Nonlinear Model for Complex Neurons in Biological Visual Visions

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Abstract: Complex cells in biological visual vision are well known to be nonlinear. In this paper, it is demonstrated that these nonlinear complex cells can be modelled under some certain conditions by a biologically inspired model which is nonlinear in nature. Our model consists of cascaded neural layers accounting for anatomical evidence in biological early visual visions. In the model proposed in this paper, the axons associated with the complex cells are considered to operate nonlinearly. We also consider the second order interaction receptive maps as directional derivatives of the complex cell's kernel along the direction of orientation tuning. Our numerical results are similar to the biologically recorded data reported in the literature.

1 INTRODUCTION

The concept of visual receptive fields is introduced in (Hartline 1938) as a region in visual field in which if visual stimuli are presented, the corresponding cell responds. The sub-regions associated with ON and OFF responses are then discovered in (Kuffler, 1953). Hubel and Wiesel introduce the orientation tuning of neurons in the primary visual cortex (Hubel and Wiesel, 2005). The receptive mapping techniques based on white noise stimuli are then exploited in (DeAngelis et al., 1995; DeAngelis and Anzai, 2004). Motion perception based on energy models is also investigated in (Adelson and Bergen, 1985) by using oriented filters in the space-time domain. In fact, biological experiments quantitatively indicate that the linear visual receptive fields are Gaussian-related kernels. In a mathematical setting, scale-space theory presents a general framework for early visual systems by postulating a set of axioms which an early visual system is expected to possess. Such a framework then leads to Gaussian-related kernels characterizing any linear visual system including early biological visual systems when they behave linearly (see e.g. Weickert et al., 1999; Lindeberg, 2011; Lindeberg, 2013; ter Haar Romeny et al., 2001; ter Haar Romeny, 2003; Koenderink, 1988; Florack, 1997). On the other hand, a model based on the anatomical and physiological properties of biological visual systems is proposed in (Mahmoodi, 2015) to derive Gaussian-related kernels in spatial as well as spatio-temporal domains. The model presented in (Mahmoodi, 2015) is not linear in nature. Therefore the conditions under which this system become linear is discussed in (Mahmoodi, 2015). Under such conditions, linear Gaussian related filters are derived (Mahmoodi, 2015). In such a model, the functionalities of Lateral Geniculate Nucleus (LGN) cells and simple cells such as linear isotropic separable, non-isotropic separable and non-separable (velocity-adapted) cells, with Gaussian related receptive fields can be explained (Mahmoodi, 2015). In this paper, the nonlinearity of this model is also considered and it is demonstrated that under certain conditions, the behaviour of nonlinear complex cells may be attributed to this non linearity of the model. Here our contribution is to explain the nonlinear nature of complex cells by using the nonlinear model of early visual system proposed in (Mahmoodi, 2015). We also demonstrate that the second order interactions of receptive maps for complex cells may be explained as directional derivatives of the neuron's kernel along the direction of orientation tuning. The structure of the rest of the paper is as follows. In section 2, our nonlinear model is explained. Section 3 presents the numerical analyses and results and finally conclusions are drawn in section 4.

2 MODEL

The nonlinear model proposed in (Mahmoodi, 2015)

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behaves linearly under some certain conditions. The linear mode of this nonlinear model is therefore fully investigated in (Mahmoodi, 2015). Linear Gaussian-related kernels associated with linear Lateral Geniculate Nucleus (LGN) cells and simple cells in strait cortex are derived for when the model behaves linearly. Complex cells on the other hand are nonlinear. Here we exploit the nonlinearity of the model presented in (Mahmoodi, 2015) to explain the nonlinear behaviour of the complex cells. According to the model presented in (Mahmoodi, 2015), neurons in early biological visual system are connected in layers which are cascaded from retina to striate cortex.

A layer of neurons is shown in Figure (1-*top*). These neurons are cascaded in a way that the axons of the neurons in the previous layer are connected to the dendrites of the neurons in the current layer. A 2D illustration of these cascaded layers are depicted in figure (1-*bottom*).

Each horizontal line in this figure, represents a neural layer and vertical layers represent axons to connect a layer (a horizontal line) to the next one (another horizontal line above the previous line). It is believed that most complex cells have a linear-nonlinear (L-N) structure. The linear part of these cells is simply a linear Gaussian-related kernel (DeAngelis and Anzai, 2004). The first step is therefore to find a formula to explain the input-output relationship of the nonlinear part of these complex cells. Axons transmitting neural spikes from a neuron to another one are modelled as transmission lines. If neuron A sends n spikes through its axon to neuron B. The potential received in the dendrites of neuron B is calculated as (see Mahmoodi, 2015 for more details):

$$w_o(z,t) = v_h(z,t)\sum_{n=1}^{N}\exp(aT_n) + \frac{v_h(z,t)}{2t}\sum_{n=1}^{N}T_n\exp(aT_n) \qquad (1)$$

where

T_n = the time the nth spike is released from neuron A

t = the time that a spike reaches to neuron B from neuron A,

N = the total numbers of spikes,

z = the length of axon,

$$v_h(z,t) = \frac{1}{2}\sqrt{\frac{1}{\pi t R_z C_z}}\exp\left(-\frac{G_z t}{C_z}\right)\exp\left(-\frac{R_z C_z z^2}{4t}\right)$$

G_z, C_z, and R_z = conductance, capacitance, and resistance of the axon (transmission line) per unit length,

and finally $a = \frac{G_z}{C_z} - \frac{R_z C_z z^2}{4t^2}$.

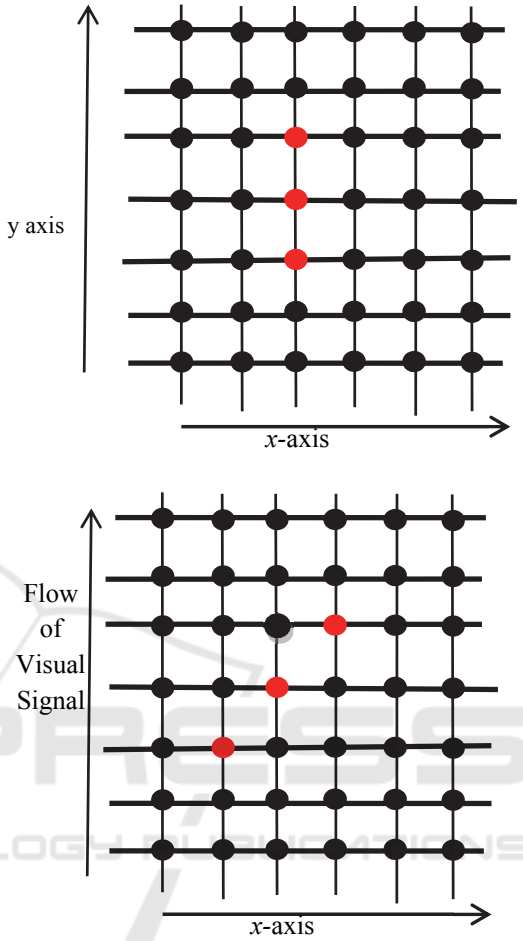

Figure 1: (*top*) the configuration of neurons in a layer with respect to spatial coordinates x and y (*bottom*) the 2D representation of the configuration of neurons in the cascaded layers of neurons.

T_n can be written as the summation of all time intervals between consecutive spikes, i.e.:

$$T_n = \sum_{m=1}^{n}\Delta T_m = n\overline{T} \qquad (2)$$

where $\overline{T}$ is the average time interval between consecutive spikes. Let us consider the case where a series of spikes are transmitted through axon from neuron A to neuron B in a time period much less than the time required for spikes to travel from neuron A to neuron B, i.e. $\max_n(T_n) << t$. By replacing (2) in (1) and assuming that $\max_n(T_n) << t$ and therefore ignoring the second term in (1), one can write:

$$w_o(z,t) = v_h(z,t)\sum_{n=1}^{N}\exp(an\overline{T}) \quad (3)$$

Equation (3) can be rewritten as:

$$w_o(z,t) = v_h(z,t)\exp(a\overline{T})\frac{e^N - 1}{e-1} \quad (4)$$

It is also reasonable to assume the average time interval between consecutive spikes is too small, i.e. $a\overline{T} << 1$. Equation (4) therefore is approximated as:

$$w_o(z,t) \approx v_h(z,t)\frac{e^N - 1}{e-1} \quad (5)$$

According to classical rectification model for neural firing rate (Carandini and Fester, 2000), the input potential of neuron A is proportional to firing rate and therefore *N*. Equation (5) determines the input-output relationship of a cell behaving nonlinearly. For small values of potentials, *N* is very small and therefore $w_o(z,t) \approx 0$

According to classical rectification model (Carandini and Fester, 2000), *N* increases linearly with respect to input potential V_{in}, i.e. $N = kV_{in}$ Therefore for large positive potentials, the input-output relationship of such a nonlinear cell will look like a half rectified power function as reported in (DeAngelis and Anzai, 2004) and shown in figure (2-*bottom*).

3 NUMERICAL RESULTS

Figure (2-*top*) shows the input-output relationship for a nonlinear cell according to equation (5) and as an example for *k*=5, i.e. for $N = 5V_{in}$. The similarity between figure (2-*top*) and figure (2-*bottom*) as reported in (DeAngelis and Anzai, 2004) is interesting and important.

The linear part of a complex cell is simply the sum of three cells with linear isotopic kernels such as the one shown in figure (1-*bottom*).The outputs of these three cells are summed by another cell whose axon behave nonlinearly governed by equation (5), i.e.:

$$\kappa(x,y,t) = \left\{h_{n-1}(x-\Delta,y)\Pi\left(\frac{2(t-\tau)}{\tau}+\frac{\tau}{2}\right)+ h_n(x,y)\Pi\left(\frac{2t}{\tau}+\frac{\tau}{2}\right)+h_{n+1}(x+\Delta,y)\Pi\left(\frac{2(t+\tau)}{\tau}+\frac{\tau}{2}\right)\right\}v(z,t) \quad (6)$$

where $h_n(x,y)$ is the isotropic kernel for layer *n* and $\Pi(t)$ is the impulse response of the cells' axons behaving linearly (Mahmoodi, 2015). The outputs of these three cells are summed by another cell according to equation (6).

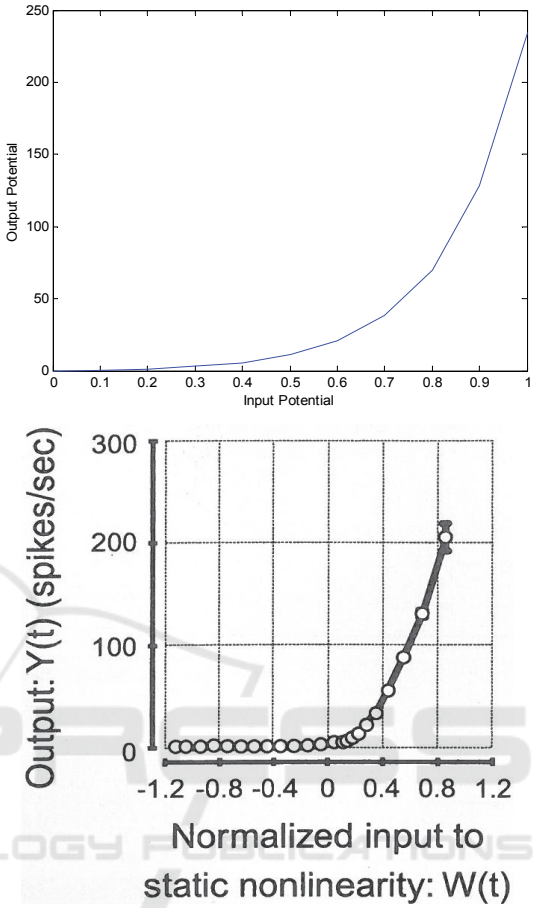

Figure 2: (*top*) The input-output relationship calculated based on equation (5) and for *k*=5, (*bottom*) Biologically recorded input-output relationship according to (DeAngelis and Anzai 2004).

The impulse response of the axon behaving nonlinearly for this cell is represented by $v(z,t)$. The spatio-temporal profile of a complex cell consisting of three simple cells with linear isotropic kernels for a single stimuli (nonlinear case) is shown in figure (3-*top*). This is similar to the biological recorded data shown in figure (3-*bottom*) in (DeAngelis and Anzai, 2004).

The response of this nonlinear complex cell for the case where two simultaneous and spatially separated stimuli (second order interactions) are presented in the visual field, is shown in figure (4-*top*).On the other hand, there are some other complex cells described by equation (7):

$$\kappa(x,y,t) = \left((h_n)_x(x,y-\Delta)+(h_n)_x(x,y)+(h_n)_x(x,y+\Delta)\right)v(z,t) \quad (7)$$

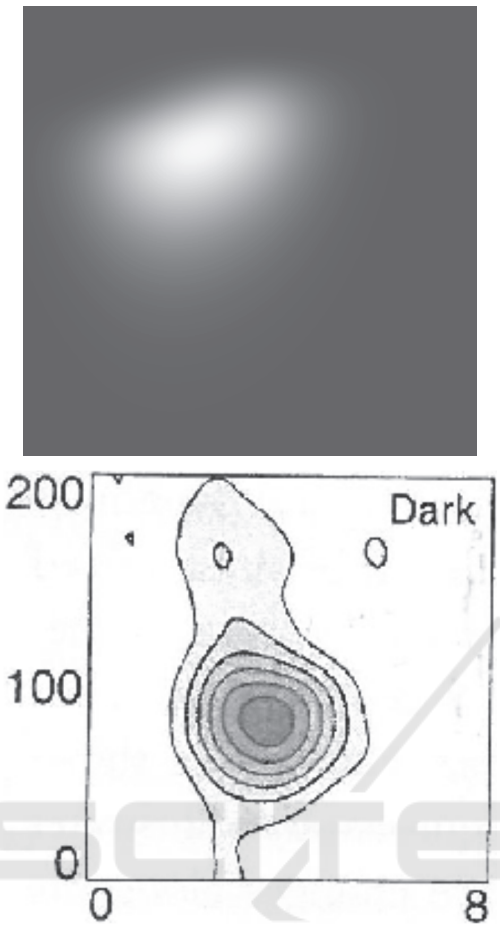

Figure 3: (*top*) The spatio-temproal profile of a nonlinear complex cell for a single stimulus. (*bottom*) The biologically recorded spatio-temporal profile according to (DeAngelis and Anzai, 2004).

The neural configuration of equation (7) is depicted in figure (1-*top*). In this configuration, the nonlinear-behaving axon of the cell summing the outputs of the three cells is represented by $v(z,t)$ in equation (7). In this paper, we hypothesize that the receptive map in the case of the second order interactions is equivalent to directional derivatives along the direction of the orientation tuning of the kernel $\kappa(x,y,t)$ in equation (6) or (7). For a complex cell whose kernel is represented by equation (7), this second order interactions is described by:

$$\frac{\partial^2 \kappa}{\partial n^2} = \frac{\partial^2 \kappa}{\partial x^2}\cos^2(\vartheta) + \frac{\partial^2 \kappa}{\partial x \partial y}\sin(2\vartheta) + \frac{\partial^2 \kappa}{\partial y^2}\sin^2(\vartheta) \tag{8}$$

where $\vec{n} = \vec{i}\cos(\vartheta) + \vec{j}\sin(\vartheta)$ is along the direction of orientation tuning of the cell. The directional derivative of equation (8) for $\vartheta = 45^o$ is calculated in figure (4-*top*).

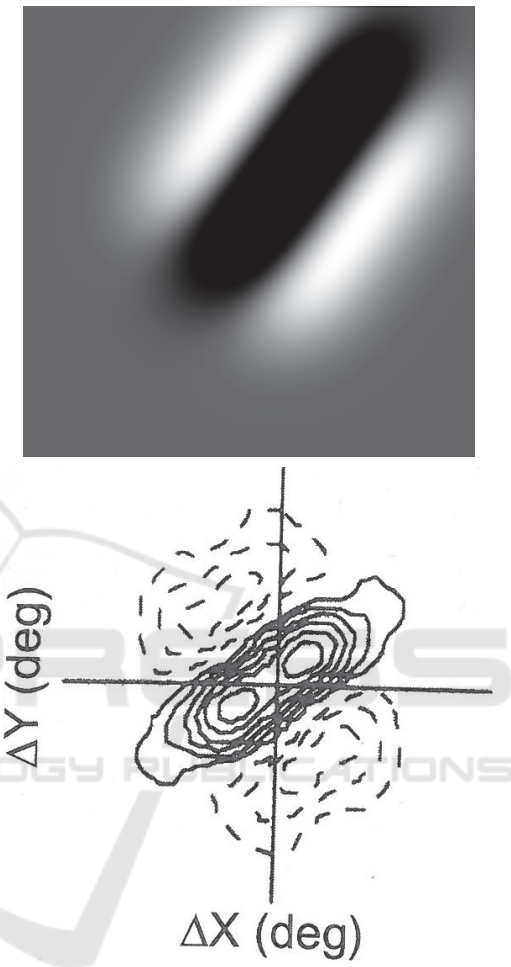

Figure 4: (*top*) A nonlinear receptive field map calculated by using equations (7) and (8) (*bottom*) the biologically recorded second order interactions of a receptive map for a complex cell as reported in (DeAngelis and Anzai, 2004).

The similarity between the map calculated in figure (4) and the biologically recorded data in figure (4-*bottom*) as reported in (DeAngelis and Anzai, 2004) is interesting.

A fourth order directional derivative in space *x* and time *t* for a complex cell represented by equation (6) is calculated in figure (5-*top*). The biologically recorded result shown as the second order interaction of a spatio-temporal profile of a complex cell shown in figure (5-*bottom*) as reported in (DeAngelis and Anzai, 2004) is similar to our result depicted in figure (5-*top*).

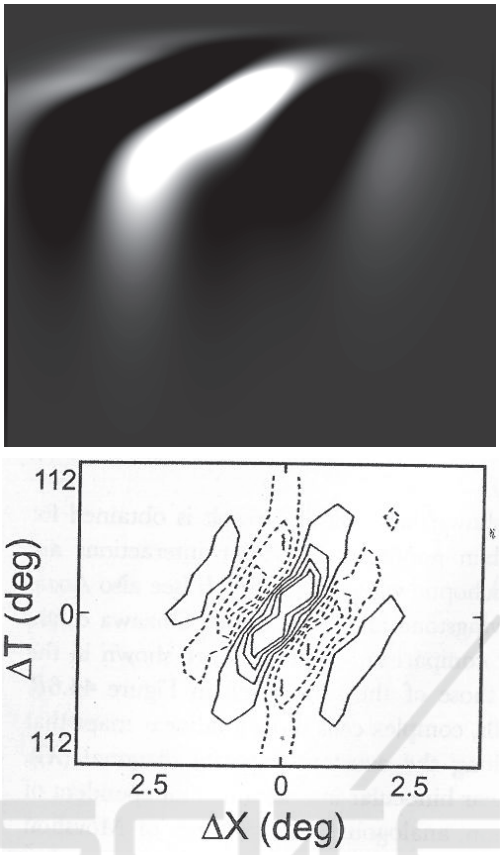

Figure 5: (*top*) Fourth order directional derivative of a spatio-temporal (*x-t*) receptive map, (*bottom*) second order interactions for *x-t* receptive map of a complex cell as reported in (DeAngelis and Anzai, 2004).

As can be seen from figure (5-*top*), there are five lobes (three positive and two negative lobes) among which the middle lobe is the strongest. This is similar to the biologically recorded data shown in figure (5-*bottom*) as reported in (DeAngelis and Anzai, 2004). This second order interaction map is reminiscent of the space-time inseparable linear maps predicting the direction selectivity of complex cells. It is noted that the fourth order directional derivatives should be calculated along the tilt of space-time response pattern.

4 CONCLUSIONS

A nonlinear model based on the model presented in (Mahmoodi, 2015) to explain the nonlinear behaviour of complex cells is proposed here. According to our model, the nonlinearity of the complex cells may be routed from the fact that the axons of neurons behave nonlinearly under certain conditions. These conditions are explained here. In this paper, some approximations to this nonlinear model of neurons are made to demonstrate that the complex cells behave like half rectified power functions corresponding to biologically recorded data. It is then shown here that the space-time calculated receptive map in our model for the complex cells when a single stimulus is presented to the neuron, is similar to the biological receptive field of the complex cells. We then hypothesize that the second order interactions of complex cells recorded in biology may be equivalent to the directional derivatives of the visual receptive map of the complex cells. Our results demonstrate that the directional derivatives of the space or space-time visual receptive maps of complex cells show similar response patterns to the biologically recorded second order interactions confirming our hypothesis.

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