Two Phases Inventory Strategy of Non-instantaneous Deteriorating

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Abstract: In this paper, we consider a replenishment model to maximize the average profits of fresh agricultural products would not immediately deterioration. The paper discuss in a replenishment cycle, demand affected only by fresh agricultural products price during "fresh-keeping period", and during "period of deterioration", demand affected by freshness and fresh agricultural products price. Numerical examples are included for illustration. The results show that decrease the rate of deterioration of fresh agricultural products would be increase the average profits of system and when T=22, the total profits and the average profits of the optimal.

1 INTRODUCTION

With the socioeconomic development and people's lifestyle change, more and more urbanites begin to pay attention to healthy eating and seasonal products are quickly becoming the top choice among them. It may be observed that the price of fresh agriculture product is no longer the only factor for urbanite's purchase, and the freshness becomes another important measurable indicator for their purchase decision. The fresh agriculture product is a kind of seasonal and fresh product which has a relatively short life cycle and liable to quick deterioration, such as vegetables, fruits, and seafood. It is a special perishable and vulnerable product that still has life activities or similar animate in a state of inventory. The demand for fresh agriculture product was easily affected by freshness because random life cycle. Consumers can get information of freshness by their sensory modalities after fresh agriculture product on hangers and we call this the sensory recognition method. Despite the freshness of fresh agriculture product will decay with the passing of time, there are time node for consumers' perceive of the fresh to old. Although it has lost its minor value in short time after fresh agriculture product on hangers, perceived the change is difficult for consumers. Fresh agricultural products would not immediately deterioration and this change process we call it "fresh-keeping period". During this time, the demand is only affected by price, both price and freshness are considered after this time node. This paper is focused on the study of different influence factors of demand in the same replenishment epochs, and it will be meaningful for retailers to adjust fresh agricultural product reasonable prices and make reasonable replenishment decision.

The rest of the paper is organized as follows. In the next section, we briefly discuss the current literature and the contributions of this paper. Section 3 is devoted to the assumptions of the modeling framework. The formulations and numerical examples are presented in section 4 and section 5. Section 6 is the paper's conclusion.

2 LITERATURE

Dan (2008) and Chen (2009) discussed fresh agricultural product supply chain coordination problem under valuable loss and physical loss, using an exponential function with downward slope, trying to denote valuable loss with greenness. Wang and Chen (2012) introduced the options contracts into

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the fresh produce supply chain ordering decision models, and the huge circulation wastages both from quantity and quality were taken into account the two stage models in one period. The paper supposed that the demand would be affected by the produce's fresh degree. Lin et al. (2011) constructed a new logarithmic freshness function and then told that the revenue-sharing contract have an influence on supply chain coordination under the time constraints. All of the paper considered freshness would affect demand. In addition, other scholars considered that the demand would be affected by price and fresh .Chen et al. (2009) developed a deteriorating inventory model with freshness in consideration and the demand depends on freshness and retail price inventory model is established. Then, an ordering policy of fresh agricultural products is studied under elastic demand, progressive price discount and loss-controlling. Gan et al. (2013) developed a demand function influenced by the freshness and price of the fresh agricultures product. Loss-averse utility function and dynamic no-cooperative game theory are applied in the model to discuss cooperation of fresh products supply chain in E-commerce. Wang and Dan (2013) according to the characteristics of freshness decrease over time of the fresh agricultural product, a time-varying consumer choice model influenced by the freshness and price of the fresh agricultures product is developed. In addition, a multi-item ordering model for various fresh agricultural products is developed to analyze the retailer's ordering policies under different unit fresh keeping cost of supplier. Yan et al. (2014) considered the coordination of a three-level fresh agricultural product supply chain under internet .Demand affect by price and freshness and built the distribution of profits model based on the improved revenue-sharing contract.

However, the above literatures either consider demand affected by price or price and freshness in a replenishment cycle. In real life, however, due to the particularity of consumer awareness, in the early stage of the fresh produce consumer perception of product freshness basic convergence. So this paper analyses the demand influence by different factors in two phases in a replenishment cycle.

The paper consider in a replenishment cycle, demand affected only by fresh agricultural products price during "fresh-keeping period" and during "period of deterioration", demand affected by freshness and fresh agricultural products price. In this view, this article trying to build different pricing model of two stages of fresh agricultural products demand function, so as to provide theory for retailers to scientific and rational pricing reference.

3 MODELING ASSUMPTIONS AND NOTATION

Assumption 1:

(1) retailers instantaneous replenishment, lead time is zero.

(2) this paper reference literature[9]about the structure of the fresh degree function and make a little change. The attenuation function for freshness is $\theta(t)=\theta_0 e^{-\eta t}$, θ_0 is initial freshness of fresh agricultural products, η is attenuation coefficient of freshness ($0<\eta<1$).

(3) When $0<t<t_1$, demand function is $D_1(t)=a_1-b_1p_1$. When $t_1<t<T$, Demand function is $D_2(t)=a_2-b_2p_2+c\theta(t)$. a_i is market capacity, b_i is price elasticity ($0<b_1<b_2$). c means the coefficient of the fresh agricultural product freshness to demand.

(4) when t belongs to $(0,t_1)$, the paper called it "fresh-keeping period". Fresh agricultural products would not immediately deterioration, so demand affected only by fresh agricultural products price. When t belongs to (t_1,T), the paper called it "period of deterioration". Demand affected by freshness and fresh agricultural products price.

In the rest of the paper, the following notation is used: p_1 denotes the price of fresh agricultural product of fresh-keeping period, p_2 denotes the price of fresh agricultural product of period of deterioration. I(t) is the retail's inventory level of time t. T means replenishment cycle, Q denotes order quantity of single cycle, A means fixed costs of single cycle. PC is purchasing cost , Cp is unit purchase of the item , HC is holding cost , h is unit holding cost, DC is deterioration cost, Cd is unit deterioration cost, SR denotes the total sales revenue, TP denotes total profits, AP means average profits. λ is deteriorating rate, θ_0 is initial freshness of fresh agricultural products, η is attenuation coefficient of freshness ($0<\eta<1$). a_i is market capacity, b_i is price elasticity ($0<b_1<b_2$). c means the coefficient of freshness to demand.

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4 MODEL FORMULATION AND SOLUTION

4.1 Model Formulation

As is shown in fig1, the initial inventory level is Q. When $0<t<t_1$, demand affected only by fresh agricultural products price and demand function is $D_1(t)=a_1-b_1p_1$. When $t_1<t<T$, demand affected by freshness and fresh agricultural products price. Demand function is $D_2(t)=a_2-b_2p_2+c\theta(t)$.

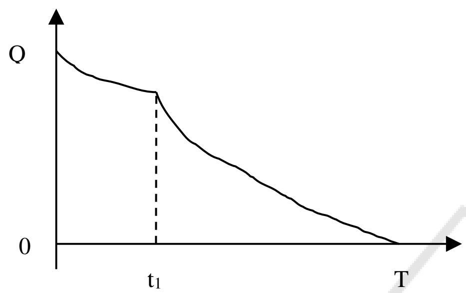

Figure 1: Fresh agricultural products two-phase inventory chart.

When $0<t<t_1$, fresh agricultural products would not immediately deterioration.

Inventory level is only affected by demand. Inventory level $I_1(t)$ satisfied:

$$\frac{dI_1(t)}{dt}=-D=-(a_1-b_1p_1) \tag{1}$$

The boundary conditions $I_1(0)=Q$, so solving equation (1), the Inventory level $I_1(t)$:

$$I_1(t)=-(a_1-b_1p_1)t+Q \tag{2}$$

When $t_1<t<T$,inventory level $I_2(t)$ satisfied：

$$\frac{dI_2(t)}{dt}=-(a_2-b_2p_2+c\,\theta(t))-\lambda I_2(t) \tag{3}$$

The boundary conditions $I_2(t)=0$, solving equations (3):

$$I_2(t)=-A-Be^{-\eta t}+(Ae^{\lambda T}+Be^{(\lambda-\eta)T})e^{-\lambda t} \tag{4}$$

$$A=(a_2-b_2p_2)/\lambda \tag{5}$$

$$B=c\theta_0/(\lambda-\eta) \tag{6}$$

By the function of continuity, we know $I_1(t)=I_2(t)$, solving equations (2)and (4), We know the function relation between order quantity and replenishment cycle

$$Q=-A-Be^{-\eta t_1}+\left(Ae^{\lambda T}+Be^{(\lambda-\eta)T}\right)e^{-\lambda t_1}+(a_1-b_1p_1)t_1 \tag{7}$$

Therefore , in a replenishment cycle T, all of the cost and profits as follows:

1)The cost of the fixed order: A (8)

2) Purchasing Cost : PC

$$PC=C_pQ$$
$$=C_p\left[-A-Be^{-\eta t_1}+\left(Ae^{\lambda T}+Be^{(\lambda-\eta)T}\right)e^{-\lambda t_1}+(a_1-b_1p_1)t_1\right] \tag{9}$$

3) Holding Cost : HC

$$HC=h\cdot\int_0^{t_1}I_1(\text{t})\,\text{dt}+h\cdot\int_{t_1}^{T}I_2(\text{t})\,\text{dt}$$
$$=h\cdot\left\{\begin{aligned}&\left(\frac{1}{2}(-a_1+b_1p_1)t_1^2+Qt_1\right)+\\&\left[-A(T-t_1)+\frac{B}{\eta}e^{-\eta(T-t_1)}-\frac{Ae^{\lambda T}+Be^{(\lambda-\eta)T}}{\lambda}\cdot e^{-\lambda(T-t_1)}\right]\end{aligned}\right\} \tag{10}$$

4) Deterioration Cost: DC

$$DC=C_d\cdot\int_{t_1}^{T}\lambda I_2(\text{t})\,\text{dt}$$
$$=C_d\lambda\left[-A(T-t_1)+\frac{B}{\eta}e^{-\eta(T-t_1)}-\frac{Ae^{\lambda T}+Be^{(\lambda-\eta)T}}{\lambda}\cdot e^{-\lambda(T-t_1)}\right] \tag{11}$$

5) Sales Revenue: SR

$$SR=p_1\int_0^{t_1}D_1(\text{t})dt+p_2\int_{t_1}^{T}D_2(\text{t})dt$$
$$=p_1\int_0^{t_1}(a_1-b_1p_1)dt+p_2\int_{t_1}^{T}\left[a_2-b_2p_2+c\cdot\theta(t)\right]dt$$
$$=p_1(a_1-b_1p_1)t_1+p_2\left[(a_2-b_2p_2)(T-t_1)-\frac{c\theta_0}{\eta}\left(e^{-\eta T}-e^{-\eta t_1}\right)\right] \tag{12}$$

6) Total Profits : TP

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$$TP = SR - (A + PC + HC + DC)$$

$$= \left\{ \begin{array}{l} \left\{ p_1(a_1 - b_1 p_1)t_1 + p_2\left[(a_2 - b_2 p_2)(T - t_1) - \frac{c\theta_0}{\eta}\left(e^{-\eta T} - e^{-\eta t_1}\right)\right]\right\} - \\ \left\{ \begin{array}{l} A + C_p\left[-A - Be^{-\eta t_1} + \left(Ae^{\lambda T} + Be^{(\lambda-\eta)T}\right)e^{-\lambda t_1} + (a_1 - b_1 p_1)t_1\right] + \\ h \cdot \left[\left(\frac{1}{2}(-a_1 + b_1 p_1)t_1^2 + Qt_1\right) + \left(\begin{array}{l} -A(T - t_1) + \frac{B}{\eta}e^{-\eta(T-t_1)} - \\ \frac{Ae^{\lambda T} + Be^{(\lambda-\eta)T}}{\lambda} \cdot e^{-\lambda(T-t_1)} \end{array}\right)\right] + \\ C_d\lambda\left[-A(T - t_1) + \frac{B}{\eta}e^{-\eta(T-t_1)} - \frac{Ae^{\lambda T} + Be^{(\lambda-\eta)T}}{\lambda} \cdot e^{-\lambda(T-t_1)}\right] \end{array} \right\} \end{array} \right\} \tag{13}$$

7) Average Profits : AP

$$AP = TP/T \tag{14}$$

And

$$\partial AP/\partial p_1 = (a_1 t_1 - 2b_1 t_1 p_1) + C_p t_1 b_1 - (1/2)hb_1 t_1^2 \tag{15}$$

$$\frac{\partial AP}{\partial p_2} = \left[(a_2 - 2b_2 p_2)(T - t_1) - \frac{c\theta_0}{\eta}\left(e^{-\eta T} - e^{-\eta t_1}\right)\right] - \left\{ \begin{array}{l} C_p \cdot \frac{b_2}{\lambda}\left(1 - e^{\lambda(T-t_1)}\right) + h\left[\frac{b_2}{\lambda}(T - t_1) + \frac{b_2}{\lambda^2}e^{\lambda t_1}\right] \\ +C_d\lambda\left[\frac{b_2}{\lambda}(T - t_1) + \frac{b_2}{\lambda^2}e^{\lambda t_1}\right] \end{array} \right\} \tag{16}$$

4.2 Model Solution

Theorem1: The fresh agricultural products profits model has the optimal solution .

Proof: 1) The necessary condition of the optimal solution is to find p_1* and p_2* that can satisfy Partial derivative is zero.

$$(a_1 t_1 - 2b_1 t_1 p_1) + C_p t_1 b_1 - (1/2)hb_1 t_1^2 = 0 \tag{17}$$

$$\left[(a_2 - 2b_2 p_2)(T - t_1) - \frac{c\theta_0}{\eta}\left(e^{-\eta T} - e^{-\eta t_1}\right)\right] - \left\{ \begin{array}{l} C_p \cdot \frac{b_2}{\lambda}\left(1 - e^{\lambda(T-t_1)}\right) + h\left[\frac{b_2}{\lambda}(T - t_1) + \frac{b_2}{\lambda^2}e^{\lambda t_1}\right] \\ +C_d\lambda\left[\frac{b_2}{\lambda}(T - t_1) + \frac{b_2}{\lambda^2}e^{\lambda t_1}\right] \end{array} \right\} = 0 \tag{18}$$

Solving equations (17) and (18), we know

$$p_1^* = 1/2\left[-(1/2)ht_1 + C_p + a_1/b_1\right] \tag{19}$$

$$p_2^* = \frac{1}{-2b_2(T - t_1)}\left\{ \begin{array}{l} \frac{b_2}{\lambda}\left\{ \begin{array}{l} C_p\left(1 - e^{\lambda(T-t_1)}\right) + \\ \left[(T - t_1) + \frac{1}{\lambda}e^{\lambda t_1}\right](h + C_d\lambda) \end{array} \right\} \\ +\frac{c\theta_0}{\eta}\left(e^{-\eta T} - e^{-\eta t_1}\right) \end{array} \right\} + \frac{a_2}{2b_2} \tag{20}$$

So, there are p_1* and p_2* that can satisfy the necessary condition of optimal solution.

According equations (15) and (16), the partial derivatives of p_1 and p_2 are as follows:

$$\partial^2 AP/\partial p_1^2 = -2b_1 t_1 < 0 \tag{21}$$

$$\partial^2 AP/\partial p_2^2 = -2b_2(T - t_1) < 0 \tag{22}$$

$$\partial^2 AP/\partial p_1 \partial p_2 = \partial^2 AP/\partial p_2 \partial p_1 = 0 \tag{23}$$

The Hessian matrix is：

$$H = \begin{bmatrix} -2b_1 t_1 & 0 \\ 0 & -2b_2(T - t_1) \end{bmatrix} \tag{24}$$

$$|H| = \frac{\partial^2 AP}{\partial p_1^2}\frac{\partial^2 AP}{\partial p_2^2} - \left(\frac{\partial^2 AP}{\partial p_1 \partial p_2}\right)^2 > 0 \tag{25}$$

Solving equations (21) to (25), the Hessian matrix negative, the maximum profits function exists.

5 NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

Table 1 shows the Optimal prices, the optimal order quantity, expected revenues, in that order, for different values of T and for given value of a_1, a_2, b_1, b_2, c, λ, η, θ_0, h, Cp, Cd. And a_1=120, a_2=100, b_1=8, b_2=15, c=100, λ=0.1, η=0.2, θ_0=0.9, h=0.05, Cp=3, Cd=0.1.

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Table 1: For different values of T.

T*	P_1*	P_2*	Q*	TP*	AP*
24	8.9	4.312	1011	4181.668	174.237
22	8.9	4.795	1086	4447.149	202.143
20	8.9	5.250	1047	3865.08	193.254
18	8.9	5.724	988	3350.61	186.145

From table 1 we see the following conclusion:

(1) As T* increase, the price of p_2* decrease. p_2 represents the price of fresh agricultural product of period of deterioration. With the increase of T*, fresh agricultural products constantly deterioration and of course price falling.

(2) As T* increase, the order quantity gradually increasing firstly. And when T=22, the order quantity at its highest point. When ordering quantity reaches a certain extreme value point, the fresh agricultural products accelerate deterioration if continue to extend the ordering cycle.

(3) As T* increase, the average profits increase and decrease trend is the same as the order quantity.

Table 2 shows: for different values of η* and for given value of a_1, a_2, b_1, b_2, c, λ, θ_0, h, Cp, Cd. T And a_1=120, a_2=100, b_1=8, b_2=15, c=100, λ=0.1, η=0.2, θ_0=0.9, h=0.05, Cp=3, Cd=0.1, T=22.

Table 2: For different values of η*.

η*	P_1*	P_2*	Q*	TP*	AP*
0.2	8.9	4.795	1086	4447.149	202.143
0.4	8.9	4.708	1023	4318.116	196.278
0.6	8.9	4.621	960	4186.116	190.413
0.8	8.9	4.534	897	4060.056	184.584

From table 2 we see that as η* increase, P_2* decrease, Q* decrease, AP* also decrease. Because θ(t) is a decreasing function and for fresh agricultural products , the higher the deterioration rate , the lower the price.

6 CONCLUSIONS

The paper considered the demand affected by different factors of two phases inventory replenishment. Due to the characteristics of non-instantaneous deteriorating of fresh agricultural products and the particularity of consumer perception, demand affected only by fresh agricultural products price during "fresh-keeping period", and during "period of deterioration", demand affected by freshness and fresh agricultural products price. Numerical examples are included for illustration. The conclusion is as follows: (1) when T=22, total profits and average profits maximum; (2) the faster the decline rate, the lower the price of P_2* and at the same time, order quantity, total profits and average profits decrease.

However, in this paper we assuming that replenishment lead time is zero, in real life, out of stock is frequent and it is a real important problem for retailer. So the article also can construct the model from the order lead time is not zero, retailers allow delayed payment etc.

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