Sheduling Mobil Medical Units for Home-healthcare Service

Ciro Alberto Amaya and John Alexander Espitia
Department of Industrial Engineering, University of Los Andes, Kr 1E 19A 40, Bogotá, Colombia

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Abstract: This paper addresses a shift-scheduling and localization problem of medical staff for home-healthcare services. In this problem a private company has to decide where to locate its mobile units and its operational times. Locations and schedules define the system capacity to attend patients, capacity produces acceptable service level but operational costs, in contrast, a capacity overestimation requires subcontracting services and consequently a decrease in expected profit. In this talk we present the situation studied and a mathematical approach to deal with. The method was used in a real situation improving net profit and increased the expected served demand.

1 INTRODUCTION

Home-healthcare service allows patients to be treated in the comfort of their own homes as long as their situation does not warrant attending a specialized emergency center. Given the complexity of decisions about organizing the scheduling of shifts and deployment of vehicles for a home care emergency service, on the one hand, and the importance of cost-benefit factors, on the other hand, in many cases empirical solutions or ones based on unverifiable assumptions do not provide a good approach and may lead to high losses for any business.

Health care providers are broadly classified into public and private sector. Due to the competitiveness of the private sector, made up of a large number of home- healthcare providers, such companies face the challenge of increasing their coverage and the quality of their services and simultaneously controlling excessive costs, that is, they need to ensure that they meet an increasing demand and boost their profits. To achieve that, they must find an ideal trade-off between per-patient income and the per-patient cost of the treatments and for the latter, take into account the availability of vehicles, scheduling of shifts and the number of doctors who attend patients.

Given the competitive nature of a private sector, it is necessary to count on a specialized staff which is available to assist its affiliates in their own homes. When a call for assistance is received, caregivers must travel to such homes in a vehicle from a predefined site. If a vehicle within range of the home is not available, the healthcare provider must rely on help from an associated company, which amounts to a failure to meet the demand and results in a loss of profits. When the available vehicles exceed the demand, on the other hand, it amounts to an additional operational cost. Thus, the problem consists of finding the ideal locations and operational intervals for both the medical staff and the vehicles, taking into account:

- Demand distribution changes during the day;
- There is a potential number of sites where vehicles can be located;
- There is a limited number of vehicles;
- Travel times between potential sites and patient locations are influenced by the time of day;
- There is a limit on response time: the maximal time permitted to reach a patient;
- The company has to both meet the demand and increase profits.

The focus of this paper is on locational and scheduling decisions made at the tactical level. More specifically, the aim is to develop and solve a model to locate qualified resources on a network and allocate this resources to specific shifts to meet demand.

This paper is organized as follows. Section 2 provides a summary of the literature on the subject. Section 3 presents two different mathematical formulations. Section 4 compares both mathematical formulations by analyzing some computational experiments. Finally, last section presents some

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conclusions.

2 LITERATURE REVIEW

One of the principal aspects studied here is the location of vehicles over a network, this situation is similar to the ambulance location problem. The aim of ambulance location models is to provide adequate coverage. In that problem a usual goal is to find the best locations, fulfilling a certain level of demand, minimizing the number of ambulances needed. Models are based on the Location Set Covering Problem (LSCP) as proposed by Toregas et al. (1971). The main characteristic of these models are that demand should be met; therefore it is likely that infeasibility will arise when no location node ensures coverage for a demand node. Church & Velle (1974) presented The Maximal Covering Locating Problem (MCLP), in which the objective is to maximize the demand covered by a fixed number of facilities. An important limitation of previous models is that coverage is binary, so a zone or demand point is to be covered in full or is not covered at all.

Previous models guaranties coverage when all ambulances are available, if different services are needed, while a vehicle is busy, other vehicles in other locations can cover that points. In this case, models with multiple coverage aimed obtaining several covers for each point of demand. For example, Church & Gerrard (2003) considered the multi-level location set-covering model (ML-LSCP) as a search for the smallest number of facilities needed to cover each demand a preset number of times. The models proposed by Gendreau, Laporte, & Semet (1997) and Karaman (2008) introduced various service times. Aringhieri et al. (2007) developed the Lower-Priority Calls Coverage Model in which they introduced priority for calls and also took the capacity of facilities into account, as Pirkul & Schilling (1989) did, but without restricting the number of vehicles in each location. It is important to note that they considered the variation in terms of demand throughout the day and solved the problem for several intervals in the day.

Another aspect dealt with in previous studies is the variability of demand and its effect on the location decisions (Drezner & Wesolowsky, 1991). Most use the information to relocate locations and thus improve operations, for example in cases of seasonal variations in the demand (Ndiaye & Alfares, 2008; Farahani et al. 2009). Recent studies have shown that the demand for services can be increased by creating routes for a fleet in a not fixed-resource environment (Halper & Raghavan, 2011). The idea of regarding the demand in terms of points rather than continuous regions has received some criticism (Yao & Murray, 2013; Franco et al. 2008) but more work needs to be done on these kinds of formulations to demonstrate their real benefits, for example, in terms of real applications and computational complexity. Nevertheless, in discussing the variability of demand, our study addresses not only the location but also with the scheduling of resources, in order to dealing with dynamic demand patterns.

Taking all of the above into account for the problem we seek to solve, it becomes evident that the factor of response time is more flexible in a home-healthcare service, because, in contrast with the coverage standard in the ambulance problem, it does not put the patient´s life at risk. This is an important difference between the two services, and it led to the decision to use a deterministic model adding three new elements: 1. An analysis of profitability and its relation to the trade-off between meeting the demand and the resulting costs of doing so. 2. Shift scheduling in order to avoid relocations. 3. The addition of three new variables to the problem: served demand, served demand by coverage, and served demand by capacity. A source of demand is defined as covered if it is located within a specified response distance or response time from a mobile unit, a sum of all sources of demand covered from a mobile unit is call “served demand by coverage”. Not necessary all the demand can be met in a given period of time, because there is a limit number of vehicles, and also because vehicles need to travel and attend patients at home. The amount of demand that vehicles can be met due to restrictions of capacity (time) is call “served demand by capacity”. The amount of demand that a vehicle can met in a period of time depends from both coverage and capacity, this demand is call “served demand”.

3 PROBLEM FORMULATION

Let suppose we have a group of demand points, each has its location and activity during a day. In order to supply the demand points, we will locate a group of vehicles and assign a preconfigured shift, within given locations. A single demand point can be supplied by a vehicle if the demand point is in the maximal time permitted to reach it. Vehicles can work on different shifts and can supply a group of demand points restricted by the time to reach and assist them. The problem can be formulated using next two mathematical formulations:

3.1 Formulation based on Binary Variables

Let us assume the following notation:

Index and sets

i: index the set of demand points **I**
j: index the set of location points **J**
s, k: index the set of shifts **S**
v,w: index the set of vehicles **V**
t: index the set of interval times **T**

Decisional variables

$x_{s,j}^{v}$: 1 if vehicle v is assigned in shift *s* to location *j*;
y_i^t : 1 if node *i* is covered in the time interval *t*;
$z_{i,j}^{t}$: The fraction of demand at point *i* that is covered by the vehicles in location *j* in the interval t.

Auxiliary Decisional variables

d^t: Expected number of clients (demand) served during the time interval *t*;
v^t: Number of vehicles active in the time interval *t*.

Parameters

L: Number of units of time for a time interval;
D_i^t : Expected demand at point *i* during interval *t*;
M_j^t : Limit on the number of vehicles at location *j* during time interval *t*;
R : Desired response time: maximum time permitted to travel to a patient;
$T_{i,j}^{t}$: Expected travel time to demand point *i* from location *j* during time interval *t*;
$C_{i,j}^{t}$: Coverage = 1 if $T_{i,j}^{t} \leq R$;
E : Expected time for a doctor to examine a patient;
u: Revenue for serving one unit of demand;
c : Cost of an activated vehicle;
A_s^t : 1 if shift s is available in time interval *t*;
B_s^t : 1 if shift s is active in time interval *t*:

First and last time intervals in a shift are considered active but unavailable, this time is used by vehicles to leave and return to depots;
$P_{s,k}$: 1 if shift s overlaps shift k;
W_j^t : Weighted average travel time at location *j* during time interval *t* =

$$w_j^t = \frac{\sum_{i\in I} D_i^t * T_{i,j}^t * C_{i,j}^t}{\sum_{i\in I} D_i^t * C_{i,j}^t} \tag{1}$$

The problem can be defined by using a graph $G = (I\ U J, A)$ consisting of a set of demand nodes I, a set of location points J and A as the set of arcs $[(j.i)$: $j \in J,\ i \in I)$. The objective is to obtain the greatest possible profit given the revenue from the expected served demand in each interval: d^t, and the costs generated by active vehicles in each interval: v^t. These costs include not only the operating expenses and maintenance of vehicles but also the salaries of medical staff assigned to them.

The mathematical model is described as:

$$maximize\colon\ u(\textstyle\sum_{t\in T} d^t) - c(\sum_{t\in T} v^t) \tag{2}$$

$s.t.$

$$\sum_{i\in I} D_i^t * C_{i,j}^t * z_{i,j}^t \leq \frac{(\sum_{v\in V}\sum_{s\in S} B_s^t * C_{i,j}^t * x_{s,j}^v) * L}{E + 2 * W_j^t},\quad \forall t \in T, \forall j \in J \tag{3}$$

$$\sum_{j\in J} z_{i,j}^t \leq 1,\ \ \forall\, t \in T, \forall\, i \in I \tag{4}$$

$$d^t \leq \sum_{i\in I} D_i^t * y_i^t,\ \forall\, t \in T \tag{5}$$

$$d^t \leq \sum_{j\in J}\left(\frac{\sum_{v\in V}\sum_{s\in S} B_s^t * x_{s,j}^v * L}{E + 2 * W_j^t}\right),\ \forall\, t \in T \tag{6}$$

$$y_i^t \leq \sum_{j\in J}\sum_{v\in V}\sum_{s\in S} B_s^t * C_{i,j}^t * x_{s,j}^v\ ,\ \forall\, t \in T, \forall\, i \in I \tag{7}$$

$$\sum_{j\in J} x_{s,j}^v + \sum_{j\in J} P_{s,k} x_{k,j}^v \leq 1,\ \ \forall v \in V,\ s \in S,\ k = s+1 \ldots S \tag{8}$$

$$\sum_{j\in J} x_{s,j}^v \leq 1,\ \ \forall v \in V,\ \forall s \in S \tag{9}$$

$$\sum_{s\in S}\sum_{v\in V} A_s^t x_{s,j}^v \leq M_j^t\ ,\ \forall t \in T,\ \forall j \in J \tag{10}$$

$$v^t = \sum_{s\in S}\sum_{j\in J}\sum_{v\in V} B_s^t\, x_{s,j}^v\ ,\ \forall\, t \in T \tag{11}$$

$$x_{s,j}^v \in \{0,1\}\ \forall s \in S,\ j \in J,\ v \in V \tag{12}$$

$$y_i^t \in \{0,1\},\ \ \forall t \in T, \forall i \in I \tag{13}$$

$$z_{i,j}^t \in [0,1],\ \ \forall t \in T, \forall i \in I, \forall j \in J \tag{14}$$

$$d^t \in \mathbb{R}^{\text{nonneg}},\ \forall t \in T \tag{15}$$

$$v^t \in \mathbb{Z}^{\text{nonneg}},\ \ \forall t \in T \tag{16}$$

Constraint (3) ensures that there are enough vehicles to meet the demand that is assigned at each location j in the interval t. Constraint (4) ensures that the maximum percentage of satisfied demand in each demand point i is 100%. Constraints (5) and (6) determine the actual amount of demand that can be served, while constraint (5) refers to coverage limits, constraint (6) to capacity limits. Constraint (7) is used to determine which nodes are covered in the desirable response time. Constraint (8) guarantees that the shifts assigned to the same vehicle do not overlap. Constraint (9) ensures that a vehicle is assigned at most to a single location in an entire shift. Constraint (10) limits the number of vehicles that can be accommodated in one location. Constraint (11) is used to determine the number of vehicles in each interval. Finally, the variables are defined in (12) to (16)

3.2 Formulation based on Integer Variables

The family of constraints (8) and binary characteristics in the previous model made its solution very difficult; as is shown in Table 1, we were able to solve to optimality only a small problem. So a new approach is proposed, with a formulation similar to the above except that the variable $x_{s,j}^{v}$ is modified by x_{sj} and constraints (8) and (9) are removed. The decision now involves the number of vehicles that should be assigned to the shift s at location j (x_{sj}).

The formulation does not match specific vehicles with shifts and also it does not have into account the maximal number of vehicles than can be used (implicitly defined by the cardinality of set **V** in the Binary Model, section 3.1). So, an additional step is needed in order to assign different shifts (that not overlapping) to the same vehicle, this determines the real number of vehicles needed. To do this, an assignment model is proposed.

3.2.1 Grouping Method

In the proposed model, the objective is to minimize the number of groups of vehicles that can be formed. Let G represents the set of groups and W the set of vehicles; note that $|W| = \sum_s \sum_j x_{sj}$. Let us define P_{vw} as the parameter that indicates whether the shift assigned to vehicle $v \in W$ overlaps the shift assigned to vehicle $w \in W$.

The model uses two binary decisional variables: x_{vg}: 1 if the vehicle v is assigned to group g; and y_g to indicate whether the group g is opened or not. It is presented below

$$minimize \sum_{g \in G} y_g \tag{17}$$

$s.t.$

$$\sum_{g \in G} x_{vg} = 1, \;\; \forall v \in W \tag{18}$$

$$\sum_{v \in W} x_{vg} \leq S * y_g, \;\; \forall g \in G \tag{19}$$

$$x_{vg} + P_{vw} * x_{vg} \leq 1, \qquad \forall v \in W, w = v+1 \ldots |G|, \; g \in G \tag{20}$$

$$y_g \in \{0,1\}, \;\; \forall g \in G \tag{21}$$

$$x_{vg} \in \{0,1\}, \; \forall v \in W, \; g \in G \tag{22}$$

Constraint (18) ensures that all vehicles are assigned to a group. Constraint (19) determines the opening of a group, a maximal number of shifts assigned to each vehicle (*S*) can be used. Constraint (20) ensures that the same group does not have two vehicles with overlapping shifts. The variables are defined in (21) and (22).

4 COMPUTATIONAL RESULTS

To test models, instances were generated by randomly changing parameters related to the number of nodes (demand points), locations and the response time. Table 1 presents the computational results. The headings in that table are: N: number of demand nodes, LOC: number of candidate locations, RT: Response time, OF: objective function obtained by running both models, IM refers to the integer model and BM to the binary model; Column DIFF compares the values of objective function for both models - in percentages. TIME represents the maximum time in seconds that the model optimizer ran; for values equal to 3600 the optimizer finished without finding an optimal solution. %Served Demand represents the percentage of served demand returned for both solution approaches.

Due to the number of variables and constraints that are required when modeling the problem directly as a mixed linear integer problem, the Binary Model is only feasible for small problem instances. We use the mixed integer solver XPRESS-MP, running on a computer with an Intel Core i5 processor at 2.53 GHz and 4 GB of RAM memory.

The results of comparing both models are presented in Table 1. For both models the solver was programmed to run for a maximum of one hour. For those problems taking a time less than 3600 in Column TIME, the solver was able to find an optimal solution.

Table 1: Comparison between the integer model (IM) and the binary model (BM).

N	LOC	RT (min)	OF			Time (Sec)		% Served Demand	
			IM	BM	DIFF	IM	BM	IM	BM
36	5	5	7,065,880	7,065,880	0%	0.1	4.1	54%	54%
36	5	15	11,313,916	11,313,916	0%	0.5	3,600	96%	96%
36	5	30	10,119,238	10,119,238	0%	0.5	3,600	92%	91%
36	10	5	13,567,621	13,567,621	0%	0.6	3,600	97%	97%
36	10	15	12,307,853	12,307,853	0%	1.0	3,600	97%	96%
36	10	30	11,973,344	11,973,344	0%	0.6	3,600	97%	97%
100	10	10	10,488,912	10,488,912	0%	4.1	3,600	84%	83%
100	10	20	10,023,735	10,023,735	0%	2.2	3,600	97%	94%
100	10	30	8,690,822	8,689,170	0.02%	9.4	3,600	95%	95%
100	20	10	11,717,514	11,716,026	0.01%	299.4	3,600	93%	91%
100	20	20	10,557,368	10,550,887	0.06%	3,600.0	3,600	97%	94%
100	20	30	9,141,004	9,137,759	0.04%	1.9	3,600	95%	94%
196	10	10	6,361,196	6,361,196	0%	1.2	3,600	58%	57%
196	10	20	9,108,291	9,097,342	0.12%	3,600.0	3,600	95%	95%
196	10	30	7,401,324	7,385,140	0.22%	1.4	3,600	97%	92%
196	20	10	10,768,594	10,768,594	0%	5.3	3,600	90%	89%
196	20	20	10,367,695	10,339,842	0.27%	410.3	3,600	97%	94%
196	20	30	8,491,289	8,491,289	0%	2.2	3,600	96%	95%
612	20	20	9,243,522	9,226,088	0.19%	300.5	3,600	96%	95%
612	20	27.5	7,623,919	7,623,919	0%	55.1	3,600	97%	90%
612	20	35	5,864,936	5,864,936	0%	4.1	3,600	94%	90%
1300	20	27.5	7,350,881	7,332,983	0.24%	300.7	3,600	85%	83%
1300	30	27.5	8,620,143	8,580,416	0.46%	1,482.8	3,600	94%	89%
			Average DIFF		0.071%				

As is shown in the Table 1, the binary model is more time consuming and in all cases except one was not able to find optimal solutions in less than an hour. Insofar as the computational time is concerned, the results show that for the binary model, the number of demand nodes and location has a strong influence, the more demand nodes and locations, the greater the time needed to solve the problem. By contrast, the same cannot be established using the integer model. Finally, the percentage of served demand for the integer model was always higher than, or at least equal to, to the result of the binary model.

In order to determine the characteristic of the solution in relation to the effect of the response time on the served demand by capacity, coverage and finally profits, we resolved ten instances in which the response time (R) varied between 12.5 minutes and 35 minutes and the number of demand nodes and locations remained constant, with values of 612 and 20 respectively. In these examples the time-lapse for an interval (L) was 60 (one hour). Figures 1 and 2 show the impact of the response time.

It is interesting to note that for low response times, the profits are low, and when the response time increases, so do the profits. The fact that profits begin to fall at a certain point implies that there is a response time that maximizes profits. This behavior can be explained as follows: for lower response times, the served demand by coverage is small because there are demand nodes that cannot be reached. By contrast, the served demand by capacity is high because the time that it takes to serve a patient is short and so a higher demand can be served per hour.

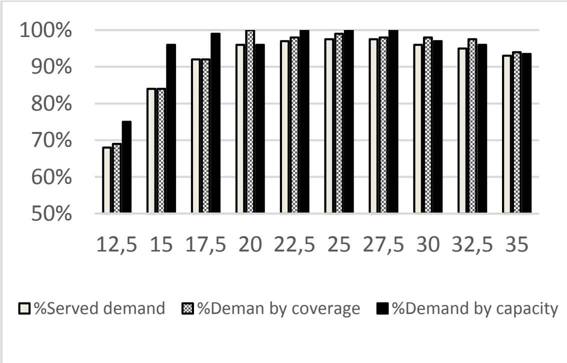

Figure 1: Relation between served demand and profits.

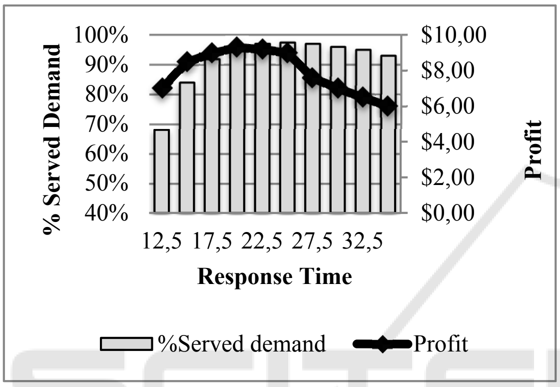

Figure 2: Relation between served demand, served demand by capacity and served demand by coverage.

On the other hand, when we have higher response times, the served demand by coverage is high (we can reach more demand nodes) and the served demand by capacity is low (we spend more time attending a patient). In conclusion, for lower response times, we have low profits because the costs are low but the revenues are low and also because we cannot cover so many patients. For higher response times, we have low profits because the costs related to the number of vehicles are high.

5 CONCLUSIONS

This study provides some initial practical insights into the location and shift problem for home-healthcare services. We employ two mathematical models to solve the problem. The results of the experiment show that our Integer model can provide much better solutions than the Binary model in terms of resolution time. In future works we want to explore solution methods for larger instances using open-source solvers.

In the model we use, profits become an objective and depend on the number of served patients and the associated costs of attending those patients, which is defined by the number of vehicles and shifts used. To describe and calculate the actual served demand, we used two variables: served demand by capacity and served demand by coverage. The former represents the amount of patients who can be served in relation to the number of available vehicles and the time needed to attend to one patient; and the latter represents the number of patients who can be reached in the response time. Similarly, we investigated the influence of the response time on these variables, concentrating on profits and how it is possible to find a value for the response time which yields the best results. For the solution strategy, we propose two mathematical models but more research is needed to deal with large problems.

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