l \leq n$, is called the subsequence of T.

Definition 3. (Piecewise Approximation Representation): Given a time series $T = \{t_1, t_2, \ldots, t_i, \ldots, t_n\}$, which is segmented into the subsequence set $\boldsymbol{S} = \{S_1, S_2, \ldots, S_j, \ldots, S_N\}$, if $\exists f$: $S_j \rightarrow V_j = [v_1, \ldots, v_m] \in \boldsymbol{R}^m$, then the set $\boldsymbol{V} = \{V_1, V_2, \ldots, V_j, \ldots, V_N\}$ is called the piecewise approximation of T.

Figure 1 shows the example of PLA representation (in red), for the stock price time series (in green) of Google Inc. (symbol: GOOG) from The NASDAQ Stock Market, which consists of the close prices at 800 consecutive trading days (2010/10/4~2013/12/5). As shown, PLA takes the linear fitting slopes and the spans of the subsequences as features to approximate the raw data, e.g., [0.5, 96] for the first subsequence.

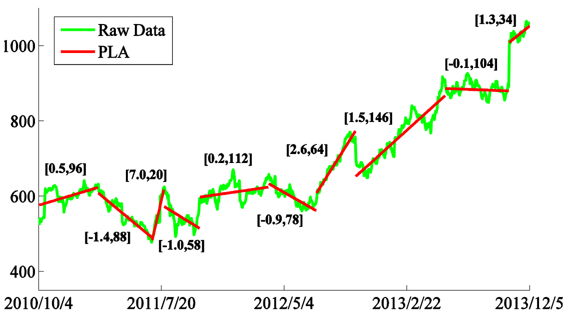


Figure 1: The PLA representation for the stock price time series.

3.1 Adaptive Segmentation

Inspired by the Marr's theory of vision (Ullman et al, 1982), we regard the turning points, where the trend of time series changes, as a good choice to segment time series. However, the practical time series is mixed with a mass of noise, which results in many trivial turning points with small fluctuation. This problem can be simply solved by the efficient moving average (MA) smoothing method (Gao et al, 2010).

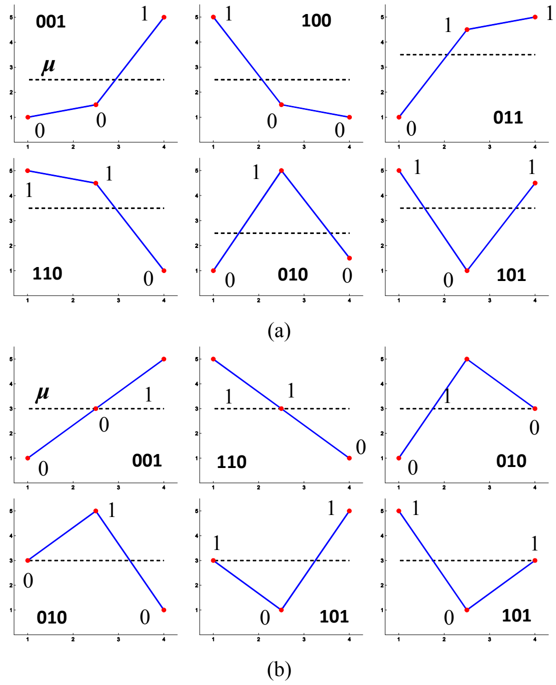


Figure 2: Three adjacent samples with the cell codes of (a) basic relationships, and (b) specific relationships.

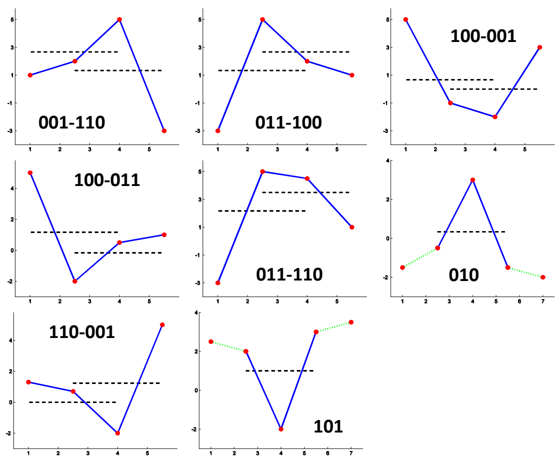


Figure 3: The minimum turning patterns composed with two cell codes.

In order to recognize the significant turning points, we first exhaustively enumerate the location relationships of three adjacent samples t_1-t_3 with their mean μ in time series, as shown in Figure 2. Six basic cell codes can be defined as Figure 2(a), which is composed by the binary codes δ_1-δ_3 of t_1-t_3, and denoted as $\Phi(t_1, t_2, t_3) = (\delta_1\delta_2\delta_3)_b$. Six special relationships that one of t_1-t_3 equals to μ are encoded as Figure 2(b).

Based on the cell codes, all the minimum turning patterns (composed with two cell codes) at the turning points can be enumerated as Figure 3. Note that, the basic cell codes 010 and 101 per se are the turning patterns. Then, we employ a sliding window with length 3 to scan the time series, and encode the samples within each window by Figure 2. In this process, all the significant turning points can be found by matching Figure 3, with which time series can be segmented into the subsequences with the adaptive lengths.

However, the above segmentation is not perfect. Although the trivial turning points can be removed with the MA, the "singular" turning patterns may exist, i.e., the turning patterns appearing very close. As shown in Figure 4, a Cricket time series from the UCR time series archive (Keogh et al, 2011) is segmented by the turning patterns (dash line), where the raw data is first smoothed with the smooth degree 10 (sd = 10).

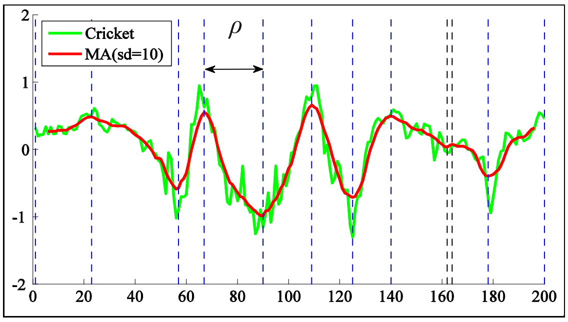


Figure 4: Segmentation for the Cricket time series.

Obviously, the dash lines can significantly segment the time series, but the two black dash lines are so close that the segment between them can be ignored. In view of this, we introduce the segment threshold ρ that stipulates the minimum segment length. This parameter can be set as the ratio to the time series length. Since the time series from a specific filed exhibit the same fluctuation characteristics, ρ is data-adaptive and can be learned from the labeled dataset. Nevertheless, the segmentation is still primarily established on the recognition of turning patterns, which determines the segment number or lengths adaptively, and is essentially different from the principles of the existing segmentation methods.

3.2 Chebyshev Factorization

At the beginning, it is necessary to z-normalize the obtained subsequences as a pre-processing step. Rather than focusing on the statistical features, PCHA will factorize each subsequence with the first kind of Chebyshev polynomials, and take the Chebyshev coefficients as features. Since the Chebyshev polynomials with different degrees represent the fluctuation components, the local fluctuation information of time series can be captured in PCHA.

The first kind of Chebyshev polynomials are derived from the trigonometric identity $T_n(\cos(\theta)) = \cos(n\theta)$, which can be rewritten as a polynomial of variable t with degree n, as Formula (1).

$$T_n(t) = \begin{cases} \cos(n\cos^{-1}(t)), & t \in [-1,1] \\ \cosh(n\cosh^{-1}(t)), & t \geq 1 \\ (-1)^n \cosh(n\cosh^{-1}(-t)), & t \leq -1 \end{cases} \quad (1)$$

For the sake of consistent approximation, we only employ the first sub-expression to factorize the subsequences, which is defined over the interval [-1, 1]. With the Chebyshev polynomials, a function $F(t)$ can be factorized as Formula (2).

$$F(t) \cong \sum_{i=0}^{n} c_i T_i(t) \quad (2)$$

The approximation is exact if $F(t)$ is a polynomial with the degree of less than or equal to n. The coefficients c_i can be calculated from the Gauss-Chebyshev Formula (3), where k is 1 for c_0 and 2 for the other c_i, and t_j is one of the n roots of $T_n(t)$, which can be get from the formula $t_j = \cos[(j-0.5)\pi/n]$.

$$c_i = \frac{k}{n}\sum_{j=1}^{n} F(t_j)T_i(t_j) \quad (3)$$

However, the employed Chebyshev polynomials are defined over the interval [-1, 1]. If the subsequences are factorized with this "interval function", they must be scaled into the time interval [-1, 1]. Besides, the Chebyshev polynomials are defined everywhere in the interval, but time series is a discrete function, whose values are defined only at the sample moments. To compute the Chebyshev coefficients, we would process each subsequence with the method proposed in (Cai et al, 2004), which can extend time series into an interval function. Given a scaled subsequence $S = \{(v_1, t_1), ..., (v_m, t_m)\}$, where $-1 \le t_1 < ... < t_m \le 1$, we first divide the interval [-1, 1] into m disjoint subintervals as follows:

$$I_i = \begin{cases} [-1, \frac{t_1 + t_2}{2}), i = 1 \\ [\frac{t_{i-1}, t_i}{2}, \frac{t_i + t_{i+1}}{2}), 2 \le i \le m-1 \\ [\frac{t_{m-1} + t_m}{2}, 1], i = m \end{cases}$$

Then, the original subsequence can be extended into a step function as Formula (4), where each subinterval $[t_i, t_{i+1}]$ is divided by the mid-point $(t_i+t_{i+1})/2$. The first half takes the value v_i, and the second half takes v_{i+1}.

$$F(t) = v_i,\ t \in I_i,\ 1 \le i \le m \tag{4}$$

After the above processing, the Chebyshev coefficients c_i can be computed. For the sake of dimension reduction, we only take the first several coefficients to approximate the raw data, which can reflect the principal fluctuation components of time series.

In the entire procedure, the time series needs to be scanned only once for the adaptive segmentation and factorization. Thus, the computational complexity of piecewise factorization is O(kn), where k is the extracted Chebyshev coefficient number and much less than the time series length n.

4 SIMILARITY MEASURE

DTW is one of the most prevalent similarity measures for time series (Serra et al, 2014), which can find the optimal alignment between time series by the dynamic programming algorithm. Given a sample space F, time series $T = \{t_1, t_2, \ldots, t_i, \ldots, t_m\}$ and $Q = \{q_1, q_2, \ldots, q_j, \ldots, q_n\}$, $t_i, q_j \in F$, a local distance measure d: $(x, y) \rightarrow \boldsymbol{R}^+$ should be first set in DTW for measuring two samples. Then, a distance matrix $\boldsymbol{C} \in \boldsymbol{R}^{m \times n}$ is computed, where each cell records the distance between each pair of samples from T and Q respectively, i.e., $\boldsymbol{C}(i, j) = d(t_i, q_j)$. There is an optimal warping path in $\boldsymbol{C}$, which has the minimal sum of the cells.

Definition 4. (Warping Path): Given the distance matrix $\boldsymbol{C} \in \boldsymbol{R}^{m \times n}$, if the sequence $p = \{c_1, ..., c_l, ..., c_L\}$, where $c_l = (a_l, b_l) \in [1 : n] \times [1 : m]$ for $l \in [1 : L]$, satisfies the conditions that:

i) $c_1 = (1, 1)$ and $c_L = (m, n)$;

ii) $c_{l+1} - c_l \in \{(1, 0), (0, 1), (1, 1)\}$ for $l \in [1 : L-1]$;

iii) $a_1 \le a_2 \le ... \le a_L$ and $b_1 \le b_2 \le ... \le b_L$;

Then, p is called warping path. The sum of cells in p is defined as Formula (5).

$$\Phi_p = \boldsymbol{C}(c_1) + \boldsymbol{C}(c_2) + \cdots + \boldsymbol{C}(c_L) \tag{5}$$

Definition 5. (Dynamic Time Warping Distance): Given the distance matrix $\boldsymbol{C} \in \boldsymbol{R}^{m \times n}$ over time series T and Q, and its warping path set $\boldsymbol{P} = \{p_1, \ldots, p_i, \ldots, p_x\}$, $i, x \in \boldsymbol{R}^+$, the minimal sum of the cells in the warping paths $\Phi_{min} = \{\Phi_\xi \mid \Phi_\xi \le \Phi_\lambda, \xi, \lambda \in \boldsymbol{P}\}$ is defined as the DTW distance between T and Q.

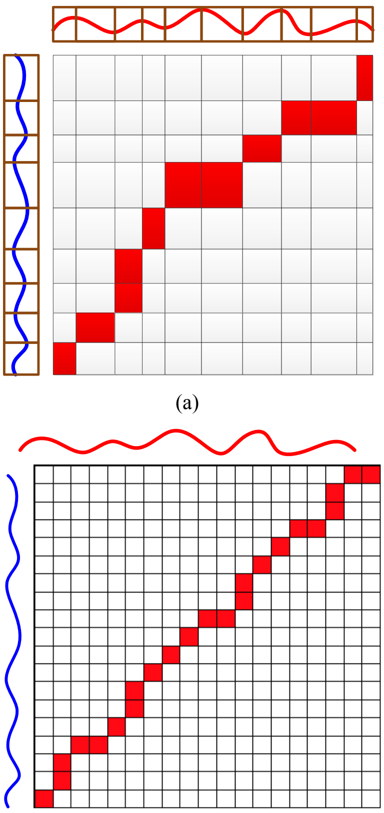
Based on PCHA, we propose a novel PA-DTW measure, named ChebyDTW. The algorithm of ChebyDTW contains two layers: subsequence matching and dynamic programming computation. Figure 5(a) shows the dynamic programming table with the optimal-aligned path (red shadow) of ChebyDTW, against that of the original DTW in Figure 5(b). In Figure 5(a), each cell of the table records the subsequence matching result over the Chebyshev coefficients. By the intuitive comparison, ChebyDTW would have much lower computational complexity than the original DTW.

With high computational efficiency, the squared Euclidean distance is a proper measure for the subsequence matching. Given d Chebyshev coefficients are employed in PCHA, for subsequences S_1 and S_2, respectively approximated as $\boldsymbol{C} = [c_1, ..., c_d]$ and $\hat{\boldsymbol{C}} = [\hat{c}_1, ..., \hat{c}_d]$, the squared Euclidean distance between them can be computed as Formula (6).

$$D(\boldsymbol{C}, \hat{\boldsymbol{C}}) = \sum_{i=1}^{d} (c_i - \hat{c}_i)^2 \tag{6}$$

Over the subsequence matching, the dynamic programming computation performs. Given that time series T with length m is segmented into M subsequences, and time series Q with length n is segmented into N subsequences, ChebyDTW can be computed as Formula (7). $\boldsymbol{C}^T$ and $\boldsymbol{C}^Q$ are the PCHA representations of T and Q respectively; C_1^T and C_1^Q are the first coefficient vectors of $\boldsymbol{C}^T$ and $\boldsymbol{C}^Q$ respectively; $rest(\boldsymbol{C}^T)$ means the rest coefficient vectors of $\boldsymbol{C}^T$ except for C_1^T; the same meaning is taken for $rest(\boldsymbol{C}^Q)$.

$$ChebyDTW\,(T, Q) = \begin{cases} 0, & \text{if } m = n = 0 \\ \infty, & \text{if } m = 0 \text{ or } n = 0 \\ D(C_1^T, C_1^Q) + \min \begin{Bmatrix} ChebyDTW\,[rest(\boldsymbol{C}^T), \boldsymbol{C}^Q], \\ ChebyDTW\,[\boldsymbol{C}^T, rest(\boldsymbol{C}^Q)], \\ ChebyDTW\,[rest(\boldsymbol{C}^T), rest(\boldsymbol{C}^Q)] \end{Bmatrix} \\ , otherwise \end{cases} \tag{7}$$



(a)

(b)

Figure 5: (a) The dynamic programming table with the optimal-aligned path (red shadow) of ChebyDTW, (b) against that of the original DTW.

5 EXPERIMENTS

We evaluate the 1NN classifier based on ChebyDTW from the aspects of accuracy and efficiency respectively. 12 real-world datasets provided by the UCR time series archive (Keogh et al, 2011) are employed, as shown in Table 1, which come from various application domains and are characterized by different series profiles and dimensionality. All datasets have been *z*-normalized and partitioned into training and testing sets by the provider. Besides, we take the 1NN classifiers respectively based on four prevalent PA-DTWs as baselines, i.e., PDTW, SDTW, APCADTW, and DSADTW. All parameters in the measures are learned on the training datasets by the DIRECT global optimization algorithm (Björkman et al, 1999), which is used to seek for the global minimum of multivariate function within a constraint domain. The experiment environment is Intel(R) Core(TM) i5-2400 CPU @ 3.10GHz; 8G Memory; Windows 7 64-bit OS; MATLAB 8.0_R2012b.

5.1 Accuracy

Table 1 shows the 1NN classification accuracy (*acc.*) based on the above PA-DTWs. The best result on each dataset is highlighted in bold. The learned parameters are also presented, which could make each classifier achieve the highest accuracy on each training dataset, including the segment threshold (ρ), the smooth degree (*sd*), and the extracted Chebyshev coefficient number (θ). For the sake of dimensionality reduction, we learn the parameter θ in the range of [1, 10] for ChebyDTW.

It is clear that, the 1NN classifier based on ChebyDTW wins all datasets and has the highest accuracy. Its superiority mainly derives from the distinctive features extracted in ChebyDTW, which can capture the fluctuation information for similarity measure. Whereas the statistical features extracted in the baselines only focus on the aggregation characteristics of time series, which would result in much fluctuation information loss.

Table 1: 1NN classification accuracy results based on five PA-DTWs.

Dataset	ρ	*sd*	θ	**ChebyDTW**	**PDTW**	**SDTW**	**APCADTW**	**DSADTW**
Adiac	0.21	22	9	**0.72**	0.61	0.34	0.28	0.38
Beef	0.18	17	5	**0.57**	0.50	0.57	0.57	0.47
CBF	0.98	8	10	**0.98**	**0.98**	0.95	0.91	0.50
ChlorineConcentration	0.73	25	8	**0.65**	0.60	0.55	0.56	0.62
CinC_ECG_torso	0.29	4	9	**0.81**	0.65	0.63	0.61	0.63
Coffee	0.51	14	9	**0.89**	0.79	0.75	0.82	0.61
ECG200	0.80	7	9	**0.89**	0.80	0.83	0.77	0.81
ECGFiveDays	0.73	17	9	**0.91**	0.79	0.68	0.68	0.57
FaceAll	0.51	29	10	**0.73**	0.63	0.50	0.63	0.71
FacesUCR	0.51	4	6	**0.80**	0.60	0.57	0.72	0.70
ItalyPowerDemand	0.51	7	5	**0.94**	0.93	0.80	0.90	0.87
SonyAIBORobotSurface	0.95	25	6	**0.80**	0.76	0.73	0.76	0.70

Table 2: The DCR results of five PA-DTWs.

Dataset	n	ChebyDTW		PDTW		SDTW		APCADTW		DSADTW	
		w	DCR	w	DCR	w	DCR	w	DCR	w	DCR
Adiac	176	3.99	**44.13**	36	4.89	13	13.54	43	4.10	70.00	2.51
Beef	470	5.18	**90.68**	61	7.70	10	47.00	61	7.70	192.32	2.44
CBF	128	1.00	**128.0**	30	4.27	27	4.74	15	8.53	46.19	2.77
ChlorineConcentration	166	2.00	**83.00**	36	4.61	29	5.72	34	4.88	64.77	2.56
CinC_ECG_torso	1639	4.00	**409.9**	103	15.91	94	17.44	84	19.51	655.49	2.50
Coffee	286	2.00	**143.0**	60	4.77	33	8.67	40	7.15	117.34	2.44
ECG200	96	1.93	**49.74**	14	6.86	19	5.05	23	4.17	35.84	2.68
ECGFiveDays	136	1.61	**84.43**	9	15.11	9	15.11	5	27.20	48.24	2.82
FaceAll	131	2.00	**65.50**	32	4.09	32	4.09	32	4.09	53.96	2.43
FacesUCR	131	2.00	**65.50**	24	5.46	32	4.09	31	4.23	54.43	2.41
ItalyPowerDemand	24	1.98	**12.13**	5	4.80	6	4.00	6	4.00	10.61	2.26
SonyAIBORobotSurface	70	1.00	**70.00**	13	5.38	9	7.78	8	8.75	27.41	2.55

5.2 Efficiency

Since the efficiency of 1NN classifier is determined by the used similarity measure, we perform the efficiency evaluation by comparing the computational efficiency of ChebyDTW against the baseline PA-DTWs. The speedup of computational complexity gained by PA-DTW over the original DTW is $O(n^2/w^2)$, where n is the time series length, and w is the segment number. It is positively correlated with the data compression rate (DCR = n/w) of piecewise approximation over the raw data. In Table 2, we present the DCRs of five PA-DTWs on all datasets, as well as n and w. Since ChebyDTW and DSADTW both employ the adaptive segmentation method, the average segment numbers on each dataset are computed for them.

As shown by the results, the DCRs of ChebyDTW are not only much larger than the baselines on all datasets, but also robust to the time series length. Thus, it has the highest computational efficiency among the five PA-DTWs. The efficiency superiority of ChebyDTW mainly derives from the precise approximation of PCHA over the raw data, and the data-adaptive segmentation method, which can segment time series into the less number of subsequences with the adaptive lengths.

6 CONCLUSIONS

We proposed a novel piecewise factorization model for time series, i.e., PCHA, where a novel adaptive segmentation method was proposed, and the subsequences were factorized with the Chebyshev polynomials. We employed the Chebyshev coefficients as features for PA-DTW measure, and thus proposed the ChebyDTW for 1NN classification. The comprehensive experimental results show that ChebyDTW can support the accurate and fast 1NN classification.

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