Anomaly Detection using B-spline Control Points as Feature Space in Annotated Trajectory Data from the Maritime Domain

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Keywords: B-spline Interpolation, Support Vector Machines, Artificial Neural Networks, Multilayer Perceptron, Gaussian Mixture Models, Anomaly Detection, Trajectories, Maritime Domain.

Abstract: The detection of anomalies and outliers is an important task for surveillance applications as it supports operators in their decision making process. One major challenge for the operators is to keep focus and not to be overwhelmed by the amount of information supplied by different sensor systems. Therefore, helping an operator to identify important details in the incoming data stream is one possibility to strengthen their situation awareness. In order to achieve this aim, the operator needs a detection system with high accuracy and low false alarm rates, because only then the system can be trusted. Thus, a fast and reliable detection system based on b-spline representation is introduced. Each trajectory is estimated by its cubic b-spline representation. The normal behavior is then learned by different machine learning algorithm like support vector machines and artificial neural networks, and evaluated by using an annotated real dataset from the maritime domain. The results are compared to other algorithms.

1 INTRODUCTION

As technology progresses, the amount of sensors and their recorded data in surveillance applications increases. Therefore, operators of such systems need to be supported to maintain an overview about all important object movements and their impact. In order to help the operators, anomaly detection systems are introduced into the surveillance systems. The aim is to identify patterns that reveal an unexpected behavior of objects, which significantly differs from normal behavior prior recorded or modeled by domain experts.

For an operator to accept the assistance of such a system, it must be reliable. This means in particular, that the system must be able to classify the behavior with a low false alarm rate and high accuracy. Else, the operator loses his faith in the system and might start to ignore it. Another important aspect is the method of displaying the unusual behavior. Only if the operator is able to clearly identify the reason for the alarm, he will be able to take the correct further steps.

Therefore, an algorithm with high precision and recall is introduced. First, related work regarding the anomaly detection, the algorithm and the application domain (here, the maritime domain) is described. Afterwards, the main idea of the algorithm is explained and an empirical evaluation with annotated real trajectory data is conducted. At the end, a conclusion and insight into future work are given.

2 RELATED WORK

The overview of general anomaly detection algorithms by Chandola et al. (2009) gives a first impression about the versatility of different approaches and domains of outlier and novelty detection algorithms. The field ranges from surveillance application to computer security and fraud detection. Morris and Trivedi (2008) give an overview about anomaly detection specially by using optical sensors. They divide the surveyed works into different applications like traffic surveillance and interactions between multiple objects, and cluster these works by the used techniques and the method of information retrieval from the optical inputs.

The idea of the proposed algorithm is to reduce the complexity of a trajectory by using b-splines. Similar to this approach, Naftel and Khalid (2006) use polynomials and other functions to approximate the trajectories, while assuming normal distribution in the co-

Anneken, M., Fischer, Y. and Beyerer, J.
Anomaly Detection using B-spline Control Points as Feature Space in Annotated Trajectory Data from the Maritime Domain.
DOI: 10.5220/0005655302500257
In *Proceedings of the 8th International Conference on Agents and Artificial Intelligence (ICAART 2016) - Volume 2*, pages 250-257
ISBN: 978-989-758-172-4

efficient feature space for the clusters. Further, they propose a Mahalanobis classifier to detect anomalies in the data. A self-organizing map is used to estimate the similarities between trajectories. The algorithm is validated by using different datasets, i.e. identification of sign language and video surveillance footage.

Melo et al. (2006) propose a feature space using low-degree polynomials for the detection and classification of road lanes. For the clustering of similar trajectories a K-Means algorithm is used. The different lanes are further classified into different categories. The proposed algorithm is tested with real data.

Dahlbom and Niklasson (2007) use splines to represent the main trajectory of a cluster. Therefore, the underlying data is clustered using the mean of the normalized distances between each trajectory point and its nearest cluster point. Afterwards, the clusters are estimated by using splines in order to reduce the complexity of the representation.

Especially in the maritime domain, anomaly detection is an active field of research in itself, e.g., de Vries and van Someren (2012) use *piecewise linear segmentation* methods to compress trajectories of maritime vessels. These compressed trajectories are then clustered and anomaly detection is performed by using kernel methods. Furthermore, expert domain knowledge is incorporated. The algorithms are validated with a dataset from the Netherlands' coast near Rotterdam.

An algorithm that estimates a mean path for normal routes is proposed by Rosen and Medvedev (2012). The mean path is defined as the trajectory which minimizes the euclidean distance to every other trajectory in the same cluster. Anomalies are then detected by comparing a new trajectory with this mean path and an anomaly score is calculated. The algorithm is evaluated by using simulated data as well as a real dataset.

Guillarme and Lerouvreur (2013) propose an unsupervised algorithm for modeling routes by using data from a satellite based Automatic Identification System (AIS). First, the recorded trajectories are partitioned by using a *stops and moves of trajectories* algorithm. The move parts of the trajectories are further divided by using *a piecewise linear segmentation* or *a sliding window* approach. This results in segments of similar movement. These segments are clustered using the OPTICS algorithm. Afterwards, hand-picked clusters are used for modeling the vessels' motion-patterns. First results for this algorithm using real data are illustrated in the paper.

Shao et al. (2014) use a fuzzy k-nearest neighbors and fuzzy c-means approach to conduct trajectory correlation and clustering. Therefore, fuzzy logic is utilized to model uncertainties in the tracks. The proposed algorithms are evaluated using different types of sensing systems.

Fischer et al. (2014) present a method to model specific situations based on dynamic Bayesian networks. The main idea is to utilize expert knowledge to describe situations of interest. For the evaluation a specific situation, namely an incoming suspicious smuggling vessel, is modeled and the results for different parameters are shown. The described situation is translated from a situational dependency network to a dynamic Bayesian network. Therefore, several parameters must be chosen. Fischer et al. present a possible approach to automatically specify these parameters.

3 ALGORITHM

A trajectory recorded by a surveillance system cannot easily be compared to another due to

- different lengths,
- different sample rates, and
- different numbers of points.

In order to compare different trajectories several approaches are possible. E.g., dynamic time warping is used by Vakanski et al. (2012) to learn trajectories demonstrated by human in order to program a robot. Laxhammar and Falkman (2011) use the Hausdorff metric to compare two trajectories. Here, a trajectory will be represented in a way, that its complexity will be reduced and direct comparison between itself and other trajectories will be possible.

A trajectory has a specific length n and consists of multiple points $p_i = (p_{i,lon}, p_{i,lat})^T$ with $i = 1, \ldots, n$, where $p_{i,lon}$ is the longitude and $p_{i,lat}$ is the latitude position of the i-th point. Therefore, a trajectory t is given by $t = \{p_i \mid i = 1, \ldots, n\}$. The idea is now to reduce the number of points, in such a way, that the resulting trajectory can be compared by using e.g. the euclidean distance.

Therefore, the trajectory is estimated by using a b-spline representation. A b-spline interpolation connects several (cubic) functions to interpolate a given set of points. To assess a new trajectory, it has to be compared with recorded normal trajectories. Hence, a normal model of the trajectory in the observed area will be generated by using machine learning algorithms.

3.1 B-spline Interpolation

In order to estimate the b-spline representation of a trajectory, the FORTRAN routine parcur from the FITPACK is used with the Python bindings provided by SciPy (Jones et al., 2001). The estimated b-spline consists of cubic functions with a prior defined amount of sections. Thus, each trajectory has the same amount of control points. Each b-spline is defined by its control points. Therefore, a trajectory with n points is reduced to the number of control points n_c resulting in a feature space of dimension $2 \cdot n_c$.

The resulting control points are used as feature vector to train the machine learning algorithms. Empirical investigation have shown, that for the used dataset 4 control points are enough to represent the trajectories. Therefore, the feature space is set to

$$x = (c_1, c_2, c_3, c_4)^T \tag{1}$$

with the control point

$$c_i = (c_{i,lon}, c_{i,lat})^T, \quad i = 1, \ldots, 4. \tag{2}$$

Figure 1 shows an example for a spline interpolation. The red line and dots are the recorded data. The idea is now to find a spline, representing the line. Therefore, the SciPy algorithms is used, to estimate the control points for the given input data. The results are the green control points and the interpolated blue line. As described by Gallier (1999), a spline is uniquely determined by its control points.

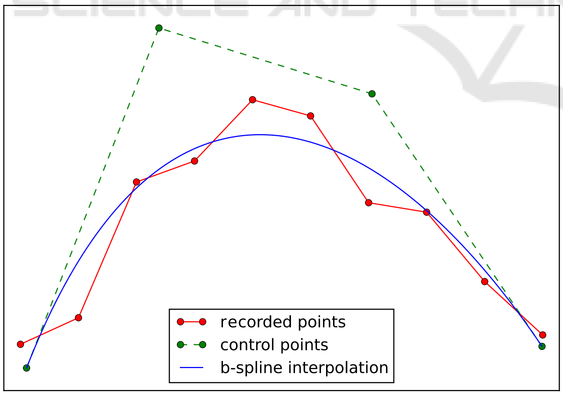


Figure 1: Example spline with control points.

3.2 Machine Learning Methods

The control points as feature vector are used to train different machine learning algorithms:

- Gaussian Mixture Models (GMM),
- Support Vector Machines (SVM), and
- Artificial Neural Networks (ANN) in form of feedforward Multilayer Perceptrons (MLP)

3.2.1 Gaussian Mixture Models

A GMM estimates the underlying probability density function of the data by using an expectation-maximization (EM) algorithm. It consists of the sum of n normal distributions with the mean vector μ_i of the dimension k and the covariance matrix Σ_i for each component given by the parameter set $\theta_i = \{\Sigma_i, \mu_i\}$. The whole probability density function is then given by

$$f(x) = \sum_{i=1}^{n} f_g(x, \theta_i) \tag{3}$$

with the normal distribution for each component given by

$$f_g(x, \theta_i) = \frac{\exp\left(-\frac{1}{2}(x-\mu_i)^T {\Sigma_i}^{-1}(x-\mu_i)\right)}{\sqrt{(2\pi)^k |\Sigma_i|}}. \tag{4}$$

The GMM and the EM-algorithm are implemented in the scikit-learn machine learning framework for Python (Pedregosa et al., 2011).

A GMM is learned unsupervised. Thus, no labels are necessary, but the algorithms will not yield an anomaly as a specific class. Therefore in order to detect an anomaly, a threshold $g_{\min}$ is set. If the evaluated log-likelihood of a new trajectory is below this threshold, the trajectory will be marked as anomaly. Anomalies are defined as occurring only sparsely; therefore, the threshold will be estimated by using a specific percentile of the trained log-likelihood results. Here, this kind of learning algorithm is called *unsup-GMM*.

Additionally as another approach, the GMM is trained by using only the normal trajectories. The threshold is then chosen as the lowest log-likelihood predicted for the training data. This algorithm will be identified as *sup-GMM*. Further information on GMMs and the learning algorithm is, e.g., given by Barber (2014).

3.2.2 Support Vector Machines

A SVM used as a classifier will divide a set of objects into classes. Therefore, the border in the feature space between the classes should be optimized, i.e. the margin between the border and each object in the training set should be as large as possible. In general, the classes need to be linear separable by a hyperplane, while the feature space may be of high dimensionality. To counter this problem, different kernels can be used for the SVM. Here, a radial basis function with the parameter γ is used. Furthermore, a penalty parameter C is available for optimizing the results of the classifier.

The parameter C balances between a simple and smooth decision surface and the misclassification of training examples, i.e., a low C results in a smooth surface a high C in mostly correctly classified examples. The influence of a single data point of the training set is regulated by the parameter γ. A high value for γ means, that training examples need to be close to each other to affect each other.

These parameters are chosen by using a grid search. Further details on SVMs are, e.g., available by Kung (2014). The implementation provided by the framework scikit-learn (Pedregosa et al., 2011) is used.

3.2.3 Multilayer Perceptron

As a non-deterministic machine learning method, a multilayer perceptron with a truncated Newton algorithm for the backpropagation based training provided by the implementation FFNET framework for Python (Wojciechowski, 2011) is used. The neural network consists of 3 layers, one input, one output and one hidden layer. As input, the control points are used. The output is the label normal or anomaly. The layers are connected as seen in Figure 2. The hidden layer consists of n_h units.

As activation function the input layer uses the identity function, whereas all other units use a sigmoid activation function. Due to its non-deterministic nature, the training is performed several times and the best model is used. Further details on ANN are, e.g., available by Shalev-Shwartz and Ben-David (2014).

3.2.4 Estimation of Optimal Parameters

For all algorithms, the optimal parameters are estimated by using a grid search with a 10-fold cross-validation. For the GMM the number of components n and the threshold $g_{\min}$, for the SVM the parameters C and γ, and for the MLP the number of units in the hidden layer n_h are estimated.

4 EMPIRICAL EVALUATION

In order to validate the algorithm, an annotated real dataset from the maritime domain is used. First, the dataset and the test setup are explained. Afterwards, the results for the different algorithms are given.

4.1 Dataset and Test Setup

As dataset for the evaluation, real ship traffic between Lolland and Fehmarn in the Baltic Sea is chosen. The tracks were recorded in a period of 7 days using the AIS system. For the validation, only the data of the tanker and cargo vessels is used, as these types have a similar behavior. All 758 unique tracks (191 tracks by tankers and 567 by cargo vessels) can be seen in Figure 3. The dataset is annotated, i.e. each point in the dataset is either marked as normal or as abnormal. Therefore, the ground truth is given for calculating the precision and recall for all algorithms.

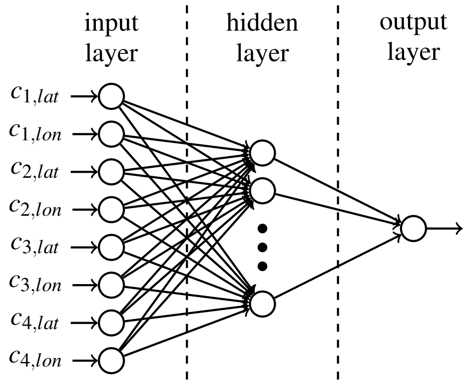


Figure 2: Structure of the feedforward MLP used for the training.

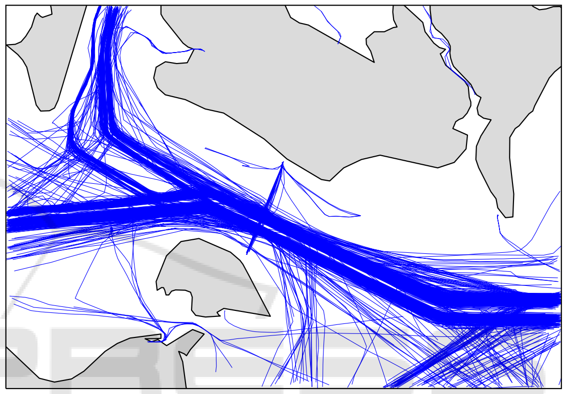


Figure 3: Whole dataset of tanker and cargo vessels between Lolland in the north and Fehmarn and the German mainland in the south. The grey polygons are landmasses, the blue lines represent the vessel traffic.

For the evaluation a 10-fold stratified cross-validation is used. This means, that the whole dataset is divided into 10 folds and each of the folds contains the same ratio of abnormal trajectories as labeled in the whole dataset. As described by Witten and Frank (2005), a 10-fold cross-validation has shown to yield the best estimate of error. Therefore, the results generated by the different folds is averaged. The whole cross-validation is performed 10 times and these results are averaged in order to get a reliable error estimate.

The results are compared to algorithms used by Laxhammar et al. (2009). In their investigation a

Table 1: Optimal parameters for the evaluation.

Algorithm	Parameters
unsup-GMM	$n = 6$; $g_{\min} = 35$th percentile
sup-GMM	$n = 30$
SVM	$C = 10000$; $\gamma = 0.1$
MLP	$n_h = 25$
L-GMM	$n = 75$; grid: 5x5
KDE	$h = 0.06$

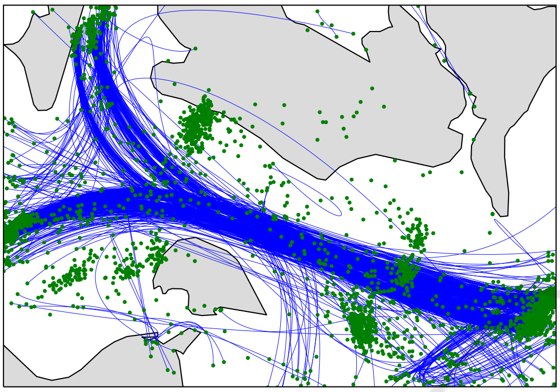


Figure 4: B-spline representations of all trajectories in the dataset. The blue lines represent the splines, the green dots represent the control points.

GMM (here named *L-GMM*) and a Kernel Density Estimation (KDE) are compared for their ability to detect anomalies in sea traffic. These algorithms are used to determine the underlying probability density function representing the distribution of data points. Therefore, these algorithms can assess each incoming data point on its own.

As described by Anneken et al. (2015) in their quantitative evaluation, the algorithms are trained with normal data only, which yields a better result for the overall f1-score as well as for the single precision and recall scores. For the GMM, the area is divided by a grid and for each cell in the grid, the anomaly detection is performed separately. For the KDE, the bandwidth h has to be chosen. A bandwidth chosen too high will result in underfitting and too low in overfitting.

For all algorithms, the optimal parameters are chosen by using a grid search. The resulting optimal parameters are shown in Table 1.

4.2 Results

In Figure 4, the b-spline representations for all trajectories are shown. Each blue line represents one trajectory. For the estimation to work properly, the raw trajectories need to contain at least 4 points. For some transformed trajectories, the course can now be seen to run over islands. This is a result of using only a limited amount of control points. The resulting trajectories are smoothed; therefore, the exact course is lost. The control points for each trajectory are depicted by green dots. The first and last point indicate the beginning and the end of the trajectory. Thus, there are clusters of green dots at the border of the figure. Furthermore, several other clusters of green dots can be seen in the image.

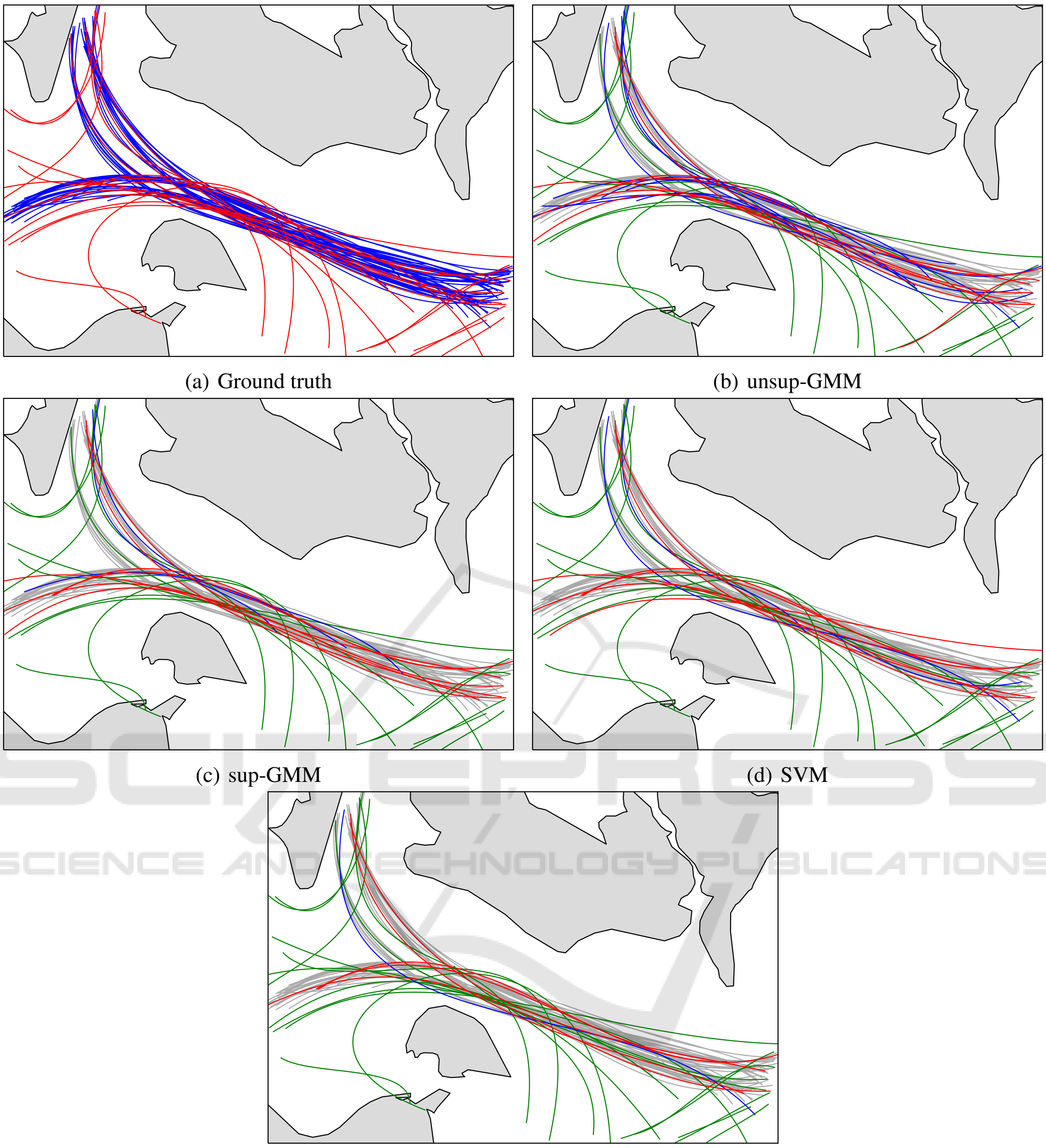
The averaged precision, recall and f1-score for the whole dataset for the b-spline feature approaches as well as the point-based approaches is shown in Table 2. These scores are given regarding the detection of anomalies and not the representation of normal data, i.e. a correctly detected anomaly is a true positive, whereas normal data marked as normal data is a true negative. Furthermore, the ground truth and the results for one test fold using the different learning algorithms for the b-spline representation is depicted in Figure 5. The blue trajectories are detected as normal, whereas the red ones are detected as anomalies.

Each sub-figure in Figure 5 underlines the scores given in Table 2. The SVM-based and MLP-based algorithms yield better results than the unsup-GMM and the sup-GMM. Most anomalies are detected correctly by the SVM and MLP approach, only a few are not found and some are detected as anomaly even though they are not.

The f1-scores for the unsup-GMM is the lowest for the b-spline feature approaches, followed by the sup-GMM (12.4% higher), the SVM (16.9% higher), and the MLP with the highest f1-score (23.8% higher). The precision and recall differ significantly for both GMM approaches. This can also be seen in Figure 5(b) and Figure 5(c). Here, the sup-GMM is able to identify the anomalies which are quite far away from the normal trajectories, but nearly no abnormal trajectory near the normal ones is marked as anomaly. Therefore, nearly all detected anomalies are true positives.

Comparing the b-spline feature methods with the L-GMM and the KDE approach, it is evident, that even the unsup-GMM with the worst results is able to outperform the point-based approaches. The f1-score for the unsup-GMM is 30.5% higher than the one for the L-GMM and 18.7% higher than the one for the KDE. Comparing the MLP approach to the L-GMM, the MLP's f1-score is even 61.6% higher than the one for the L-GMM.

Furthermore, the false positive rate (FP rate) is given for each algorithm in Table 2. The best FP rate is achieved with the b-spline approach using a SVM and a MLP, and the L-GMM. By far the unsup-GMM



(a) Ground truth

(b) unsup-GMM

(c) sup-GMM

(d) SVM

(e) MLP

Figure 5: The results for tanker and cargo vessels for one fold. In Figure 5(a), the blue lines are marked as normal trajectories, whereas the red ones are detected as anomalies. For all other sub-figures, anomalies detected as anomalies are depicted with green lines, anomalies detected as normal with red lines, normal trajectories detected as anomalies with blue lines, and normal trajectories detected as normal with grey lines. The grey polygons are landmasses (in the north Lolland and other Danish islands and in the south Fehmarn and the German mainland), the white background is the Baltic Sea.

has the worst FP rate of the tested algorithms. This is a result of using all the available data (normal and abnormal) as training data. The sup-GMM has a FP rate in between the L-GMM and the KDE.

The results of the unsup-GMM and the sup-GMM are far worse than the ones for the SVM and the MLP. The precision for the unsup-GMM is lower than the one for the sup-GMM, SVM, and MLP, whereas the recall is as high as the one for the SVM. The precision for the sup-GMM is 3.9% lower than the SVM one, but the recall is the lowest of the b-spline feature methods. For the unsup-GMM, the problem is founded in the usage of all available training data during the learning phase. Even abnormal data is used

Table 2: Averaged results of the anomaly detection for tanker and cargo vessels.

Score	B-spline features				Laxhammar et al. (2009)	
	unsup-GMM	sup-GMM	SVM	MLP	L-GMM	KDE
precision	0.6793	0.8923	0.9282	0.9398	0.6547	0.6250
recall	0.7650	0.7401	0.7692	0.8468	0.4763	0.5883
FP rate	0.1643	0.0407	0.0271	0.0248	0.0256	0.0545
f1-score	0.7196	0.8090	0.8412	0.8908	0.5514	0.6061

for learning the underlying distribution, resulting in lower precision and recall. Comparing the unsup-GMM results in Figure 5(b) with the ground truth in Figure 5(a), the false classification of normal and abnormal data can be seen. Several trajectory labeled as normal are classified as anomaly and vice versa.

Comparing Figure 5(d) and Figure 5(e) with each other, the difference is not that significant. There are some occasions, where the MLP classifies the data differently than the SVM, resulting in a better overall classification for the MLP (the f1-score is 5.9% higher for the MLP than for the SVM).

5 CONCLUSIONS

Depending on the machine learning methods, the proposed algorithm has a high precision and recall score. The introduced algorithm, especially based on the MLP, outperforms the algorithms used by Laxhammar et al. (2009) by far.

Nevertheless, the algorithm has some drawbacks. First, due to the smoothing of the trajectories, small anomalies cannot be detected by this approach. Second, the whole trajectory must be known to use this algorithms for anomaly detection. Therefore, it can only be used for post-processing, or in case that the execution time for each trajectory is rather short and therefore the assessment will be available near real-time. Third, the whole trajectory may not be too complicated. This is also a result of the smoothing, as the estimation for long and complicated trajectories is rather bad.

6 FUTURE WORK

To improve the shown drawbacks, several solutions are plausible. In order to make the algorithm usable for real-time analysis, the possible endpoint of a trajectory can be estimated and by using this estimation, a b-spline representation can be predicted.

Furthermore, the trajectories can be segmented, e.g., by dividing the trajectories at turns. These segments can be used to build a Markov Chain or similar to represent the transition between the segments. This will tackle two problems at once. First, smaller anomalies might be detectable, as the trajectories are shorter and therefore the smoothing has less impact. Second, a real-time prediction for vessels is possible, as for each vessel a path can be predicted and thus anomalies can be detected earlier.

ACKNOWLEDGEMENTS

The underlying projects to this article are funded by the WTD 81 of the German Federal Ministry of Defense. The authors are responsible for the content of this article.

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