Generating Metamodel Instances Satisfying Coverage Criteria via SMT Solving

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Abstract: One of the challenges for using metamodels in Model Driven Engineering is to automatically generate metamodel instances. Each instance should satisfy many constraints defined by a metamodel. Such instances can then be used for verifying or validating metamodels. Recent studies have already shown that this can be tackled by using SAT/SMT solvers. However, such instance generation does not take coverage criteria into account, and instances satisfying specified coverage criteria could be useful for testing model transformation. In this paper, we present an approach consisting of two techniques for coverage oriented metamodel instance generation. The first technique realises the standard coverage criteria defined for UML class diagrams, while the second technique focuses on generating instances satisfying graph-based criteria. With our approach, both kinds of criteria are translated to SMT formulas which are then investigated by an SMT solver. Each successful assignment is then interpreted as a metamodel instance that provably satisfies a coverage criteria or a graph property. We have already integrated this approach into our existing tool to demonstrate the feasibility.

1 INTRODUCTION

A *model* provides a representation of aspects of a system. This can include design models such as UML class or sequence diagrams, or implementation models, such as source code in a programming language. A *metamodel* is a model that is used to describe the structure of other models, modelling languages or domain specific languages. Each *instance* of a metamodel is then a model that can be regarded as a test case. These test cases are important not just for validating a metamodel itself, but also useful for testing the tools and frameworks that process the models defined by that metamodel such as model transofrmation.

For example, given a domain specific language *L*, say, a metamodel would usually define the abstract syntax and static semantics of the language. A typical representation of the metamodel would be as a UML class diagram (using a subset of the constructs) with constraints specified using the Object Constraint Language (OCL). A set of instances of this metamodel would be programs written in language *L*, and would allow language engineers to check that they had specified the relevant constructs correctly.

A number of approaches and tools have already provided a way of generating these instances (Ehrig et al., 2009; González Pérez et al., 2012; Cabot et al., 2014). However, these instances are not measured via any criteria. At least, meeting some criteria such as standard coverage criteria for UML class diagram would help users to increase their confidence in designing or validating metamodels. Furthermore, users may also wish to generate instances that possess certain coverage metrics for other testing purposes such as using depth of inheritance tree for testing inheritance relationships. Thus, naively generating instances from a metamodel without taking account of coverage criteria or other properties is not very adequate.

This paper addresses the issue of generating metamodel instances satisfying coverage criteria. More specifically, this paper makes the following contributions:

- A technique that enables metamodel instances to be generated so that they satisfy partition-based coverage criteria.
- A technique for generating metamodel instances which satisfy graph properties.

Both two techniques that encode coverage criteria and graph properties into a set of SMT formulas. These formulas are then combined with the formulas generated from our previous work, and solved by using an

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external SMT solver (Wu et al., 2013). Each successful assignment for the formulas is interpreted as an instance. We have already automated this process into a tool to demonstrate the feasibility of this approach.

2 BACKGROUND

In this section, we briefly review the standard coverage criteria defined for UML class diagram, notations we use for expressing a metamodel as a graph, and basic SMT encodings from our previous work. Formally, we consider all metamodels in this paper as being presented as UML class diagrams, and represented as graphs.

2.1 Metamodel Coverage Criteria

A metamodel is a structural diagram and can be depicted using the UML class diagram notation. Thus, the coverage criteria defined for UML class diagram can also be borrowed for metamodels. In particular, we focus on the coverage criteria presented in (Andrews et al., 2003) (Ghosh et al., 2003), especially the work focused on testing the structural elements of a UML class diagram. These coverage criteria are standard criteria for testing a UML class diagram and they are defined as follows:

- Generalisation coverage (*GN*) which describes how to measure inheritance relationships.
- Association-end multiplicity coverage (*AEM*) which measures association relationships defined between classes.
- Class attribute coverage (*CA*) which measures the set of representative attribute value combinations in each instance of class.

AEM and *CA* are partition-based testing criteria which means that testing results depend on the choice of a representative value from each partition (Ostrand and Balcer, 1988). Therefore, the value domain is partitioned into several equivalence classes, and each value from an equivalent class is expected to have the same results. The partitions can also be decided using domain knowledge-based partitioning.

For example, to satisfy the *CA* criterion for the metamodel in Figure 1, we may assume a user could choose a representative value of 18 [1], and this allows the attribute *age* in the abstract class *Person* to be divided into 3 partitions which are $18 < age$, $age = 18$ and $age > 18$. The hypothesis is that any single value from one of the three partitions is expected to have the same results for all other values from that partition (Myers and Sandler, 2004). Similarly, to satisfy the *AEM* criterion, the binary association (*employees*) in the metamodel can also be divided into two partitions: a $Department$ that has no $Manager$ or a $Department$ that has multiple $Manager$s. The multiple number of *Managers* can be a boundary value chosen by a user. For example, it can be the maximum value that an integer can hold or 5 if it is determined by specific domain knowledge about the model. Finally, to satisfy the *GN* criterion, the inheritance relation can be covered by ensuring the creation of an instance of $Manager$.

In this paper, we focus on generating instances meeting *CA* and *AEM* criteria by providing a general SMT encoding. For *GN*, it has already been incorporated into our previous work. Our previous work takes a metamodel, presented as a class diagram with OCL constraints, augmented with quantitative constraints, and uses an SMT solver to generate instances. Our earlier work also supports a subset of OCL, this includes: constraints on an attribute, navigation over an association, and nested quantifiers over a collection of instances. To facilitate the transformation from class diagrams with OCL constraints to SMT formulas we use a *bounded typed graph* as an intermediate representation.

2.2 Bounded Typed Graphs

Our previous work considered classes in a metamodel as nodes, and relationships between classes are edges linking one node to another (Wu et al., 2013). Thus, we can formally define a graph, namely typed graph (TG) as: $TG = (V_T, E_T)$, where V_T and E_T represents a set of nodes (classes) or edges (associations and inheritances).

Each valid instance of a metamodel is also a graph: $G = (V_G, E_G)$, where V_G is set of nodes (objects) and E_G is set of edges (links), but preserve extra type information about the classes in a metamodel. Thus, we now can define a mapping between two graphs TG and G: $type = (type_V, type_E)$ where $type_V : V_G \rightarrow V_T$ and $type_E : E_G \rightarrow E_T$.

Now, we formally define a bounded typed graph as: $TG_b = (V_T, E_T, b)$ where b is a bound function $b : V_T \rightarrow Z^+$ that maps a typed node (non-abstract) to an integer. This integer specifies an upper bound of the same typed node that an instance may contain. Therefore, using this bound function, we can bound our search space to guarantee the termination for metamodel instance generation.

For example, Figure 1 and 2 show a meta-

[1] A user may choose a different representative value, this depends on the knowledge about a specific domain.

model represented as a bounded typed graph and an instance of it. The bounded typed graph (metamodel) and graph (model) can be related with *type* such that $type_V(ComputerScience) = Department$, and $type_V(John) = type_V(Willam) = type_V(Robert) = Manager$. Similarly,$type_E(e1) = type_E(e2) = employ$.

To generate valid instances from a metamodel, we form a finite universe based on each bound defined for a typed node. This includes generating all possible links based on a particular association (edge) defined within the bound. We then use corresponding translation rules to encode them into SMT formulas. For example, for the typed graph shown in Figure 1, we form a finite universe containing 1 *Department* (*ComputerScience*) and 3 *Managers* (*John*,*William*,*Robert*). For the association *employ*, we form an adjacent matrix that describes all possible connections between *Department* and *Manager*. Each entry in the matrix is an SMT boolean variable indicating whether a link is selected or not. We then disjunct each entry in the matrix. The following steps show this basic translation for the metamodel in Figure 1 to the SMT formulas:

1. Form a finite universe: $\{ComputerScience, John, William, Robert\}$
2. For association *employ*, we form an adjacency matrix:

	John	*William*	*Robert*
ComputerScience	e_1	e_2	e_3

3. Generate SMT formula: $e_1 \vee e_2 \vee e_3$

The SMT formula captures the meaning of association *employ*: each *Department* is associated with at least one of the *Mangers*. Figure 2 shows an example of only e_1, e_2 and e_3 are assigned to be *true* by an SMT solver, representing *John*, *William* and *Robert* are employed by the *ComputerScience* department.

In this paper, we assume all OCL constraints defined over a metamodel are not conflicted with both criteria. For example, a representative value of 18 is chosen for the attribute *age*, and an OCL constraint is defined as $self.age <> 18$.

3 PARTITION-BASED INSTANCE GENERATION

The main idea for generating instances that satisfying *CA* and *AEM* coverage criteria is by adding additional constraints expressed as SMT formulas to block irrelevant instances during the search. Each successful

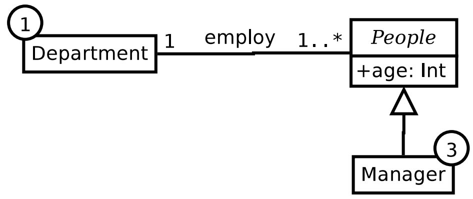


Figure 1: An example of a metamodel, represented as a bounded typed graph. The bounds for *Department* and *Manager* are depicted with a number in a circle on each class. In this case, they are 1 and 3 for *Department* and *Manager* respectively. No bound is specified for *People* as it is an abstract class.

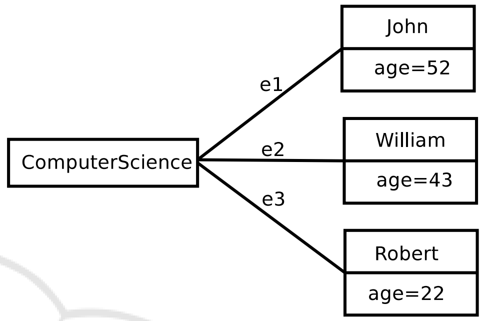


Figure 2: An instance of metamodel in Figure 1. This instance contains 1 instance of *department* and 3 instances of *manger*.

assignment is then interpreted as an instance that satisfies the coverage criteria. In the following sections, we show how these constraints can be expressed as SMT formulas.

For the set of features P in a metamodel, the general form of a constraint for each feature P_i in P can be expressed by:

$$\bigvee_{j=1}^{|P_i|} (T_i = V_j) \wedge F_i$$

where

- $|P_i|$ denotes the total number of partitions of a feature P_i.
- T_i is a *partition switch*, determines when a particular partition is to be switched on or off based on V_j.
- V_j indicates the jth partition of a feature P_i. This implies that the value for V_j chosen by an SMT solver determines which particular partition is selected.
- F_i is a *criteria formula* that is connected with a partition switch, indicating that when a partition switch is on the *criteria formula* must be applied.

For different partition-based criteria, ensuring that the instances generated by the SMT solver achieve

- When an attribute d is an integer type: $((T_i = 0) \wedge (d < p)) \vee ((T_i = 1) \wedge (d = p))$ $\vee ((T_i = 2) \wedge (d > p))$
- when an attribute d is a boolean type: $((T_i = 0) \wedge (d = false)) \vee ((T_i = 1)$ $\wedge (d = true))$

Figure 3: SMT encoding for partitioning integer and boolean type attribute.

those criteria, depends on criteria formulas in each constraint. These criteria formulas are captured by the corresponding SMT encoding, and we elaborate these encodings in the Section 3.1 and 3.2.

3.1 Partitioning for Class Attributes

Achieving CA coverage requires that a constraint covers every partition created for each attribute, and this is controlled by criteria formula. In other words, the criteria formulas determine what value is to be assigned for an attribute in each instance.

Our current approach supports two types of attributes: integer and booelan, and the SMT encodings for those two types of attribute are presented in Figure 3.

As shown in Figure 3, for each *ith* attribute in a class, a partition switch (T_i) is created. For an integer type attribute, T_i has a value of 0, 1 or 2 indicating 3 partitions: $> p$, $= p$ and $< p$, where p is a representative value chosen by a user, each partition has a corresponding criteria formula. If no particular value is given, a value $p = 0$ will be chosen as default. These three partitions are directly expressed into SMT formulas. Similarly, an SMT encoding for a boolean type attribute is formed except that the partition switch is either 0 or 1, since a boolean value can only be $true$ and $false$.

3.2 Partitioning Associations

Associations between classes are an important part of a metamodel, and it is desirable that generated instances should also cover these associations for different partitions. The standard coverage criteria for associations, known as Association-end multiplicity (AEM), has already been defined in (Andrews et al., 2003), and in this section we show how this can be extended to metamodels and incorporated into our approach.

3.2.1 Partitioning Unidirectional Associations

To implement AEM coverage for unidirectional association, we specify *criteria formulas* corresponding to the most frequently used association types defined in a metamodel based on their multiplicities. These criteria formulas determine how each node (object) in an instance can be linked to others. For an association, we form all possible links from typed node to another based on the bound defined for each class at both association ends, and we then apply the corresponding translation rule to form a set of SMT formulas.

As shown in section 2.2, we use an adjacent matrix E_{ref} to represent all possible links for an association ref between class A and B. The rows of the matrix, denoted as E_{row}, represent all links from one instance of A to one instance of B. The columns of the matrix, denoted as E_{col}, represent all links from one instance of B to one instance of A. Each entry $e_{i,j}$ in E_{ref} is an SMT boolean variable.

Figure 4 summarises 4 rules for common unidirectional association patterns. For each rule, there is a partition switch that is either 0 or 1 indicating that the association is divided into two partitions. For example, the second formula in Figure 4 shows an example of the translation rules for unidirectional association pattern $1..*$. This rule states that this association can be partitioned into two partitions, and each partition is controlled by a partition switch (T) and a criteria formula. One partition is that for each instance of A is associated with exactly one instance of B, and the other is for each instance of A is linked with a k number of instances of B. In order to know the exact k number of instances of B that can be associated with an instance of A, both criteria formulas (associated with T) consist of an auxiliary matrix (Aux), where each element ($Aux_{i,j}$) in that matrix is an integer SMT variable. Each $Aux_{i,j}$ uses either 1 or 0 to denote whether a link in E_{ref} is selected or not. To compute k number of instances of B that connect to an instance of A, we add up all $Aux_{i,j}$s in the same row to k. For example, Figure 5 shows a possible assignment found by an SMT solver. Each instance of A is connected to 3 instances of B, since each row in the array is added up to 3. Once an $Aux_{i,j}$ is chosen to be one, the corresponding $e_{i,j}$ in the matrix E_{ref} is also switched on (set to $true$). This indicates that a relevant link is presented in the instance.

3.2.2 Partitioning Bidirectional Associations

A bidirectional association distinguishes a unidirectional association by counting links in two directions. Therefore, a translation rule for a bidirectional association constrains both E_{row} and E_{col}. In general, the translation rules in Figure 6 are similar to unidirectional except that we need to correctly calculate the possible maximum number of instances of B that an instance of A connects to.

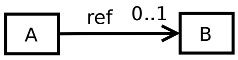
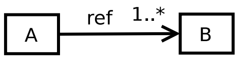
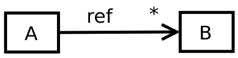
<table>
<tr><th>Association Pattern (Unidirectional)</th><th>Translation Rule</th></tr>
<tr><td></td><td>(1) $((T=0) \quad \wedge \quad \bigwedge_{i=1}^{|E_{row}|} \bigvee_{j=1}^{|E_{col}|} \neg e_{i,j})$
$\vee \quad ((T=1) \quad \wedge \quad (\bigwedge_{i=1}^{|E_{row}|} (\bigvee_{j=1}^{|E_{col}|} (\bigwedge_{k=1,k\neq j}^{|E_{col}|} \neg e_{i,k}) \wedge e_{i,j})))$</td></tr>
<tr><td></td><td>(2) $((T=0) \quad \wedge \quad \bigwedge_{i=1}^{|E_{row}|} (\sum_{j=1}^{|E_{col}|} Aux_{i,j}) = 1)$
$\vee \quad ((T=1) \quad \wedge \quad \bigwedge_{i=1}^{|E_{row}|} (\sum_{j=1}^{|E_{col}|} Aux_{i,j}) = k \quad \wedge \quad |E_{col}| > 1)$
where $1 < k \leq |E_{col}|$</td></tr>
<tr><td></td><td>(3) $((T=0) \quad \wedge \quad \bigwedge_{i=1}^{|E_{row}|} \bigvee_{j=1}^{|E_{col}|} \neg e_{i,j})$
$\vee \quad ((T=1) \quad \wedge \quad \bigwedge_{i=1}^{|E_{row}|} (\sum_{j=1}^{|E_{col}|} Aux_{i,j}) = k \quad \wedge \quad |E_{col}| \geq 1)$
where $1 \leq k \leq |E_{col}|$</td></tr>
</table>

Figure 4: Translation rules for unidirectional association patterns.

$$Aux = \begin{array}{c|ccccc} & b_1 & b_2 & b_3 & b_4 & b_5 \\ \hline a_1 & 0 & 1 & 0 & 1 & 1 \\ a_2 & 1 & 0 & 1 & 0 & 1 \\ a_3 & 0 & 1 & 1 & 1 & 0 \end{array}$$

Figure 5: An example of a possible assignment found by an SMT solver for the auxiliary matrix.

For example, the second rule in Figure 6 is for a bidirectional association pattern $1 \leftrightarrow 1..*$ [2]. This pattern is partitioned into two partitions. One is for each instance of A is connected to exactly one instance of B, and one instance of B can only connect to one instance of A. The other partition allows one instance of A to connect to multiple instances of B. Both partitions are controlled by a partition switch (T), and have two different criteria formulas. The first criteria formula specifies the first partition is one instance of A connects exactly one instances of B, and one instances of B can only connect to one instances of A.

Since the second partition allows k number of instances of B to be connected to an instance of A, the criteria formula needs to compute the maximum possible number of instances of B that an instance of A can connect to. We compute this number k by calculating the difference between the bound of B and the bound of A, and adding 1. Thus, $|E_{row}|$ specifies $b(A)$ while $|E_{col}|$ gives $b(B)$. To understand how k gets calculated, we consider the following three scenarios:

1. $|E_{col}| = |E_{row}|$: we have equal number instances of A and B. Since the multiplicities for two association-ends (1 and $1..*$) tell us that one instance of A must be connected to at least one instance of B, and one instance of B can only be linked to one instance of A, this scenario now implies that each instance of A connects each instance of B, vice versa. Thus, the maximum number of instances of B that an instance of A can connect to is $k = 1$.
2. $|E_{col}| > |E_{row}|$: we have more instances of B than A. We first connect every instance of A to one instance of B, and every instance of B connects to only one instance of A. Now, we can add the remaining number instances of B to one of the existing connections between instance of A and B, and counts one link as already having been established. Thus, k gives the maximum number of Bs that one of the instance of A can connect to.
3. $|E_{col}| < |E_{row}|$: we have less instances of B than A. However, this scenario violates the constraint implied by the multiplicities, and is thus ruled out.

In all possible cases the minimum number of instances of B has to be equal to the number instances of A. Figure 7 shows an example where one instance of A can connect to at most 3 (here we set $k = 3$) instances of B.

Similarly, rule 1 in Figure 6 states that when the first partition is chosen (T is 0), no links encoded by $e_{i,j}$ are chosen. When the second partition is chosen (T is 1), only one $e_{i,j}$ is chosen. This indicates that a link ($e_{i,j}$) between each instance of A and B is al-

[2]We use $x \leftrightarrow y$ to denote a bidirectional association with two multiplicities x and y at two association ends

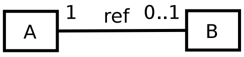
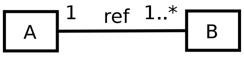
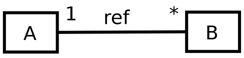
Association Pattern (Bidirectional)	Translation Rule
	(1) $((T=0) \wedge \bigwedge_{i=1}^{\lvert E_{row}\rvert}\bigvee_{j=1}^{\lvert E_{col}\rvert} \neg e_{i,j})$ $\vee \quad ((T=1) \wedge \bigvee_{i=1}^{\lvert E_{row}\rvert}(\sum_{j=1}^{\lvert E_{col}\rvert} Aux_{i,j}) = 1)$
	(2) $((T=0) \wedge \bigvee_{i=1}^{\lvert E_{row}\rvert}(\sum_{j=1}^{\lvert E_{col}\rvert} Aux_{i,j}) = 1)$ $\vee \quad ((T=1) \wedge \bigvee_{i=1}^{\lvert E_{row}\rvert}(\sum_{j=1}^{\lvert E_{col}\rvert} Aux_{i,j}) = k) \wedge \lvert E_{row}\rvert < \lvert E_{col}\rvert$ where $1 < k \leq \lvert E_{col}\rvert - \lvert E_{row}\rvert + 1$
	(3) $((T=0) \wedge \bigwedge_{i=1}^{\lvert E_{row}\rvert}\bigvee_{j=1}^{\lvert E_{col}\rvert} \neg e_{i,j})$ $\vee \quad ((T=1) \wedge \bigvee_{i=1}^{\lvert E_{row}\rvert}(\sum_{j=1}^{\lvert E_{col}\rvert} Aux_{i,j}) = k) \wedge \lvert E_{row}\rvert \leq \lvert E_{col}\rvert$ where $1 \leq k \leq \lvert E_{col}\rvert - \lvert E_{row}\rvert + 1$

Figure 6: Translation rules for bidirectional association patterns.

$$Aux = \begin{array}{c|ccccc} & b_1 & b_2 & b_3 & b_4 & b_5 \\ \hline a_1 & 1 & 0 & 0 & 1 & 1 \\ a_2 & 0 & 0 & 1 & 0 & 0 \\ a_3 & 0 & 1 & 0 & 0 & 0 \end{array}$$

Figure 7: An example of an assignment found by an SMT solver for an association between two classes A and B, where the bounds for A and B are 3 and 5, respectively. In this example an instance of A is linked with a maximum of 3 instances of B. Since the sum of each column is 1, an instance of B is only connected to a single instance of A.

lowed to be presented. Rule 3 for the association pattern ($1 \leftrightarrow *$) has a criteria formula that combines the first criteria formula from the first association-pattern ($1 \leftrightarrow 0..1$) with the second criteria formula from the second association-pattern ($1 \leftrightarrow 1..*$) as they indicate that either no links are chosen at all or choose exactly k number of links between an instance of A and B.

4 INSTANCE GENERATION USING GRAPH-BASED CONSTRAINTS

The techniques described in the previous sections allow coverage criteria to be achieved for the class attributes and associations in a metamodel. As such they are standard coverage criteria for UML class diagrams, and may be applied to any metamodel. However, it is reasonable to suppose that a user will have more sophisticated requirements, and wish to direct the generation of instances so as to satisfy other constraints. For this reason, we present a new technique that allows instance generation to meet the following graph-based properties.

4.1 Directed Acyclic Graphs

Directed acyclic graphs (DAGs) are commonly used in many areas, for example, the topology of a network, data flow diagrams, etc. Regarding metamodeling, one may require a program to have a particular depth of inheritance tree, or a particular call depth. Thus, to ensure the generation of a DAG from a reflexive association in a metamodel, we only enable the elements that are in the upper triangle of the adjacency matrix, and disable the rest of the $e_{i,j}s$ in the matrix, breaking all the cycles in the graph.

4.2 Sharing and Non-sharing Nodes

Since we use an adjacency matrix to capture all possible links (within the bounds) for an association, we can also manipulate this matrix to form new formulas that can express how links are connected to each other. In particular, we can facilitate the specification of *graph-based constraints* by the user that will direct instance generation. In order to facilitate instance generation with such properties, we introduce the following new properties.

In a graph, some nodes may have all their outgoing edges going to the same node and some may not. We consider these nodes having sharing and non-sharing properties. Sharing and non-sharing properties can only be applied to a *non-reflexive* association. Before we precisely define sharing and non-sharing properties, we first define two functions (f and g).

1. Function f is an out-adjacency function (Adj^+) that computes a set of nodes from all out-going edges of a particular node. $f : V_G \rightarrow 2^{V_G}$, where V_G is the set of nodes, and 2^{V_G} is the power set of V_G.
2. Function g is an in-adjacency function (Adj^-) that computes a set of nodes from all in-coming edges of a particular node. $g : V_G \rightarrow 2^{V_G}$, where V_G is the set of nodes, and 2^{V_G} is the power set of V_G.

With functions f and g, we are able to calculate a set of nodes based on their in-coming and out-going edges. Now we can use these two functions to define the following sharing and non-sharing properties:

- A set of nodes $L = \{N_1, N_2, ..., N_j\}$, where $|L| \geq 2$, are said to be *strong sharing nodes* iff $(\bigcap_{i=1}^{j} f(N_i)) \neq \emptyset$, $\forall L_x \in \bigcup_{i=1}^{j} f(N_i)$ and $g(L_x) \subseteq L$.
- A set of nodes $L = \{N_1, N_2, ..., N_j\}$, where $|L| \geq 2$, are said to be *weak sharing nodes* iff $(\bigcap_{i=1}^{j} f(N_i)) \neq \emptyset$, $\exists L_x \in \bigcup_{i=1}^{j} f(N_i)$ and $L \subset g(L_x)$.
- A set of nodes $L = \{N_1, N_2, ..., N_j\}$, where $|L| \geq 2$, are said to be *strong non-sharing nodes* iff $\forall L_i \in L$, $|f(N_i)| = 1$ and $f(N_a) \cap f(N_b) = \emptyset$, where $1 \leq a < b \leq j$.
- A set of nodes $L = \{N_1, N_2, ..., N_j\}$, where $|L| \geq 2$, are said to be *weak non-sharing nodes* iff $\forall N_i \in L$, $|f(N_i)| > 1$ and $f(N_a) \cap f(N_b) = \emptyset$, where $1 \leq a < b \leq j$.

To understand these definitions, we use two examples to illustrate sharing and non-sharing properties. In Figure 8, a solid line is used to denote the existing links and a dashed line is used to represent possible links. The set of nodes $n1$ and $n2$ (with solid lines) are considered as strong sharing nodes since both their out-adjacency functions return $n3$ ($f(n1) = f(n2) = \{n3\}$), and $n3$'s in-adjacency function returns $n1$ and $n2$ ($g(n3) = \{n1, n2\}$) . In other words, $n3$ can only be accessed by both $n1$ and $n2$ and no other nodes. However, if a link from $n4$ to $n3$ is connected, then the set of nodes $n1$ and $n2$ are regarded as weak sharing nodes because $n3$'s in-adjacency function this time returns three nodes: $g(n3) = \{n1, n2, n4\}$. Thus, the set of nodes $n1$, $n2$ and $n4$ are considered as strong sharing nodes ($f(n1) \cap f(n2) \cap f(n4) = n3$), and $g(n3) \subseteq \{n1, n2, n4\}$).

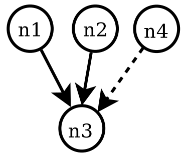


Figure 8: An example of sharing nodes in a graph.

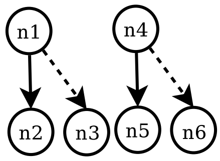


Figure 9: An example of non-sharing nodes in a graph.

Similarly, in Figure 9 the solid lines between nodes $n1$, $n2$ and $n4$, $n5$ make the set of nodes $n1$ and $n4$ strong non-sharing nodes in the graph ($|f(n1)| = |f(n4)| = 1$, and $f(n1) \cap f(n4) = \emptyset$). If $n1$ also connects to $n3$ (a possible link), and $n4$ connects to $n6$, then the set of nodes $n1$ and $n2$ are weak non-sharing nodes, since they all connect to more than one other node ($|f(n1)| = |f(n4)| > 1$).

Figure 11 shows a matrix for capturing an association in a metamodel. Suppose we want to give strong sharing property to a set of nodes $L = \{a_1, a_4\}$. This indicates that at least one of the $b's$ must be shared by them. For example, $e_{1,1}$ and $e_{1,4}$ could be selected at the same time, or $e_{2,1}$ and $e_{2,4}$ are chosen ($(e_{1,1} \wedge e_{1,4}) \vee (e_{2,1} \wedge e_{2,4})$). This represents that a_1 and a_4 they both have out-going edges to b_1 or b_2. This is captured by the first sub-formula of rule (2) from Figure 10. Now, suppose $e_{1,1}$ and $e_{1,4}$ are selected, then anything between them cannot be selected otherwise they are not strong sharing nodes. Thus, $e_{1,2}$ and $e_{1,3}$ are disabled when $e_{1,1}$ and $e_{1,4}$ are selected ($(e_{1,1} \wedge e_{1,4}) \rightarrow (\neg e_{1,2} \wedge \neg e_{1,3})$). This is captured by the second sub-formula of rule (2) from Figure 10.

Similarly weak sharing, strong and weak non-sharing properties are captured in rule (1) (3) and (4) in Figure 10. In each formula listed in Figure 10, we use L to denote a set of nodes to be assigned with one of the four properties, and $|L| \geq 2$. For weak sharing property, the Formula is similar to the Formula for strong sharing property except that we drop the second sub-formula. Instead, we add a formula that states that at least one of the $a's$ not specified in L can be linked to the $b's$. The formula for the strong non-sharing property indicates that only one link can be selected according to specified nodes in L. It indicates that as long as one link is selected all other links in the same row and column are switched off. Similarly, for weak non-sharing property, the formula indicates that there could be multiple links selected according to a

Property	SMT Formula
(1) Weak sharing	$\bigvee_{i=1}^{\lvert E_{row}\rvert} \bigwedge_{k=1}^{\lvert L\rvert} e_{i,L_k} \wedge \bigwedge_{i=1}^{\lvert E_{row}\rvert} \bigvee_{j=1,j\notin L}^{\lvert E_{col}\rvert} e_{i,j}$
(2) Strong sharing	$(\bigvee_{i=1}^{\lvert E_{row}\rvert} \bigwedge_{k=1}^{\lvert L\rvert} e_{i,L_k}) \wedge (\bigwedge_{i=1}^{\lvert E_{row}\rvert} (\bigwedge_{k=1}^{\lvert L\rvert} e_{i,L_k} \rightarrow \bigwedge_{j=1,j\notin L}^{\lvert E_{col}\rvert} \neg e_{i,j}))$
(3) Strong non-sharing	$(\bigwedge_{k=1}^{\lvert L\rvert} \bigvee_{i=1}^{\lvert E_{row}\rvert} (\bigwedge_{j=1,j\neq i}^{\lvert E_{row}\rvert} \neg e_{j,k}) \wedge e_{i,L_k}) \wedge (\bigwedge_{i=1}^{\lvert E_{row}\rvert} (\bigwedge_{k=1}^{\lvert L\rvert} e_{i,L_k} \rightarrow \bigwedge_{j=1,j\notin L}^{\lvert E_{col}\rvert} \neg e_{i,j}))$
(4) Weak non-sharing	$(\bigwedge_{k=1}^{\lvert L\rvert} \bigvee_{i=1}^{\lvert E_{row}\rvert} e_{i,L_k}) \wedge (\bigwedge_{i=1}^{\lvert E_{row}\rvert} (\bigwedge_{k=1}^{\lvert L\rvert} e_{i,L_k} \rightarrow \bigwedge_{j=1,j\notin L}^{\lvert E_{col}\rvert} \neg e_{i,j}))$

Figure 10: The SMT formulas for capturing sharing/non-sharing properties.

	a_1	a_2	a_3	a_4
b_1	$e_{1,1}$	$e_{1,2}$	$e_{1,3}$	$e_{1,4}$
b_2	$e_{2,1}$	$e_{2,2}$	$e_{2,3}$	$e_{2,4}$
b_3	$e_{3,1}$	$e_{3,2}$	$e_{3,3}$	$e_{3,4}$

Figure 11: An example of matrix for illustrating sharing and non-sharing properties. In this example, each $e_{i,j}$ is represented as a link from a_j to b_i.

specific node in L. This indicates that a node can connect to at least one or more nodes. Since connections to multiple nodes are allowed, all other nodes in the same row must be disabled.

5 EVALUATION

In this section, we first briefly describe a tool that is extended with partition-based and graph-based instance generation, then we present our initial evaluation, and finally we discuss its capabilities and limitations.

5.1 Implementation

We have implemented and integrated the partition-based and graph-based criteria described in this paper in our existing tool, ASMIG [3]. ASMIG takes in a metamodel in ecore format (with a bound defined for each class in that metamodel) and an OCL file, outputs all consistent models (if it has any) within those bounds. To enumerate "all possible" instances, ASMIG blocks all previously generated instances by adding the negation of satisfiable assignments found by an SMT solver one at a time until no more satisfiable assignment is possible. ASMIG is purely written in Java and it consists of about 22000 (excluding UI) lines of code (LOC) with about 8300 LOC dedicated to the techniques described in this paper. To construct ASMIG, we re-engineered a parser extracted from the USE tool (Kuhlmann et al., 2011), adapting it to use as our front-end for reading the OCL invariants. The current version of ASMIG uses Z3 as its default back-end SMT solver (De Moura and Bjørner, 2008), supports generating formulas in SMT2 standard (Barrett et al., 2010).

5.2 Results

The evaluation for both partition-based and graph-based criteria is performed on a machine with a 2.93GHz Intel Core 2 Duo and 4GB memory, the results and time are recorded in Figure 12 and 13. More detailed results, along with further examples, are available at our website[8].

Figure 12 shows the results for partition-based criteria based on a total of 15 metamodels. For each metamodel, we record translation time and average instance generation time. To evaluate scalability, we select a variety of metamodels such as general purpose programming languages, domain specific languages, ranging from small size to large size. We believe these metamodels are good representatives in terms of their usage in different domains. To effectively evaluate partition-based technique, we change ASMIG's internal configuration in order to set a large enough upper bound for each non-abstract class, this would guarantee each non-abstract class to be initialised at least once and up to that upper bound. The *total bounds* column in Figure 12 shows the sum of each bound for every non-abstract class in the metamodel. The *total instances* column indicates the number of instances generated in order to achieve full coverage for the CA and AEM criteria. For the class attributes coverage criteria (*CA*), ASMIG allows users

[3]available at: https://bitbucket.org/classciwuhao/asmig

[4]Available at: http://www.emn.fr/z-info/atlanmod/

[5]From Eclipse Modeling Framework Royal and Loyal Example Project

[6]Extracted from Eclipse Modeling Framework

[7]Available at: http://www.jamopp.org/

[8]http://www.cs.nuim.ie/∼haowu/ASMIG/Results/

	Number of			Total		Time in ms	
Metamodel	Classes	Assocs	Attribs	Bounds	Instances	Translation	Avg Finding
Finite State Machine 1.0 [4]	16	7	0	16	4	485	30
DOT (Graphviz) 1.0 [4]	26	20	0	21	4	512	50
Royal & Loyal [5]	15	41	2	40	8	535	65
Ant [4]	48	27	0	56	4	673	71
CPL 1.0 [4]	32	16	0	38	4	513	91
Maven (maven.xml) 0.3 [4]	58	32	0	65	4	559	108
Ecore [6]	22	40	0	31	4	585	195
UML2 Class Diagram [6]	40	26	46	35	8	721	966
HTML [4]	59	7	0	59	4	673	1562
Java [7]	233	104	1	183	8	971	2629
Company [8]	7	6	6	13	12	487	36
C++ 1.0 [4]	16	4	5	20	8	499	22
GraphML [4]	11	13	2	20	8	496	53
Hierarchical State Machine 1.0 [4]	15	16	0	33	4	510	72
BibTexML1.2 [4]	28	4	0	18	4	510	28

Figure 12: Results of using our tool to generate instances of 15 publicly-available metamodels. For each metamodel we show its size (in terms of classes, associations and attributes), the bound on the number of classes in a metamodel, the number of instances generated, and two measures of the time taken.

to choose a representative value, but for general purpose, we choose the default value 0 to obtain three partitions (< 0, $= 0$ and > 0) for each integer type attribute. For the association-end multiplicity criteria (*AEM*), we set 3 instances as an upper bound for each association that has a multiplicity of $*$. We choose this upper bound because it is easy for us to distinguish this from a one-to-one multiplicity for both association ends.

Figure 13 shows the results for ASMIG to generate 100 instances of a subset of Java programming language metamodel based on specific bounds for four different CK metrics (Chidamber and Kemerer, 1994). We choose this metamodel because the graph-based criteria focus on a specific association such as an inheritance relationship must be a directed acyclic graph. We choose CK metrics because these metrics possess good graph properties. For simplicity, we consider the complexity of WMC as the sum of the methods in a class. To correctly calculate LCOM value, we first use an SMT solver to select a list of nodes to be connected, then apply sharing/non-sharing properties to those nodes. There are a number of possible definitions of the LCOM metric which are supported by ASMIG. We choose LCOM3 here which represents the methods accessing common fields as a connected graph. Thus, a value of 2 for LCOM3 means 2 connected graphs with respect to method accessing fields. For each of the four metrics in Figure 13 we specified three different metric values (shown in the *values* column) causing the calculated bounds to vary between 3 to 10. For DIT, we directly apply the method described in Section 4.1. For NOC, we first fix the number of links between two nodes via an auxiliary matrix as shown in Section 3.2, then apply strong sharing properties to these nodes. All the detailed instances generated for above metrics are available at our website [8].

5.3 Discussion

Capabilities. (1) The partition-based technique in section 3.1 and section 3.2 allows one to achieve a full equivalence partitioning testing by iteratively selecting a different representative value. Equivalence partitioning testing is considered as one of the important techniques for testing object oriented system (Binder, 1999) (Gutjahr, 1999). For *CA*, equivalence partitioning testing for a valid range of 0..100 can be achieved via two steps. Firstly, pick 0 as the representative value covering < 0, $= 0$ and > 0, then choose 100 covering < 100, $= 100$ and > 100. Sim-

CK Metric	Metric Value	Total Bound	Time in ms Translate	Time in ms Finding
WMC	2	3	446	31
	3	6	444	59
	5	10	461	120
DIT	2	3	456	53
	4	6	457	54
	8	10	460	180
NOC	2	3	469	64
	4	6	481	65
	8	10	489	180
LCOM	2	3	476	63
	2	6	470	42
	3	10	484	138

Figure 13: Results of generating 100 instances which satisfy constraints based on four of the CK metrics. Each metric was constrained using three values, and the calculated bounds are shown, as well as two measures of the time taken to generate appropriate instances.

ilarly, for *AEM*, one can set a larger bound just outside the boundary (the boundary can be decided by using knowledge of the problem domain) for each class at two ends of an association. (2) Having instances meeting graph-based constraints provides a way of analysing or measuring a software system such as generating a control flow graph via specifying sharing/non-sharing properties on specific nodes. Viewing a metamodel or an instance as a graph brings one kind of diversity of instance generation for those who require models that are based on particular shape of a graph.

Limitations. (1) Currently, our SMT formulas do not fully support a graph criteria that constrains over more than a single association. For example, metrics like response for a class (RFC) or coupling between object classes (CBO) typically constrain over two different associations. However, this can be avoided via a sequence of SMT solving, and use the assignment from previous successful solving as the input to the next SMT solving. For example, for RFC, one could fix a set of methods first, then use SMT solver to distribute the number of methods directly called by that set of methods, finally apply sharing/non-sharing properties to those methods are selected from previous SMT solving. (2) We admit that both partition-based and graph-based criteria may not be sufficient enough to fulfill users' expectation of the diversity of instances. One may certainly require a different criteria for validating a metamodel based on different testing strategies. *CA* and *AEM* criteria both are part of standard coverage criteria for UML class diagrams, and graph-based criteria provides a way of generating instances based on describing graph properties. With both kinds of instance generation, we can at least provide a certain degree of confidence in designing, testing or validating a metamodel. In the future, we will investigate a more general approach that would allow language engineers to describe customised coverage criteria via a simple domain specific language.

6 RELATED WORK

One of the challenges with metamodelling is that it is difficult to instantiate a metamodel since instances have to conform to both the metamodels' structural constraints and additional semantic constraints written in a language such as OCL. Although much recent research has endeavoured to instantiate metamodels using different approaches and techniques (Anastasakis et al., 2007; Ehrig et al., 2009; González Pérez et al., 2012; Macedo and Cunha, 2013), the ability to coverage criteria directed instance generation is still quite limited.

One of the earliest approaches to generating programs using coverage criteria is Purdom's algorithm, based on generating programs that cover all the rules in a context free grammar (CFG) (Purdom, 1972). However, a metamodel captures more than a CFG because the static semantics can be defined, e.g. using extra OCL constraints. Though work has been done on extending Purdom's approach to attribute grammars (Harm and Lämmel, 2000), thus incorporating semantic constraints, the core generation framework is still based on rule coverage, and more general coverage criteria are not considered.

The most closely related research to our work is the model finding tool *Alloy* (Jackson, 2002). Alloy translates a relational specification into formulas for a SAT solver, and each successful assignment for the SAT instances can be mapped back to the problem domain, and much of the research built around Alloy facilitates instance generation. In previous work we have used Alloy to generate instances of metamodels using the Eclipse Modeling Framework, and then applied test-suite reduction techniques in order to pick out instances contributing to a coverage criteria (McQuillan and Power, 2008). However much of our work (and that of others) was limited by the capabilities of Alloy, particularly in relation to coverage oriented generation and quantitative constraints (Anastasakis et al., 2007; Bordbar and Anastasakis, 2005; Kuhlmann et al., 2011; Sen et al., 2009; Kuhlmann and Gogolla, 2012).

The latest version of Alloy employs a powerful relational engine called *kodkod* (Torlak and Jackson, 2007), which outperforms previous versions of Alloy on large-scale problem solving. However, a major disadvantage is the dependence on SAT solvers which perform poorly when dealing with numeric quantities, using calculations such as addition, multiplication, comparison, etc.

Graph grammars offer a natural way to describe the instance generation process and so have an advantage for generating metamodel instances (Ehrig et al., 2009; Hoffmann and Minas, 2011). Though graph grammars deal with graphs, it is more difficult to user their grammars to quantify a set of nodes than using first-order logic. Parsing a graph is expensive because graph matching is not always deterministic. Thus, the cost of using graph grammars to produce instances that meet graph-based constraint could be very high.

Cabot et al. propose a detailed systematic procedure that reduces the problem of UML class digram instantiation to a Constraint Satisfaction Problem (CSP) (González Pérez et al., 2012; Cabot et al., 2008; Cabot et al., 2014). The main advantage is that CSP provides a high-level language so that a particular constraint problem is programmable. Our approach distinguishes with theirs by reducing it to an SMT problem. SMT encoding provides a much better expressiveness power than SAT, and it is more natural to encode a problem into SMT formulas. Much research has been made in improving SMT solvers' expressiveness and performance (Barrett et al., 2010; De Moura and Bjørner, 2008; Barrett et al., 2011; Cimatti et al., 2013), which make them more suitable for complicated tasks such as verification, test case generation, program synthesis, etc (Büttner et al., 2012; Felbinger and Schwarzl, 2014; Tillmann and De Halleux, 2008; Gulwani, 2010).

Soeken et al. encode a UML class diagram in a set of operations on bit-vectors which can be solved by SMT solvers using bit-vector theory (Soeken et al., 2010). A successful assignment for each bit-vector can be interpreted as an instance of the UML class diagram. They also propose an approach to encode a subset of OCL constraints as bit-vectors, and provide a list of corresponding mappings between OCL collection data types and bit-vector operations (Soeken et al., 2011). However, their approach does not support structural constraints on the metamodel, especially the quantitative constraints for associations. Furthermore, it is unlikely to use their approach to generate instances satisfying graph-based constraints because they do not represent a metamodel as a graph and provide no tool support.

7 CONCLUSION

In this paper, we have presented a new approach to improve metamodel instance generation by considering two kinds of coverage criteria: standard coverage criteria defined for UML class diagram and graph-based criteria. Both kinds of criteria are translated to SMT formulas and solved using an external SMT solver. We have already implemented and integrated our techniques into a tool, and results reveal both its capabilities and limitations. In the future, we plan to improve expressiveness of graph properties to allow users to be able to describe more complicated graph shapes. We will also investigate a way of detecting the conflicts between coverage criteria and OCL invariants defined for a metamodel.

REFERENCES

Anastasakis, K., Bordbar, B., Georg, G., and Ray, I. (2007). UML2Alloy: A challenging model transformation. In *ACM/IEEE 10th International Conference on Model Driven Engineering Languages and Systems*, pages 436–450, Nashville, TN. Springer.

Andrews, A., France, R., Ghosh, S., and Craig, G. (2003). Test adequacy criteria for UML design models. *Software Testing, Verification and Reliability*, 13(2):95–127.

Barrett, C., Conway, C. L., Deters, M., Hadarean, L., Jovanović, D., King, T., Reynolds, A., and Tinelli, C. (2011). CVC4. In *The 23rd International Conference on Computer Aided Verification*, pages 171–177. Springer.

Barrett, C., Stump, A., and Tinelli, C. (2010). The SMT-LIB Standard: Version 2.0. In *8th International Workshop on Satisfiability Modulo Theories*, Edinburgh, UK. Elsevier Science.

Binder, R. (1999). *Testing Object Oriented Systems: Models, Patterns and Tools*. Addison-Wesley.

Bordbar, B. and Anastasakis, K. (2005). UML2Alloy: A tool for lightweight modelling of discrete event systems. In *International Conference on Applied Computing*, pages 209–216, Algarve, Portugal. IADIS.

Büttner, F., Egea, M., and Cabot, J. (2012). On verifying ATL transformations using 'off-the-shelf' SMT solvers. In *15th International Conference on Model Driven Engineering Languages and Systems*, pages 432–448.

Cabot, J., Clarisó, R., and Riera, D. (2008). Verification of UML/OCL class diagrams using constraint programming. In *IEEE International Conference on Software Testing Verification and Validation Workshop*, pages 73–80, Berlin, Germany. IEEE Computer Society.

Cabot, J., Clarisó, R., and Riera, D. (2014). On the verification of UML/OCL class diagrams using constraint programming. *Journal of Systems and Software*, 93:1–23.

Chidamber, S. and Kemerer, C. (1994). A metrics suite for object oriented design. *IEEE Transactions Software Engineering*, 20(6):476–493.

Cimatti, A., Griggio, A., Schaafsma, B. J., and Sebastiani, R. (2013). The mathSAT5 SMT solver. In *The 19th International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 93–107, Rome, Italy.

De Moura, L. and Bjørner, N. (2008). Z3: an efficient SMT solver. In *14th International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 337–340, Budapest, Hungary. Springer.

Ehrig, K., Küster, J. M., and Taentzer, G. (2009). Generating instance models from meta models. *Software and Systems Modeling*, 8(4):479–500.

Felbinger, H. and Schwarzl, C. (2014). Suitability analysis of CSP- and SMT-solvers for test case generation. In *The 6th International Workshop on Constraints in Software Testing, Verification, and Analysis*, pages 40–49. ACM.

Ghosh, S., France, R., Braganza, C., and Kawane, N. (2003). Test adequacy assessment for UML design model testing. In *14th International Symposium on Software Reliability Engineering*, pages 332–343.

González Pérez, C. A., Buettner, F., Clarisó, R., and Cabot, J. (2012). EMFtoCSP: A tool for the lightweight verification of EMF models. In *Formal Methods in Software Engineering: Rigorous and Agile Approaches*, Zurich, Suisse.

Gulwani, S. (2010). Dimensions in program synthesis. In *The 12th International ACM SIGPLAN Symposium on Principles and Practice of Declarative Programming*. ACM.

Gutjahr, W. J. (1999). Partition testing vs. random testing: The influence of uncertainty. *IEEE Transactions on Software Engineering*, 25(5):661–674.

Harm, J. and Lämmel, R. (2000). Testing attribute grammars. In *3rd Workshop on Attribute Grammars and their Applications*, pages 79–99, Ponte de Lima, Portugal.

Hoffmann, B. and Minas, M. (2011). Generating instance graphs from class diagrams with adaptive star grammars. In *3rd International Workshop on Graph Computation Models*.

Jackson, D. (2002). Alloy: a lightweight object modelling notation. *ACM Transactions on Software Engineering Methodologies*, 11(2):256–290.

Kuhlmann, M. and Gogolla, M. (2012). Strengthening SAT-based validation of UML/OCL models by representing collections as relations. In *Modelling Foundations and Applications*, volume 7349 of *Lecture Notes in Computer Science*, pages 32–48. Springer.

Kuhlmann, M., Hamann, L., and Gogolla, M. (2011). Extensive validation of OCL models by integrating SAT solving into USE. In *49th International Conference on Objects, Models, Components, Patterns*, pages 290–306, Zurich, Switzerland. Springer.

Macedo, N. and Cunha, A. (2013). Implementing QVT-R bidirectional model transformations using Alloy. In *The 16th International Conference on Fundamental Approaches to Software Engineering*, pages 297–311. Springer, Rome, Italy.

McQuillan, J. A. and Power, J. F. (2008). A metamodel for the measurement of object-oriented systems: An analysis using Alloy. In *1st International Conference on Software Testing Verification and Validation*, pages 288–297, Lillehammer, Norway. IEEE Computer Society.

Myers, G. J. and Sandler, C. (2004). *The Art of Software Testing*. John Wiley & Sons.

Ostrand, T. J. and Balcer, M. J. (1988). The category-partition method for specifying and generating fuctional tests. *Communications of the ACM*, 31(6):676–686.

Purdom, P. (1972). A sentence generator for testing parsers. *BIT Numerical Mathematics*, 12(3):366–375.

Sen, S., Baudry, B., and Mottu, J.-M. (2009). Automatic model generation strategies for model transformation testing. In *2nd International Conference on Theory and Practice of Model Transformations*, pages 148–164. Springer.

Soeken, M., Wille, R., and Drechsler, R. (2011). Encoding OCL data types for SAT-based verification of UML/OCL models. In *5th International Conference on Tests and Proofs*, pages 152–170, Zurich, Switzerland. Springer.

Soeken, M., Wille, R., Kuhlmann, M., Gogolla, M., and Drechsler, R. (2010). Verifying UML/OCL models using boolean satisfiability. In *Design, Automation Test in Europe Conference Exhibition*, pages 1341–1344, Dresden, Germany.

Tillmann, N. and De Halleux, J. (2008). Pex: White box test generation for .NET. In *The 2nd International Conference on Tests and Proofs*, pages 134–153.

Torlak, E. and Jackson, D. (2007). Kodkod: a relational model finder. In *13th International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 632–647, Braga, Portugal. Springer.

Wu, H., Monahan, R., and Power, J. F. (2013). Exploiting attributed type graphs to generate metamodel instances using an SMT solver. In *7th International Symposium on Theoretical Aspects of Software Engineering*, Birmingham, UK.