

Fast Direction-of-Arrival Estimation for Single Source Near- and Far-Field Approaches for 1D Source Localization

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Abstract: The new approaches for a single narrowband source direction-of-arrival estimation in a far-field scenario and both direction-of-arrival and range estimation a near-field scenario are proposed. The main idea is to estimate the spatial frequency directly along the uniform linear array aperture from the single-shot measurement. The algorithm based on the autoregressive moving average model of the sinewave is applied for the frequency estimation. The effectiveness of proposed methods is analysed via computer simulations.

1 INTRODUCTION

The problem of direction-of-arrival (DOA) estimation of multiple plane waves generated by narrowband signal sources have attracted considerable interest in the literature due to a variety of applications in communication, seismology, oceanography, radar, acoustics, and so on. This problem is considered in the framework of the array signal processing and signal parameter estimation in particular. Usually the objective is to estimate parameters, such as azimuth, elevation, range, center frequency etc. associated with each signal.

Localization problem can be generally divided into two types, based on the distance between the source and the antenna array: far-field (when $r \geq 2D^2/\lambda$, r is the range between the source and the array reference point, D is the array aperture, λ is the wavelength of the source signal), and near-field localization. In far-field case, the wavefront of the signal impinging on the array is assumed to be planar (Johnson, 2006). When the source is located in the Fresnel region ($0.62\sqrt{D^3/\lambda} < r < 2D^2/\lambda$) or even closer in the near-field $r \leq 0.62\sqrt{D^3/\lambda}$ the wave front gets some curvature. It is reasonable to split processing algorithms onto ones based on the planar wave assumption and ones for the circular wavefront.

For the far-field estimation there are a lot of methods that can be separated into three categories. The first one is beamforming algorithms like delay-and-sum or minimum variance distortionless response (Bai, Ih, Benesty, 2013), which obtain a nonparametric spatial spectrum by application of a data-adaptive spatial filtering. The subspace algorithms like MUSIC (Stoica, Nehorai, 1989), ESPRIT (Gao, Gershman 2005) use the low-rank structure of the noise-free signal. The maximum likelihood methods (Wax, 1982), (Stoica, Besson, 2000), (Chen, Lorenzelli, Hudson, Yao, 2008) work with statistical properties, but require precise initialization to ensure convergence to a global minimum. Due statistical nature, they need sufficiently big data amount for accurate estimation.

In the case of single source localization, direction finding of the narrowband signal can be interpreted as a problem of a sinewave signal parameter estimation, particularly estimation of the spatial frequency. Besides, reduction of the problem allows using of simplified algorithms. (Wu, Liu, So, 2009).

In the near-field scenario it is necessary to estimate simultaneously two position parameters: a pair of coordinates or DOA and range. Therefore, traditional approaches like MUSIC must be extended to a two-dimensional field. Swindlehurst and Kailath (1988) suggest a quadratic (Fresnel) approximation of the wavefront in the near-field.

Using this approximation, the rotational invariance property can be used with the symmetric subarrays to estimate the DOA by ESPRIT (Zhi, Chia, 2007). In the paper of (Grossocki, Abed-Meraim, Hua 2005) position is obtained through estimation of two angles by weighted linear prediction. Another approach of transformation near-field localization problem to far-field one via interpolation is considered in (Yang, Shi, Liu, 2009).

In this work, we focus on the problem of estimation the DOA of a single source in both far- and near-field situations and the alternative approach of single-shot direct estimation of the spatial frequency from the one source is considered.

2 DOA ESTIMATION ALGORITHMS

2.1 Far-field Scenario

Let us consider a single narrowband signal $s(t)$ that comes from far-field and its source is located far enough to assume a wavefront as a linear one. The signal is received from direction θ by a uniform linear array (ULA) of M sensors. In order to avoid spatial aliasing distance d between them must be lesser than a signal wavelength λ_c .

The narrowband signal can be simply written as the next time-harmonic dependence

$$s(t) = A(t)\exp(\omega_c t), \quad (1)$$

where $A(t)$ is the baseband signal, ω_c is the signal center angular frequency.

The signal from far-field received by the microphone array can be written as the next vector

$$\mathbf{x}(t, \theta) = \begin{bmatrix} x_1(t, \theta) \\ \vdots \\ x_M(t, \theta) \end{bmatrix} = \begin{bmatrix} e^{j2\pi d \cdot 0 \sin(\theta)/\lambda} \\ \vdots \\ e^{j2\pi d \cdot (M-1) \sin(\theta)/\lambda} \end{bmatrix} \cdot s(t) + \begin{bmatrix} \eta_{c1}(t) \\ \vdots \\ \eta_{cM}(t) \end{bmatrix} = \mathbf{a}(\theta) \cdot s(t) + \boldsymbol{\eta}_c(t). \quad (2)$$

$x_m(t, \theta)$ is a signal captured by m th sensor, $\boldsymbol{\eta}_c(t)$ is a corresponding complex additive noise assumed as a white Gaussian noise with zero mean, $\mathbf{a}(\theta)$ is an array manifold vector or the steering vector (Bai, et al., 2013) that depends on the DOA θ . One can see that captured signals $x_m(t)$ have constant phase shift between each other $\omega = kd \sin(\theta)$, $k = 2\pi/\lambda_c$. This shift is a spatial frequency that has to be estimated.

In many real situations the received signal is not complex or sensors record only a real part of it. If we assume that the received signal is a single-tone one with an angular frequency ω_c , amplitude A and is sampled onto N samples with sampling interval τ than in any discrete moment of time $t_n = \tau(n-1)$, $n = \overline{1, N}$ can be described as

$$\mathbf{x}(t_n, \theta) = \begin{bmatrix} A \sin(\omega_c t_n + 0 \cdot \omega) \\ \vdots \\ A \sin(\omega_c t_n + (M-1) \cdot \omega) \end{bmatrix} + \begin{bmatrix} \eta_1(t_n) \\ \vdots \\ \eta_M(t_n) \end{bmatrix}. \quad (3)$$

where $\eta_m(t_n)$ are real parts of the noise vector $\boldsymbol{\eta}_c(t)$ in the equation.

The minimal sufficient information for frequency estimation is contained in only the one vector $\mathbf{x}(t_n)$ taken in any arbitrary moment of the discrete time. Hence, for simplicity we can chose the $n=1$ and write the corresponding measurement vector

$$\mathbf{x}(\theta) = A \sin(\omega(m-1)) + \eta_m, \quad m = \overline{1, M}. \quad (4)$$

For estimation of the spatial frequency in the signal (4) the algorithm considered in the paper of Prokopenko, Omelchuk, Chyrka (2012) is used. It is based on the representation of the noised sinuswave signal as an autoregressive moving average model of the second order. The DOA estimation procedure with frequency estimation steps can be summarized as follows:

a) calculation of the signal statistics

$$B(\bar{x}) = \sum_{m=2}^{M-1} [(x_{m+1} + x_{m-1})^2 - 2x_m^2] / \left[2 \sum_{m=2}^{M-1} (x_{m+1}x_m + x_mx_{m-1}) \right]$$

b) obtaining of two values of the autoregressive model parameter

$$\hat{\alpha}_{1(2)} = B(\bar{x}) \pm \sqrt{B(\bar{x})^2 + 2};$$

c) spatial frequency estimation

$$\hat{\omega}_{1(2)} = \arccos(\hat{\alpha}_{1(2)} / 2);$$

d) choice of the value $\hat{\omega}$ that is located in the zone of the method uniqueness $(0, \pi/2)$ that is a working range;

e) final calculation of the direction angle as

$$\theta = \arcsin(\omega \lambda_c / 2\pi d).$$

2.2 Near-field Scenario

In the near-field case the received signal is not a plane wave anymore and can be described as

$$\mathbf{x}(t, \mathbf{r}) = \begin{bmatrix} x_1(t, r_1) \\ \vdots \\ x_M(t, r_M) \end{bmatrix} = \begin{bmatrix} \frac{e^{j2\pi r_1/\lambda}}{r_1} \\ \vdots \\ \frac{e^{j2\pi r_M/\lambda}}{r_M} \end{bmatrix} \cdot s(t) + \begin{bmatrix} \eta_{c1}(t) \\ \vdots \\ \eta_{cM}(t) \end{bmatrix} = \quad (5)$$

$$= \mathbf{a}(\theta) \cdot s(t) + \boldsymbol{\eta}_c(t).$$

Here $r_m = |\mathbf{s}_0 - \mathbf{s}_m|$ is a distance between the source point \mathbf{s}_0 and the m th sensor position \mathbf{s}_m .

In the case of a real signal it can be written in the form similar to (4) as

$$\mathbf{x}(\mathbf{r}) = A_m \sin(kr_m) + \eta_m, \quad m = \overline{1, M}, \quad (6)$$

where $A_m = A/r_m$. In the near-field scenario the signal comes to each sensor from some direction that can be roughly defined as $\sin\theta_m \approx (r_m - r_{m-1})/d$, $m = \overline{2, M}$. The corresponding spatial frequency at the sensor's position is $\omega_m = kd \sin(\theta_m)$. We can estimate these local frequencies and use these angles to find the source position.

A single frequency can be calculated on the basis of at least three data samples, therefore we should take at least three consecutive samples x_{m-1}, x_m, x_{m+1} , $m = \overline{2, M-1}$, assume that on this short interval the signal (6) is sinusoidal and apply mentioned earlier estimation method for obtaining the set of ω_m . Note that frequencies ω_1, ω_M can not be estimated due to limitations of this approach. If one consider this approach in the framework of the array processing, it means splitting of the real array onto several overlapping subarrays. On the other hand, it can be considered as an instantaneous frequency estimation in the running window.

Having a set of local frequencies one can calculate a set of direction angles θ_m and draw several rays from corresponding points like it is illustrated in the Fig. 1.

In this example, the array consists of 31 sensors that give us 29 estimates of angles. Here the black cross in the center illustrates the source position. There are some missed rays at the picture that means that it was impossible to estimate a frequency by the considered algorithm and some beams are pointed

far away from the true source position. These facts can be explained by failures and errors of the estimation algorithm due to noise action.

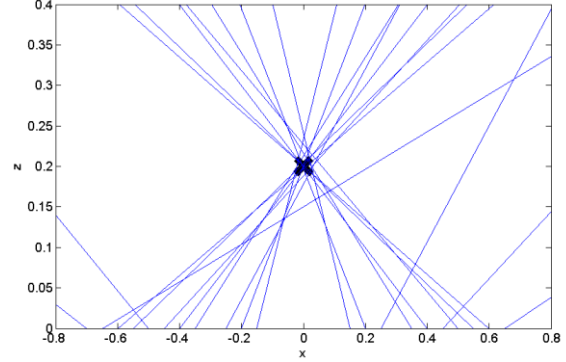


Figure 1: Plot of estimated local directions of arrival for the source located at the boresight.

If we look at the points of rays crossing, we can see that the spatial distribution of them is torn with multiple outliers, but the biggest density is around the true source position. To find the source position the distribution peak position must be estimated as median of all crossing points coordinates.

3 SIMULATION RESULTS

The effectiveness of two proposed approaches was analyzed and the far-field algorithm additionally compared to the ML single-tone estimator and Cramer-Rao lower bound (Rife, Boorstyn, 1974).

Statistical simulations by the Monte-Carlo approach were done under the next conditions: number of sensors in the ULA for a far-field case $N=11$, for near-field $N=31$; due to limited range of the used estimation algorithm, sensors spacing distance is $d = \lambda_c/4$; number of independent runs with single-shot measurements for each plot is 1000.

The first experiments (Fig. 2) represent performance of methods for different directions of arrival under signal-to-noise ratio $\text{SNR}=20$ dB.

One can see that proposed approach works pretty well in the range 0.2–1.4 rad. On the other hand its performance decreases when source is located at boresight or endfire positions because they corresponds to boundaries of the estimation range.

Figure 3 shows performance for different SNR at direction of arrival $\theta = 45^\circ$. One can see, that the proposed method almost reaches the ML-estimator, especially at high SNR.

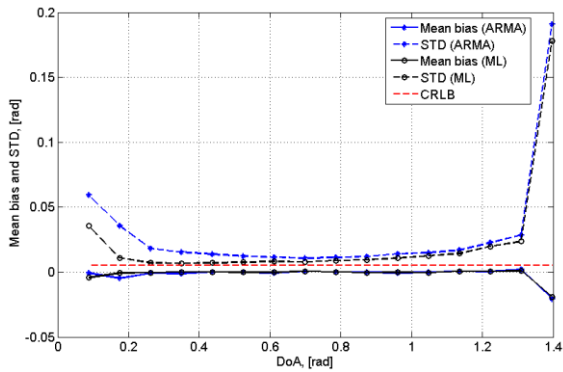


Figure 2: Dependence of the DOA estimation precision of far-field algorithms on its value.

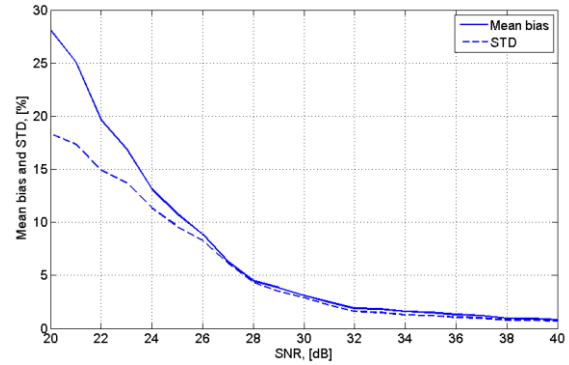


Figure 4: Dependence of the range estimation precision of the near-field algorithm on the SNR.

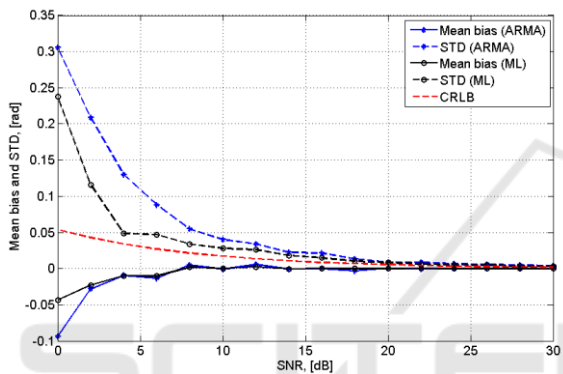


Figure 3: Dependence of the DOA estimation precision of far-field algorithms on the SNR.

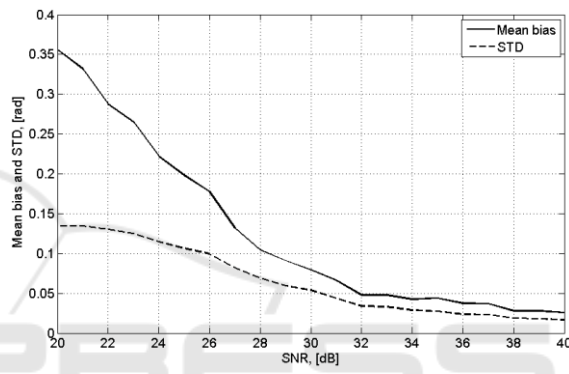


Figure 5: Dependence of the DOA estimation precision of the near-field algorithm on the SNR.

The near-field estimation algorithm was analysed under the SNR=20 dB for the source located $2\lambda_c$ forward and $2\lambda_c$ right from the beginning of the ULA. The signal was simulated as a real part of the model (Bai et al., 2013, p. 15). Figures 4 and 5 shows precision indicators for the range and the DOA. One can see, that the proposed approach requires quite high SNR (>30 dB) for decent estimation quality, even with comparatively big number of sensors. This can be explained by the fact that local spatial frequency is estimated only in 3-point running window and under this condition the algorithm is pretty sensitive to the noise action. On the bigger distance to the source using of bigger windows becomes possible and precision increases.

4 CONCLUSIONS

The proposed far-field method shows performance close to the maximum likelihood estimator in the range between boresight and endfire source positions, when SNR is bigger than 5 dB for few sensors. The near-field method generally requires bigger amount of sensors in comparison to far-field method and gives relatively unbiased estimates only at SNR higher than 30 dB.

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