

Dynamics of Interacting Bragg Grating Solitons in a Semilinear Dual-core System with Cubic-quintic Nonlinearity

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Abstract: The interaction dynamics of in-phase Bragg grating gap solitons in a semilinear dual-core optical waveguide, where one core has cubic-quintic nonlinearity and equipped with Bragg grating and the other is linear, are investigated. The model supports two disjoint families of Bragg grating solitons (referred as Type 1 and Type 2). It is found that the interactions of two stable in-phase ($\Delta\theta = 0$) quiescent solitons result in several outcomes. The possible interaction outcomes between two solitons may include symmetric or asymmetric separation, merger into one quiescent or moving soliton, destruction of one or both solitons and the formation of three solitons. It is found that the outcomes of the interactions are dependent upon the strength of quintic nonlinearity (q), initial separation (Δx) of the solitons, coupling-coefficient (κ) between the cores and the group velocity term (c) in the linear core.

1 INTRODUCTION

Fiber Bragg gratings (FBGs) are generated through the periodic variation of the refractive index of the core in an optical fiber. In recent years, FBGs based devices have played an important role in optical systems due to their applications in high speed switching, pulse compression, high-bit-rate optical communications, filtering, sensing and signal processing (Radic et al., 1995; Christiansen et al., 2000; Loh et al., 1996; Kashyap, 1999).

One of the key characteristics of FBGs is that the cross coupling between the forward- and backward-propagating waves gives rise to strong dispersion (Desterke and Sipe, 1994). At high intensities, the strong FBG-induced dispersion and the nonlinearity may be balanced leading to the formation of Bragg grating (BG) solitons. BG solitons have been investigated extensively in Kerr nonlinear media both theoretically (Christadoulides and Joseph, 1989; Aceves and Wabnitz, 1989; Mak et al., 2003; Neill et al., 2007; Neill and Atai, 2006) and experimentally (Eggleton et al., 1997; Taverner et al., 1998; Mok et al., 2006). More recently, the existence and stability of BG solitons have been investigated in other nonlinear systems such as quadratic nonlinearity (Conti et al., 1997) and cubic-quintic nonlinearity (Atai and Malomed, 2001; Atai, 2004; Dasanayaka and Atai, 2013b; Dasanayaka and Atai, 2013a). They have also

been studied in more sophisticated structures such as dual-core fibers where Bragg grating exists in one or both cores (Mak et al., 1998; Atai and Malomed, 2000).

Couplers with dissimilar cores have received much attention due to their potential applications in switching and signal processing (Bertolotti et al., 1995; Atai and Chen, 1992; Atai and Chen, 1993; Nistazakis et al., 2002). The presence of a Bragg grating in such couplers results in a system that supports BG solitons whose stability properties are governed by other parameters such as the strength of the coupling coefficient and relative group velocity in the linear core (Atai and Malomed, 2000). As a result, such systems exhibit a very rich dynamics.

In this paper, we investigate the interactions of BG solitons in a dual-core system that is composed of a linear core coupled to a nonlinear core with a Bragg grating and cubic-quintic nonlinearity.

2 THE MODEL

Starting with the model put forward in (Atai and Malomed, 2000) and employing the approach described in (Atai and Malomed, 2001), the following dimensionless coupled-mode equations for two linearly coupled cores can be derived, assuming BG is present only in the nonlinear core and the other being

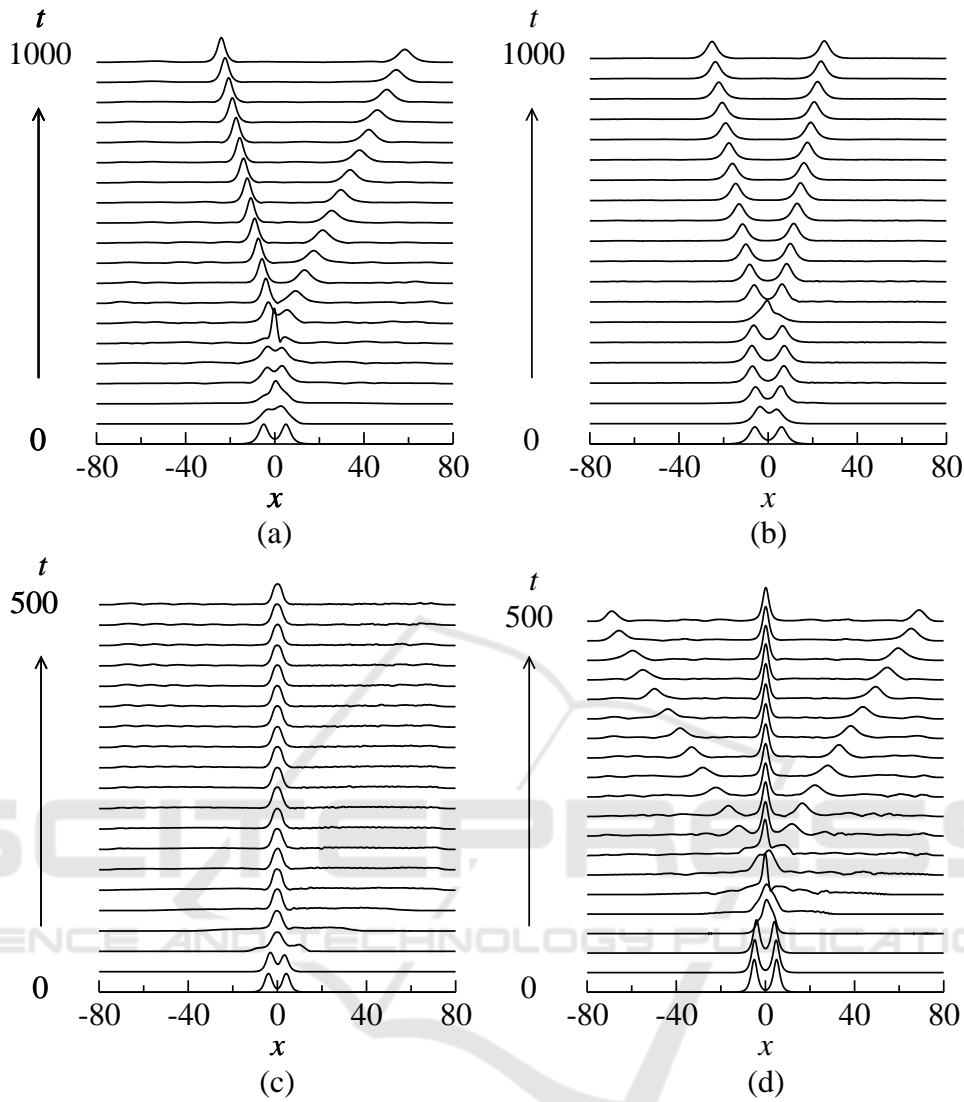


Figure 1: Typical examples of the interactions of quiescent solitons. (a) Asymmetric separation for $\kappa = 0.10$, $c = 0.1$, $\Delta x = 10.0$, $q = 0.35$, $\omega = 0.85$; (b) symmetric separation for $\kappa = 0.50$, $c = 0.20$, $\Delta x = 12.0$, $q = 0.16$, $\omega = 1.10$; (c) merger into a quiescent soliton for $\kappa = 1.0$, $c = 0.0$, $\Delta x = 8.0$, $q = 0.40$, $\omega = 1.35$ and (d) formation of three solitons (a quiescent and two moving solitons) for $\kappa = 0.1$, $c = 0.1$, $\Delta x = 10.0$, $q = 0.26$, $\omega = 0.60$. In all figures only the u component is shown.

linear:

$$\begin{aligned}
 & iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2 \right] u - \\
 & q \left[\frac{1}{4}|u|^4 + \frac{3}{2}|u|^2|v|^2 + \frac{3}{4}|v|^4 \right] u + v + \kappa\phi = 0, \\
 & iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2 \right] v - \\
 & q \left[\frac{1}{4}|v|^4 + \frac{3}{2}|v|^2|u|^2 + \frac{3}{4}|u|^4 \right] v + u + \kappa\psi = 0, \\
 & i\phi_t + ic\phi_x + \kappa u = 0, \\
 & i\psi_t - ic\psi_x + \kappa v = 0,
 \end{aligned} \tag{1}$$

where u and v are the forward- and backward-propagating waves in the nonlinear core and ϕ and ψ are their counterparts in the linear core, respectively. $q > 0$ is a real parameter that controls the strength of the quintic nonlinearity and κ is the coefficient of linear coupling between the cores. Also, c represents the relative group velocity in the linear core (group velocity in the nonlinear core has been set to 1). It is worth noting that the cubic-quintic nonlinearity has been observed in various organic materials and chalcogenide glass (Boudebs et al., 2003; Zhan et al., 2002; Lawrence et al., 1994). Assuming a typical value of $\Delta n = 5 \times 10^{-4}$ and using the values of nonlinear coefficients from these references, it is

found that q can range from 0.05 to 0.6. As a result, in our simulations we have assumed that q varies in the range $0 \leq q \leq 1$.

Substituting $u, v, \phi, \psi \sim \exp(ikx - i\omega t)$ into the system of Eqs. (1) and linearizing, a dispersion equation for $\omega(k)$ can be obtained as (Islam and Atai, 2015):

$$\omega^4 - [1 + 2\kappa^2 + (1 + c^2)k^2] \omega^2 + \kappa^4 + (c^2 - 2c\kappa^2)k^2 + c^2k^4 = 0. \quad (2)$$

Analyzing this equation, for $c = 0$ it is easy to conclude that the spectrum contains two set of disjoint bandgaps; one in upper half and the other in lower half of the spectrum and the limits of the gaps are (Islam and Atai, 2015):

$$\begin{cases} -\frac{1}{2} + \sqrt{\frac{1}{4} + \kappa^2} \leq \omega \leq \frac{1}{2} + \sqrt{\frac{1}{4} + \kappa^2} & \omega > 0, \\ -\frac{1}{2} - \sqrt{\frac{1}{4} + \kappa^2} \leq \omega \leq \frac{1}{2} - \sqrt{\frac{1}{4} + \kappa^2} & \omega < 0. \end{cases} \quad (3)$$

For $c \neq 0$, the shapes of the branches of the dispersion diagram change, and a central gap (which is a genuine gap) is formed. In this case, the lower and upper gaps overlap with one branch of continuous spectrum and therefore they are not genuine bandgaps. It has been found that soliton solutions exist in the upper and lower gaps only (Islam and Atai, 2015).

3 INTERACTION OF SOLITONS

The model of Eqs. 1 is nonintegrable. Thus, the interactions between solitons are more complex. To simulate the interaction of solitons we have utilized a symmetrized split-step Fourier method to numerically solve Eqs. 1 subject to the following initial conditions:

$$\begin{aligned} u(x, 0) &= u\left(x + \frac{\Delta x}{2}, 0\right) + u\left(x - \frac{\Delta x}{2}, 0\right) e^{i\Delta\theta}, \\ v(x, 0) &= v\left(x + \frac{\Delta x}{2}, 0\right) + v\left(x - \frac{\Delta x}{2}, 0\right) e^{i\Delta\theta}, \\ \phi(x, 0) &= \phi\left(x + \frac{\Delta x}{2}, 0\right) + \phi\left(x - \frac{\Delta x}{2}, 0\right) e^{i\Delta\theta}, \\ \psi(x, 0) &= \psi\left(x + \frac{\Delta x}{2}, 0\right) + \psi\left(x - \frac{\Delta x}{2}, 0\right) e^{i\Delta\theta}, \end{aligned} \quad (4)$$

where Δx and $\Delta\theta$ are the initial separation and the phase difference between the two quiescent solitons, respectively and $u(x, 0)$, $v(x, 0)$, $\phi(x, 0)$ and $\psi(x, 0)$ belong to the stable regions. We have previously found that Eqs. 1 support two disjoint families of solitons, namely Type 1 and Type 2, and that only Type 1 solitons are stable (Islam and Atai, 2015). Therefore, we have only considered the interactions of Type 1 solitons.

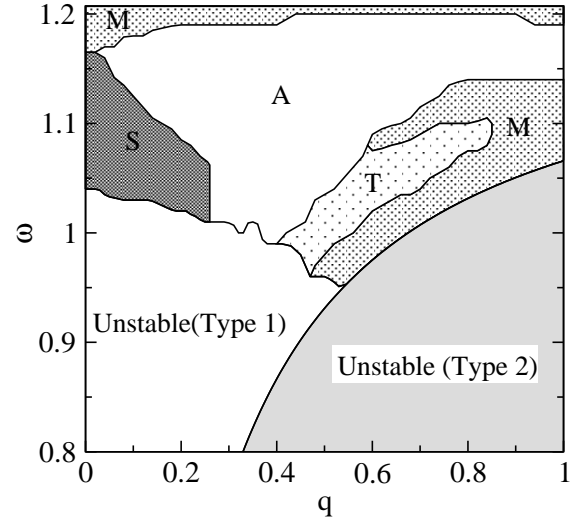


Figure 2: Results of the interactions of in-phase ($\Delta\theta = 0$) quiescent solitons in (q, ω) plane for $\kappa = 0.5$, $c = 0.2$ and $\Delta x = 12.0$. The labeled regions are asymmetric separation (A), symmetric separation (S), merger into a quiescent soliton (M) and the formation of three solitons (T) i.e. a quiescent soliton and two moving solitons.

We have conducted a systematic investigation of the interactions of in-phase quiescent solitons. Typical examples of the interaction outcomes are displayed in Figure (1). The interactions may lead to the destruction of one or both solitons, generation of two asymmetrically (Figure 1(a)) or symmetrically (Figure 1(b)) separating solitons, merger of solitons into a single quiescent soliton (Figure 1(c)) and the formation of three solitons (Figure 1(d)) [one quiescent soliton and two moving solitons with equal velocities].

Figure 2 displays the soliton interaction outcomes in the (q, ω) plane for $\kappa = 0.50$. In the region A where the outcome of the interactions is generation of two asymmetrically separating solitons (e.g. Figure 1(a)), the solitons may undergo multiple collisions and then separate with unequal velocities and magnitudes, followed by subsequent radiation of energy. In case of symmetric separation (i.e. region S), the two solitons temporarily merge into a single one and then splits into two moving solitons with equal velocities. An interesting feature of the interactions is that region S is principally observed in the upper bandgap. On the other hand, the transformation of $2 \rightarrow 3$ solitons occurs in both upper and lower gaps. It is also found that the interaction regions are greatly affected by Δx , c , and κ . The interplay of these parameters and their effect on the outcomes are currently under investigation.

4 CONCLUSIONS

In this paper, we have investigated the interaction dynamics of two in-phase quiescent BG solitons in a semilinear dual core system, where one core has cubic-quintic nonlinearity with BG and the other is linear. By means of systematic numerical simulations, the soliton interactions have been studied for different soliton parameters. It is found that, the interactions between stable Type 1 solitons may produce diverse results such as destruction of one or both the solitons, symmetric or asymmetric separation, merger into one quiescent or moving soliton and even the transformation of $2 \rightarrow 3$ solitons. It is also observed that the interaction outcomes strongly dependent on the strength of quintic nonlinearity (q), coupling coefficient (κ) between the cores and the the group velocity term (c) in the linear core. The interactions may result in the formation of three solitons (a quiescent soliton and two moving ones) in both the upper and lower bandgaps.

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