Particle Convergence Time in the PSO Model with Inertia Weight

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Abstract: Particle Swarm Optimization (PSO) is a powerful heuristic optimization method being subject of continuous interest. Theoretical analysis of its properties concerns primarily the conditions necessary for guaranteeing its convergent behaviour. Particle behaviour depends on three groups of parameters: values of factors in a velocity update rule, initial localization and velocity and fitness landscape. The paper presents theoretical analysis of the particle convergence properties in the model with inertia weight respectively to different values of these parameters. A new measure for evaluation of a particle convergence time is proposed. For this measure an upper bound formula is derived and its four main types of characteristics are discussed. The way of the characteristics transformations respectively to changes of velocity equation parameters is presented as well.

1 INTRODUCTION

Particle swarm optimization (PSO) (Kennedy and Eberhart, 1995) belongs to a big family of modern heuristic optimization methods. A number of versions of PSO has already been proposed sharing the same paradigm of stochastic, population-based method of exploration in the given space of solutions in searching for the best one. In our research we selected one of the earlier versions of PSO proposed in (Shi and Eberhart, 1998). Like in other methods, the population consists of members called here particles which represent solutions from the given space. Particles are also equipped with memories which store attractors, that is, solutions best found so far by the particles. A working group of particles controlled by the method is called a swarm. After the initialization of a swarm the cycle of iterations performs the search process. The distinctive features of PSO are: (1) application of particle memory as well as the mechanism of memory sharing by groups of neighbouring solutions, (2) the method of finding new solutions based on the idea of displacement originated from the real-world. Unlike other metaheuristics, every iteration consists of two main steps: particles memory update and the displacement of particles within the space of solutions. In PSO less-fit particles do not die, that is, there is no "survival of the fittest" mechanism typical for the evolutionary approach. The rules of displacement make use of the information from the memory and are expressed by equations which may differ to each other for different versions of PSO. Particularly, in the version of PSO which we selected for analysis the rules of displacement use the inertia weight parameter.

Numerous applications of PSO confirmed its usefulness and potential but also motivate for studying their theoretical properties. Particularly, a particle stability analysis is a subject of great interest. One of the main aims is estimation of particle parameter ranges guaranteing the convergent movement within the given boundaries of the search space. For the purpose of theoretical analysis some assumptions concerning randomness have always to be made. The most restricted deterministic approach simply eliminates stochastic coefficients from the velocity equation (Clerc and Kennedy, 2002). Other approaches implement expected values of the particle locations (Trelea, 2003; van den Bergh and Engelbrecht, 2006) (which is called a first order stability analysis), or the variance of the locations (a second order stability analysis) (Poli, 2009; Liu, 2015; Bonyadi and Michalewicz, 2015).

In the presented research we study behaviour of a particle which parameters belong to the ranges guaranteing the convergent movement, particularly, we evaluate the time necessary for a particle to enter the convergent state. This kind of a swarm property was already investigated for swarms consisting of a number of particles (Cleghorn and Engelbrecht, 2014b). In a series of experiments for different particle configurations authors evaluated number of iterations nec-

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essary to satisfy the assumed convergence condition. However, in our paper we propose a new method of evaluation of a particle convergence time based on the first order stability model of PSO with inertia weight (van den Bergh and Engelbrecht, 2006) and a new convergence condition. This means that the analysis concerns a particle model based on the following assumptions:

1. the particle moves in one-dimensional search space — there is no need to consider n-dimensional velocity vectors due to the fact, that all the velocity parameters are evaluated individually for each of the search space coordinates and they do not influence to each other in any way,
2. random values in the velocity equation are replaced by their expected values (e.g., for $r \sim U(0,1)$ it is 0.5), thus the rules of the particle movement become deterministic,
3. both the local and the global attractor remain in the same place of the search space over the entire time of the modelled particle behaviour,
4. there is just one particle to observe — due to the previous assumption that global attractor remains unchanged, no communication between particles exists in fact,
5. values of parameters in the velocity equation belong to the ranges guaranteeing convergent movement of the modelled particle.

Thus stability is defined as:

$$\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{y} \tag{1}$$

where $\mathbf{y}$ is a constant point in the search space.

The selected model based on the five assumptions allows to generate convergent trajectories of a particle over space. However, it has to be stressed that the shape of the trajectory does not influence the proposed measure and the only important information is the number of steps necessary for the particle to get and stay in the sufficiently close neighborhood of $\mathbf{y}$.

The paper consists of five sections. In Section 2 the model of PSO with inertia weight is briefly described. Section 3 presents the proposed new measure of particle convergence time. Discussion of the new measure properties can be found in Section 4. Section 5 concludes the paper.

2 THE PSO MODEL

The PSO model with inertia weight implements the following velocity and position equations:

$$\begin{cases} v_{t+1} &= w \cdot v_t + \varphi_1(y_t - x_t) + \varphi_2(y_t^* - x_t), \\ x_{t+1} &= x_t + v_{t+1} \end{cases} \tag{2}$$

where $\varphi_1 = r_1 c_1$, $\varphi_2 = r_2 c_2$, and c_1,c_2 represent acceleration coefficients, $r_1, r_2 \sim U(0,1)$. In the further analysis the stochastic components φ_1 and φ_2 are substituted by their expected values being equal $c_1/2$ and $c_2/2$ respectively. We also assume that both attractors are constant over time.

From this pair of equations a recursive formula can be derived (van den Bergh and Engelbrecht, 2006):

$$x_{t+1} = (1 + w - \varphi_1 - \varphi_2)x_t - wx_{t-1} + \varphi_1 y + \varphi_2 y^* \tag{3}$$

which allows to evaluate the particle location, assuming that its two previous locations and its attractor are known. This way a basic simplified dynamic system can be defined:

$$\mathbf{P}_{t+1} = M \times \mathbf{P}_t, \tag{4}$$

where:

- $\mathbf{P}^t$ — the particle state made up of its current position x_t and the previous one x_{t-1}.
- M — the dynamic matrix whose properties determine the transformations of the particle state.

Results from dynamic system theory say that the transformations of the particle state depend on the eigenvalues of M. Further analysis of the dynamic matrix originated from Eq. (3) allowed to define the region in the parameters space were eigenvalues of M are smaller than 1. All the configuration parameters sets originated from this region guarantee that the particles do not diverge during the process of search.

In (van den Bergh and Engelbrecht, 2006) authors show that the particle equilibrium point is a weighted average of its personal best y and global best y^* positions: $\frac{\varphi_1 y + \varphi_2 y^*}{\varphi_1 + \varphi_2}$. However, just for simplicity of calculations and without loss of generality we can assume, that $y^* = y$. In this case we can substitute ϕ for $\varphi_1 + \varphi_2$ and Eq. (3) is reformulated as follows:

$$x_{t+1} = (1 + w - \phi)x_t - wx_{t-1} + \phi y \tag{5}$$

Eventually, the following *stable region*, that is, a set of convergent configurations satisfies the following system of inequalities was derived:

$$\begin{cases} w > 0 \wedge w < 1, \\ \phi > 0, \\ w > 0.5\phi - 1 \end{cases} \tag{6}$$

Since the first presentation of the above-mentioned boundaries of the stable region a number of publications appeared discussing the problem of boundaries definition based on different assumptions concerning stochastic components in the velocity equations and stability of attractors. For more details the reader is referred to (Kadirkamanathan et al., 2006; Poli, 2009; Gazi, 2012; Cleghorn and Engelbrecht, 2014a; Liu, 2015). Particularly, in (Cleghorn and Engelbrecht, 2014a) a set of inequalities coinciding with Ineq. (6) has been derived. In our research presented in the further text we implement the stable region as it is defined by Ineq. (6) having in mind that constraint $w > 0$ represents just the intuitive assumption that inertia of a moving object should not be negative.

3 THE PROPOSED MEASURE

3.1 Particle Convergence Time

Even if the stable region is given, it is also interesting to know the number of steps necessary for the particle to obtain its stable state for different configurations (ϕ, w). In this case "obtaining stable state" means that the distance between current and the next location of the particle is never greater than the given threshold value δ.

Lets define a set of natural numbers $S(\delta)$ for a given $\delta > 0$ such that:

$$s \in S(\delta) \iff |x_{t+1} - x_t| < \delta \text{ for all } t \geq s. \quad (7)$$

We define the *particle convergence time* (*pct*) for given $\delta > 0$ as follows:

$$pct(\delta) = \min\{s \in S(\delta)\}. \quad (8)$$

The particle convergence time *pct* is the minimal number of steps necessary for the particle to obtain its stable state as defined above. For estimation of the particle convergence time we use Eq. (3).

3.2 Upper Bound Formula for *pct*

Recurrent equations are difficult to analyse, however, an explicit closed form of the recurrence relation Eq. (5) is also known (van den Bergh and Engelbrecht, 2006):

$$x_t = k_1 + k_2\lambda_1^t + k_3\lambda_2^t, \quad (9)$$

where

$$k_1 = y, \quad (10)$$

$$k_2 = \frac{\lambda_2(x_0 - x_1) - x_1 + x_2}{\gamma(\lambda_1 - 1)}, \quad (11)$$

$$k_3 = \frac{\lambda_1(x_1 - x_0) + x_1 - x_2}{\gamma(\lambda_2 - 1)}, \quad (12)$$

$$x_2 = (1 + w - \phi)x_1 - wx_0 + \phi y, \quad (13)$$

$$\lambda_1 = \frac{1 + w - \phi + \gamma}{2}, \quad (14)$$

$$\lambda_2 = \frac{1 + w - \phi - \gamma}{2}, \quad (15)$$

$$\gamma = \sqrt{(1 + w - \phi)^2 - 4w}. \quad (16)$$

Thus, the distance between two subsequent values of the particle locations x_{t+1} and x_t equals:

$$|x_{t+1} - x_t| = |k_2\lambda_1^t(\lambda_1 - 1) + k_3\lambda_2^t(\lambda_2 - 1)|. \quad (17)$$

From the triangle inequality it follows that:

$$|x_{t+1} - x_t| \leq |k_2||\lambda_1|^t|\lambda_1 - 1| + |k_3||\lambda_2|^t|\lambda_2 - 1|. \quad (18)$$

We are interested in the minimal number of steps s after which the condition

$$|x_{t+1} - x_t| < \delta \quad (19)$$

is satisfied for all $t \geq s$. To obtain this we employ the fact, that:

$$|a| < \delta/2 \wedge |b| < \delta/2 \Rightarrow |a + b| < \delta \quad (20)$$

where $|\cdot|$ is the absolute value.

Thus, we look for such t_1 and t_2, that:

$$|k_2||\lambda_1|^{t_1}|(\lambda_1 - 1)| < \delta/2, \quad (21)$$

$$|k_3||\lambda_2|^{t_2}|(\lambda_2 - 1)| < \delta/2. \quad (22)$$

and we get:

$$t_1 > \frac{\ln\delta - \ln(2|k_2||\lambda_1 - 1|)}{\ln|\lambda_1|}, \quad (23)$$

$$t_2 > \frac{\ln\delta - \ln(2|k_3||\lambda_2 - 1|)}{\ln|\lambda_2|}. \quad (24)$$

Now, we define $s = \max(t_1, t_2)$, where t_1 and t_2 are minimal natural number satisfying Ineq. (23) and (24) respectively. From (20), (21) and (22) it follows that for all $t \geq s$ the condition (19) is satisfied.

In the case where γ is a complex number consisting of just an imaginary value, that is, when $(1 + w - \phi)^2 < 4w$, the reasoning presented above may be simplified. In this case the following is satisfied: $|\lambda_1| = |\lambda_2|$ and $|\lambda_1 - 1| = |\lambda_2 - 1|$. Let's denote: $|\lambda| = |\lambda_1| = |\lambda_2|$ and $|\lambda - 1| = |\lambda_1 - 1| = |\lambda_2 - 1|$. Then, Ineq. (18) can be expressed as:

$$|x_{t+1} - x_t| \leq |\lambda|^t|\lambda - 1|(|k_2| + |k_3|). \quad (25)$$

In this case we look for such t that:

$$|\lambda|^t|\lambda - 1|(|k_2| + |k_3|) < \delta, \quad (26)$$

which is equivalent to

$$t > \frac{\ln\delta - \ln(|\lambda - 1|(|k_2| + |k_3|))}{\ln|\lambda|}, \quad (27)$$

Now, we define s as a minimal natural number t satisfying Ineq. (27). From (25) an (27) it follows that for all $t \geq s$ the condition (19) is satisfied.

For both cases, that is, real and imaginary value of γ, the defined number of steps s satisfies condition (7). Due to the fact, that $pcs(\delta)$ is defined as a minimal number satisfying condition (7), we get $pcs(\delta) \leq s$.

Thus, Ineq. (23), (24) and (27) give us the analytic upper bounds for the particle convergence time, which is denoted as $pctb(\delta)$. The explicit formula for $pctb(\delta)$ is

$$pctb(\delta) = \max\left(\frac{\ln\delta - \ln(2|k_2||\lambda_1 - 1|)}{\ln|\lambda_1|}, \frac{\ln\delta - \ln(2|k_3||\lambda_2 - 1|)}{\ln|\lambda_2|}\right) \quad (28)$$

for real value of γ and

$$pctb(\delta) = \frac{\ln\delta - \ln(|\lambda - 1|(|k_2| + |k_3|))}{\ln|\lambda|} \quad (29)$$

for imaginary value of γ.

4 VISUALIZATIONS OF *PCTB* CHARACTERISTICS

Particle convergence time depends on three groups of parameters: values of factors in a velocity update rule, initial localization and velocity and fitness landscape. Parameters from the first group, that is, ϕ and w define character (or temperament) of a particle. An example graph of $pctb(\phi, w)$ is presented in the subsection below. The next subsection presents example graphs of $pctb(x_0, x_1)$, that is, convergence times of particles with selected characters respectively to their starting conditions. Particle trajectories for respective types of character are also presented. The third subsection shows how $pctb(x_0, x_1)$ and $pctb(x_0, v)$ graphs vary respectively to the changes in a particle character.

4.1 Particle Convergence Time for Different Types of Particles

The characteristics of *pctb* as a function of particle configuration parameters ϕ and w share common shape presented in Figure 1. The Figure depicts the $pctb(\phi, w)$ characteristic obtained from a grid of evaluation points starting from a configuration [$\phi = 0.025$, $w = 0.044$] and changing with step 0.05 in both directions. This choice of method for the function graph generation is due to the fact, that γ appears in the denominator of Eq. (11) and (12), so, it cannot equal zero. Unfortunately, this is the case, when $w = 1 + \phi - 2\sqrt{\phi}$, that is, there exist points in the stable region for which the upper bound for their convergence time can be evaluate neither with formula (28) nor (29).

For better visibility the $pctb(\phi, w)$ axis has logarithmic scale and the evaluation points from outside the stable region have assigned the constant value 5000.

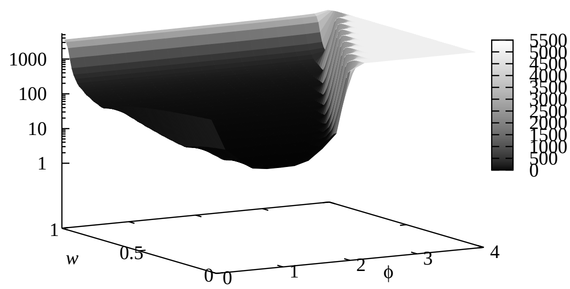


Figure 1: Particle convergences $pctb(\phi, w)$ for example starting conditions: $x_0 = 1$ and $x_1 = -8.1$.

Figure 1 shows that when the inertia weight w is low the convergence times are also low and increase as the inertia grows. Additionally, *pctb* increases also for the cases when acceleration coefficient ϕ approaches boundary values, both left and right, however, for the right boundary the increase is much higher than for the left.

4.2 *pctb* as a Function of Initial Location and Velocity

For ϕ and w values satisfying Ineq. (6) the shapes of $pctb(x_0, x_1)$ can be classified into four main types. Their representatives for $\delta = 0.0001$ are depicted in Figure 2:

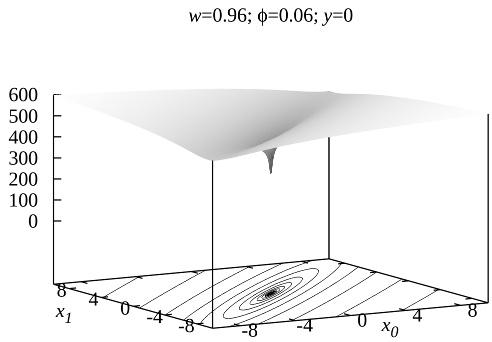
A: convergence is fast when the velocity is low (x_1 close to x_0) and the initial location x_0 is irrelevant in every case;

B: a transitional state between states A and C;

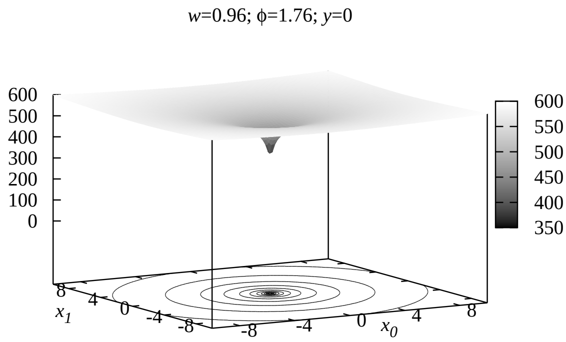
C: convergence is fast when the velocity is adjusted to the location and directed toward the attractor;

D: the particle has almost no inertia, so, the less distance from x_1 to the attractor, the less value of *pctb*.

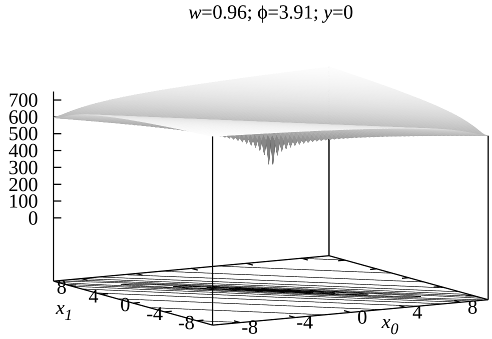
Figure 3 shows subsequent locations of particles over time for particle configurations selected for presentation in Figure 2 and for three different starting locations each. Graphs of particle trajectories similar to the ones presented in Figure 3 can be also found in (Trelea, 2003), however, in that case they were obtained for different particle parameter space. Graphs



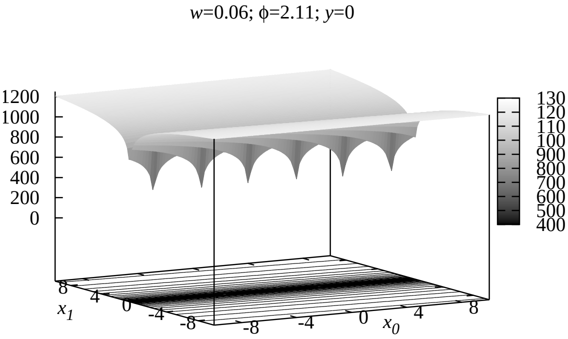
(a) type A



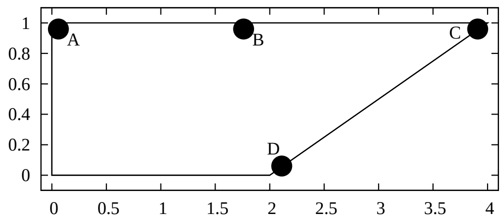
(b) type B



(c) type C



(d) type D

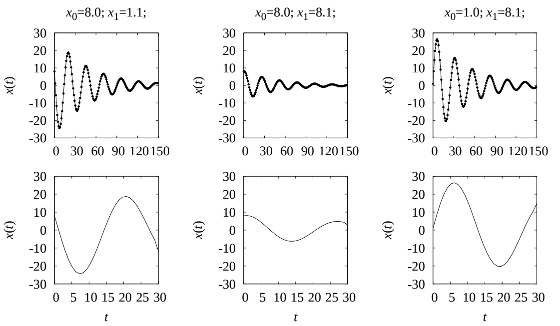


(e) localizations of selected configurations for types A, B, C and D in the configuration space (ϕ, w)

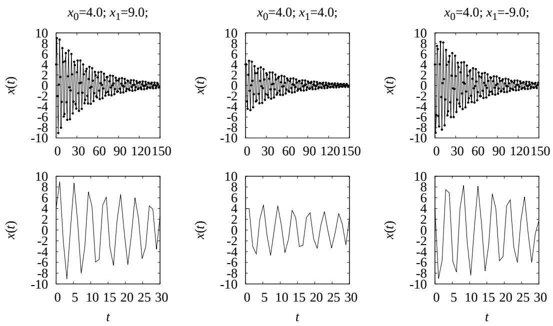
Figure 2: Graphs of $pctb(x_0, x_1)$ for selected configurations (ϕ, w) which represent four types of characteristics: A, B, C and D.

with trajectories can be also found in other publications, particularly in (van den Bergh and Engelbrecht, 2006), however, they are not classified respectively to the subarea in the stable region of the configuration space they appear.

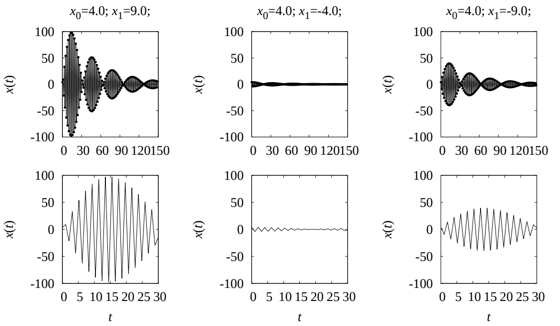
In Figure a ”A” particles are represented by three cases: with low (starting points x_0 and x_1 at (8,8.1)) and high initial velocity: ((8,1.1) and (1,8.1)). High inertia and weak attraction toward y make the movement smooth and the subsequent steps short in every case. For the high initial velocity oscillations around the attractor are higher. In the case of ”B” particles (Figure b) oscillations appear in every graph, however, the length of subsequent steps is irregular: when the particle moves away from y with high velocity, sometimes the attracting force almost stops it, velocity decreases and the particle turns back slowly, whereupon runs toward the attractor with a high velocity again. Figure c presents a ”zig-zag” trajectories of ”C” particles which amplitude cyclically increases and decreases. The amplitude of oscillations is less when the initial velocity is adjusted to the initial location and directed toward the attractor. Clearly, the fastest convergence of $pctb$ is obtained when x_1 has the same absolute value as x_0 but the opposite sign. Figure d also presents a ”zig-zag” trajectories of ”D” particles but without cycles in the magnitude of amplitude. In this case particle also converges to the attractor faster when the initial velocity is adjusted to the initial location, however, in this case the veloc-



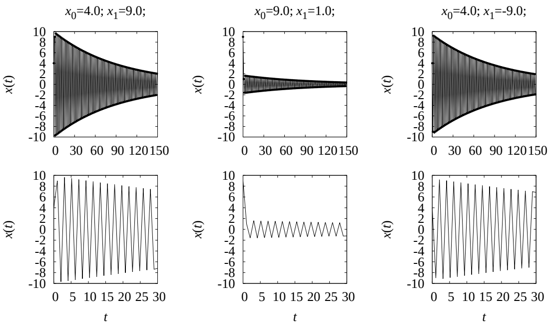
(a) type A: $(x_0, x_1) \in \{(8, 1.1), (8, 8.1), (1, 8.1)\}$



(b) type B: $(x_0, x_1) \in \{(4, 9), (4, 4), (4, -9)\}$



(c) type C: $(x_0, x_1) \in \{(4, 9), (4, -4), (4, -9)\}$



(d) type D: $(x_0, x_1) \in \{(4, 9), (9, 1), (4, -9)\}$

Figure 3: Particle trajectories for the four types of characteristics: A, B, C and D, and for three example starting locations; a view of 150 locations (top figures) and a close-up of the first 30 locations (bottom figures).

ity has to be adjusted so as to locate x_1 in the nearest neighborhood of the attractor. Finally, it is worth noting that different types of trajectories appear for different types of particle characteristics, which confirms the proposed selection of types and allows one to assume that none of the selected types is a subtype of any other.

4.3 Transformations of *pctb* Characteristics

The four types of characteristics transform smoothly from one to another when the ϕ and w parameters vary. Example series Q1, Q2 and Q3 of *pctb* graph pairs: $pctb(x_0, x_1)$ and $pctb(x_0, v)$ for $\delta = 0.0001$ are presented in Figures 5, 6 and 7 respectively. Localizations of selected series of configurations: Q1, Q2 and Q3 in the configuration space (ϕ, w) are depicted in Figure 4.

In Figure 5 the first series of figures called Q1 shows the transformations when the inertia weight w is high, that is, $w = 0.96$ and ϕ varies from minimal to maximal values within the stability region: $\phi \in \{0.06, 0.46, 2.46, 3.91\}$. For small values of ϕ the most important for *pctb* is the initial velocity: when

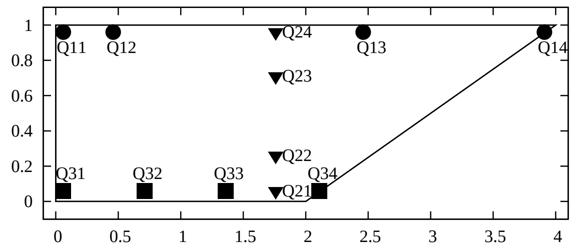
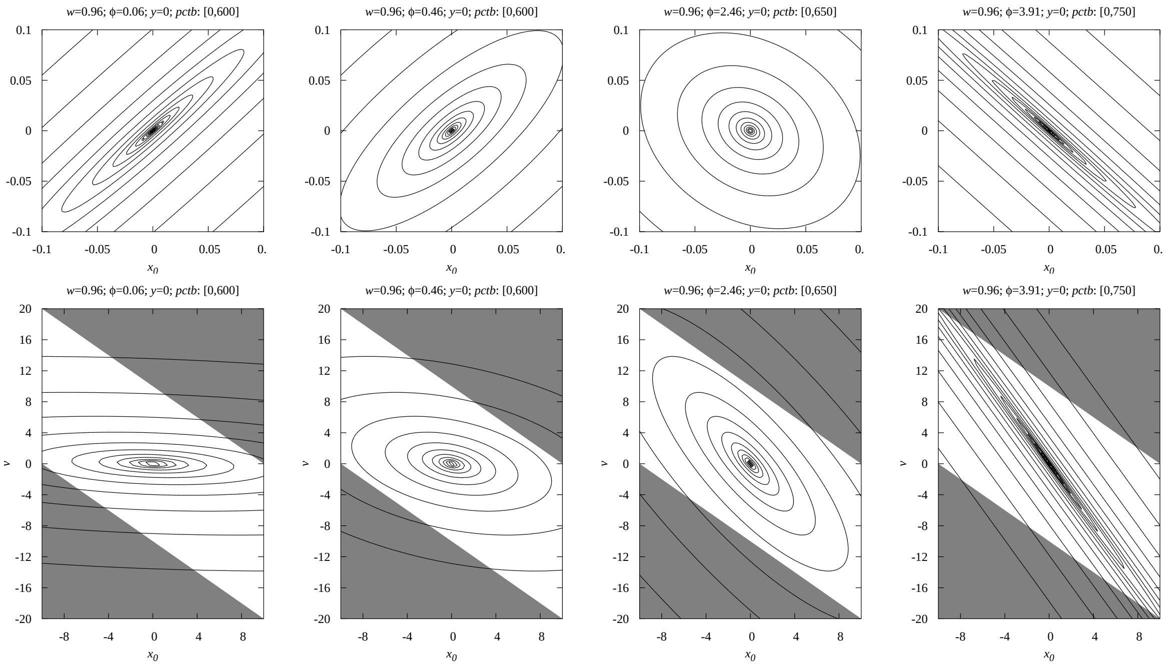


Figure 4: Localizations of configuration series presented in top three rows of pictures: Q1 (marked as circles), Q2 (triangles), and Q3 (squares) in the configuration space (ϕ, w).

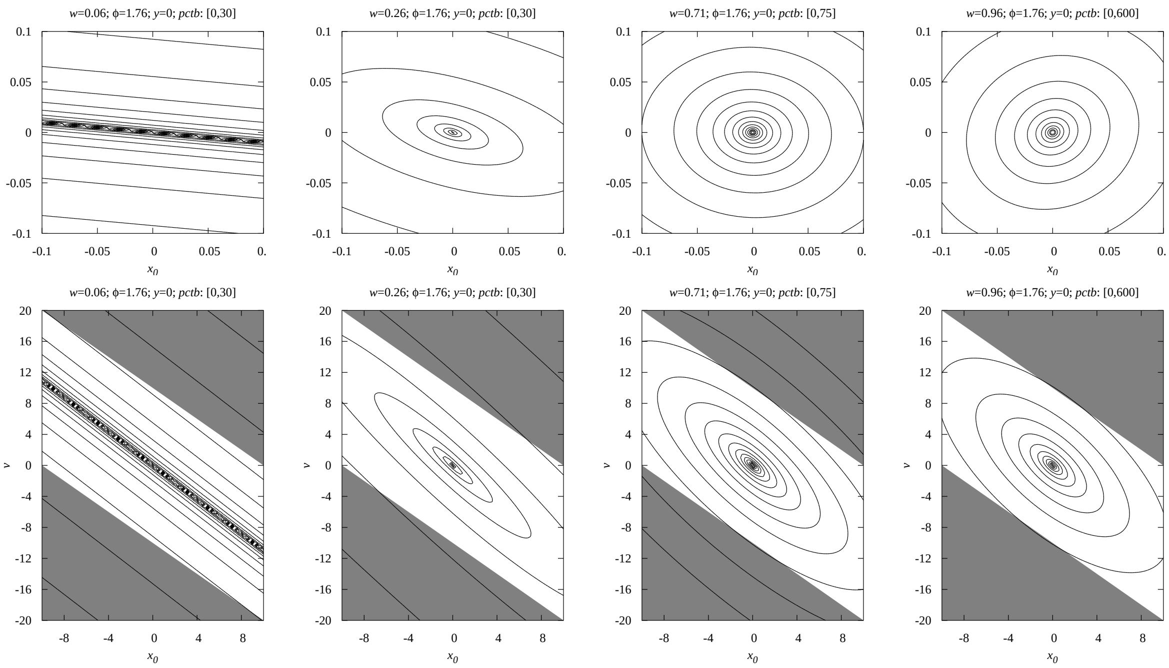
it is small, the *pctb* is low, otherwise, the number of steps necessary to reach the attractor grows rapidly. On the opposite end of series Q1 one can observe the case when for small values of *pctb* the velocity should be adjusted to the distance to the attractor. The further is the particle from the attractor, the higher initial velocity is needed to reach the attractor in small number of steps. In every case the velocity must be directed toward the attractor.

In Figure 6 the series Q2 is presented. The attractor coefficient is fixed, that is, $\phi = 1.76$ and the inertia weight varies: $w \in \{0.06, 0.26, 0.71, 0.96\}$. In every case for the sake of *pctb* minimization the ini-



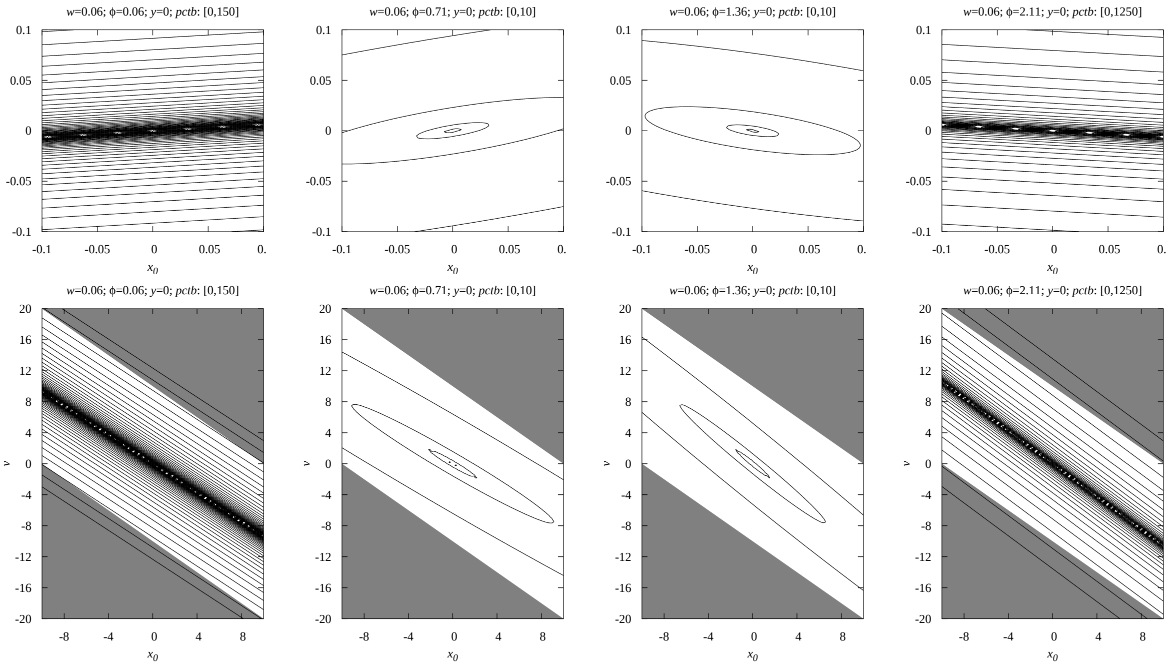
(a) Q11 (b) Q12 (c) Q13 (d) Q14

Figure 5: Particle convergence times *pctb* for a series Q1: fixed $w = 0.96$ and $\phi \in \{0.06, 0.46, 2.46, 3.91\}$; the top figures: $pctb(x_0, x_1)$; the bottom figures: $pctb(x_0, v)$; the white area in figures for $pctb(x_0, v)$ maps to the domain defined for $pctb(x_0, x_1)$.



(a) Q21 (b) Q22 (c) Q23 (d) Q24

Figure 6: Particle convergence times *pctb* for a series Q2: fixed $\phi = 1.76$ and $w \in \{0.06, 0.26, 0.71, 0.96\}$; the top figures: $pctb(x_0, x_1)$; the bottom figures: $pctb(x_0, v)$; the white area in figures for $pctb(x_0, v)$ maps to the domain defined for $pctb(x_0, x_1)$.



(a) Q31 (b) Q32 (c) Q33 (d) Q34

Figure 7: Particle convergence times *pctb* for a series Q3: fixed $w = 0.06$ and $\phi \in \{0.06, 0.71, 1.36, 2.11\}$; the top figures: $pctb(x_0, x_1)$; the bottom figures: $pctb(x_0, v)$; the white area in figures for $pctb(x_0, v)$ maps to the domain defined for $pctb(x_0, x_1)$.

tial velocity should be adjusted to the initial location of the particle. However, for small values of the inertia weight a small error in adjustment causes large increase of *pctb* value, whereas, large values of inertia make this change less abrupt, that is, the system is more stable.

The series of characteristics Q3 is depicted in Figure 7. In this case the inertia weight w is low, that is, $w = 0.06$ and $\phi \in \{0.06, 0.71, 1.36, 2.11\}$. As it is in the series Q1, when ϕ is small the initial location is almost negligible and the most influential parameter is velocity: when v is close to zero, the *pctb* is the smallest. In the Q3 series the boundary cases represent configurations sensitive to the error of velocity vs. location adjustment, that is, the stability of these configurations is low. The most stable configurations are the ones in the middle of the range.

Finally, note, that the three series have two shared configurations. Q21 may belong also to Q3: this configuration can be located between Q33 and Q34. Q24 may belong to Q1 and located between Q12 and Q13.

When we take a look at all the series, one can also observe that in most cases the *pctb* is sensitive to an error in the adjustment particularly for the largest values of ϕ both for small and high values of w (particularly, the examples Q14, Q21, and Q34). The most stable configurations, that is, resistant to lack of appropriate adjustment of parameters can be found in the middle of the series Q2, particularly Q24. It is worth noting here, that one of the popular choices of particle parameters: $c_1 = c_2 = 1.49445$ and $w = 0.72984$ (in (Eberhart and Shi, 2000) authors showed that the two values lead to satisfying results for a series of benchmark functions) belongs to the area of such a stable configurations. On the other side, for the smallest values of ϕ the initial location of a particle has no significant influence and *pctb* depends on just the velocity: the smaller v the less *pctb*.

5 CONCLUSIONS

In the presented research for a model of PSO with inertia weight we propose a new measure of particle convergence time (*pct*). The measure evaluates number of steps necessary for a particle to obtain a stable state defined with any precision. For this measure an upper bound formula (*pctb*) is derived and its properties are studied. Particularly, for the particle configurations from the convergence region of the (ϕ, w) space four main types of characteristics are identified. Additionally, we show the way of transformation between the characteristic shapes when the parameters ϕ and w vary.

In the future work we can use the obtained results, for example, for development of heterogenous particle swarm optimizers. The idea of swarms where particles may vary its behaviour during the process of search can be found in the literature (see, e.g, (Engelbrecht, 2010; Li and Yang, 2010; Nepomuceno and Engelbrecht, 2013a; Nepomuceno and Engelbrecht, 2013b)). Now, using the measure presented in this paper it can be easier to identify requested particle properties and develop strategies of particle configuration adaptation respectively to the search progress and current state of particles in a swarm.

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