On the Decomposition of Min-based Possibilistic Influence Diagrams

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Abstract: Min-based possibilistic influence diagrams allow a compact modelling of decision problems under uncertainty. Uncertainty and preferential relations are expressed on the same structure by using ordinal data. Like probabilistic influence diagrams, min-based possibilistic influence diagrams contain three types of nodes: chance, decision and utility nodes. Uncertainty is described by means of possibility distributions on chance nodes and preferences are expressed as satisfaction degrees on utility nodes.
In many applications, it may be natural to represent expert knowledge and preferences separately and treat all nodes similarly. This paper shows how an influence diagram can be equivalently represented by two possibilistic networks: the first one represents knowledge of an agent and the second one represents agent's preferences. Thus, the decision evaluation process is based on more compact possibilistic network.

1 INTRODUCTION

Decision making under uncertainty (Whalen, 1984), (Denardo et al., 2012), (Anzilli, 2013), (Dubois et al., 2013) plays an important role in Artificial Intelligence (AI) applications. Several decision making tools have been developed to assist decision makers in their tasks: simulation techniques, dynamic programming (Sniedovich, 2010), logical decision models (Dubois et al., 1998) and graphical decision models (Zhang, 2013), (Garcia and Sabbadin, 2006). Using graphical models, authors in (Boutouhami and Khellaf, 2015) have proposed an approximate approach for computing optimal qualitative possibilistic optimistic decision in the context of optimistic criteria. They showed that it comes down to computing a normalization degree of the moral graph associated to the resulting graph obtained by merging preferences and knowledge represented by two min-based possibilistic networks.

This paper also focuses on graphical decision models which provide efficient decision tools by allowing a compact representation of decision problems under uncertainty (Shenoy, 1994). Most of decision graphical models are based on Influence Diagrams (ID) (Howard and Matheson, 1984), (Zhang, 1998) for representing decision maker's beliefs and preferences on sequences of decisions to be made under uncertainty. The evaluation of Influence Diagrams ensures optimal decisions while maximizing the decision maker's expected utilities (Tatman and Shachter, 1990), (Zhang, 1998), (Dubois and Prade, 1988). Min-based (or qualitative) possibilistic Influence Diagrams "PID" (Garcia and Sabbadin, 2006) allow a gradual expression of both agent's preferences and knowledge. The graphical part of possibilistic Influence Diagrams is exactly the same as the one of standard Influence Diagrams. Uncertainty is expressed by possibility degrees and preferences are considered as satisfaction degrees.

Unlike probabilistic decision theory which is based on one expected utility criteria to evaluate optimal decisions, a qualitative possibilistic decision theory (Dubois et al., 1999), (Dubois et al., 2001) offers several qualitative utility criteria for decision approaches under uncertainty. Among these criteria, one can mention the pessimistic and optimistic utilities proposed in (Dubois and Prade, 1995), the binary utility proposed in (Giang and Shenoy, 2005), etc. As standard Influence Diagrams, direct (Garcia and Sabbadin, 2006) and an indirect methods (Garcia and Sabbadin, 2006), (Guezguez et al., 2009) have been proposed to evaluate a min-based PID. Besides, Influence Diagrams represent agent's beliefs and preferences on the same structure and they operate on three types of nodes: chance, decision and utility nodes. In practice, it will be easier for an agent to express its knowledge and preferences separately. Further-

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more, it is more simple to treat all nodes in the same way. In (Benferhat et al., 2013), authors have proposed a new compact graphical model for representing decision making under uncertainty based on the use of possibilistic networks. Agent's knowledge and preferences are expressed in qualitative way by two distinct qualitative possibilistic networks. This new representation, for decision making under uncertainty based on min-based possibilistic networks, benefits from the simplicity of possibilistic networks.

In this paper, we show how to decompose an initial min-based Influence Diagram into two min-based possibilistic networks: the first one represents agent's beliefs and the second one encodes its preferences. Then, we define the required steps for splitting a qualitative Influence Diagram into two min-based possibilistic networks preserving the same possibility distribution and the same qualitative utility. This procedure allows us to obtain a more compact (in terms of dependence) qualitative possibilistic network for computing optimal decisions. This decomposition process provides also the opportunity to exploit the inference algorithms (Ajroud et al., 2012), (Amor et al., 2003) developed for min-based possibilistic networks to solve qualitative Influence Diagrams.

The rest of this paper is organized as follows: next section briefly recalls basic concepts of possibility theory, min-based possibilistic networks and min-based PID. Section 3 describes how the decomposition process can be efficiently used for encoding an Influence Diagram into two possibilistic networks. Section 4 gives related works. Section 5 concludes the paper.

2 BACKGROUND

2.1 Basic Concepts of Possibility Theory

This section gives a brief refresher on possibility theory (Dubois and Prade, 1988) which is issued from fuzzy sets theory (Zadeh, 1978).

Let $\mathcal{X} = \{X_1, ..., X_N\}$ be a set of variables. We denote by $\mathbb{D}_{X_i} = \{x_{i1}, ..., x_{in}\}$ the domain associated with the variable X_i. x_{ij} denotes the jth instance of X_i. The universe of discourse is denoted by $\Omega = \times_{X_i \in \mathcal{V}} \mathbb{D}_{X_i}$, which is the Cartesian product of all variables domain in $\mathcal{X}$. Each element $\omega \in \Omega$ is called an interpretation which represents a possible state of Ω. It is denoted by $\omega = (x_{1i}, ..., x_{Nj})$. ϕ, ψ... represent events, namely subsets of Ω.

A basic element in a possibility theory is the notion of possibility distribution π which corresponds to a mapping from Ω to the scale $[0,1]$. This distribution encodes available knowledge on real world. $\pi(\omega) = 1$ means that ω is completely possible and $\pi(\omega) = 0$ means that it is impossible for ω to represent the real world. A possibilistic scale can be interpreted in ordinal or numerical way. A possibility distribution π is said to be normalized, if $\max_{\omega} \pi(\omega) = 1$.

Given a possibility distribution π on the universe discourse Ω, two dual measures are defined for each event $\phi \subseteq \Omega$: *Possibility measure* $\Pi(\phi)$ and *Necessity measure* $N(\phi)$. The first one evaluates to what extent ϕ is consistent with our knowledge encoded by π, namely $\Pi(\phi) = \max_{\omega \in \Omega} \{\pi(\omega) : \omega \models \phi\}$. The second one, evaluates at which level ϕ is certainly implied by our knowledge represented by π, namely $N(\phi) = 1 - \Pi(\neg\phi)$.

The possibilistic conditioning consists in the revision of our initial knowledge, encoded by a possibility distribution π by the arrival of a new certain information $\phi \subseteq \Omega$. The initial distribution π is then replaced by another one, denoted $\pi' = \pi(. \mid \phi)$. The two interpretations relative to the possibilistic scale (qualitative and quantitative) induce two definitions of possibilistic conditioning (Amor et al., 2002), (Bouchon-Meunier et al., 2002): product-based conditioning and min-based conditioning. In this paper, we use the last one defined by:

$$\pi(\omega \mid_{min} \phi) = \begin{cases} 1 & \text{If } \pi(\omega) = \Pi(\phi) \text{ and } \omega \models \phi \\ \pi(\omega) & \text{If } \pi(\omega) < \Pi(\phi) \text{ and } \omega \models \phi \\ 0 & \text{oterhwise} \end{cases} \quad (1)$$

Similarly, possibility theory offers several definitions of independence relation (Amor et al., 2002), (de Campos and Huete, 1999). As we interpret the uncertainty scale in ordinal manner, we will used the so-called min-based independence relation, initially defined as a non-interactivity relation by Zadeh (Zadeh, 1978). This relation is obtained by using the min-based conditioning (Equation 1) and it is defined by:

$$\forall x, y, z \; \Pi(x \wedge y \mid z) = \min(\Pi(x \mid z), \Pi(y \mid z)). \quad (2)$$

2.2 Min-based Possibilistic Networks

In a possibility theory framework, there are two ways to define possibilistic networks according to the possibilistic conditioning. In this paper, we only focus on min-based possibilistic networks. A min-based possibilistic network (Borgelt et al., 1998) over a set of variables $\mathcal{X}$ denoted by $\Pi G_{min} = (G, \pi)$ is characterized by:

1. **A Graphical Component:** which is represented by a Directed Acyclic Graph (DAG) where nodes correspond to variables and arcs represent dependence relations between variables.

2. **Numerical Components:** these components quantify different links in the DAG by using local possibility distributions for each node X in the context of its parents denoted by $Par(X)$. More precisely:
   - For every root node X $(Par(X) = \emptyset)$, uncertainty is represented by a priori possibility degree $\pi(x)$ for each instance $x \in \mathbb{D}_X$, such that $\max_{x\in\Omega} \pi(x) = 1$.
   - For the rest of the nodes $(Par(X) \neq \emptyset)$, uncertainty is represented by the conditional possibility degree $\pi(x \mid u_X)$ for each instance $x \in \mathbb{D}_X$ and for any instance $u_X \in \mathbb{D}_{Par(X)}$ (where $\mathbb{D}_{Par(X)}$ represents the Cartesian product of all variables domain in $Par(X)$), such that $\max_{x\in\Omega} \pi(x \mid u_X) = 1$, for any u_X.

The set of a priori and conditional possibility degrees induces a unique joint possibility distribution π_G defined by:

$$\pi_G(X_1,...,X_N) = \min_{i=1..N} \pi(X_i \mid U_i). \tag{3}$$

The most common task performed on possibilistic networks is the *possibilistic inference* which consists in determining how the realization of specific values of some variables called (observations or evidence) affects the remaining variables (Huang and Darwiche, 1996).

2.3 Min-based Possibilistic Influence Diagrams

Like standard Influence Diagrams (Lauritzen and Nilsson, 2001)(Zhang, 2013), PID_s have two components: the graphical part which is exactly the same as the one of standard Influence Diagrams and the numerical part which consists in evaluating different links in the graph. The uncertainty is expressed by possibility degrees and preferences are considered as satisfaction degrees.

In a min-based possibilistic Influence Diagram (qualitative possibilistic ID), denoted by $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$, both agent's knowledge and preferences are expressed in a qualitative setting. This is achieved by ordering the different states of the world and providing a preference relation between different consequences.

1. **A Graphical Component:** which is represented by a DAG, denoted by $G_{ID} = (\mathcal{X}, \mathcal{A})$ where $\mathcal{X} = \mathcal{C} \cup \mathcal{D} \cup \mathcal{U}$ represents a set of variables containing three different kinds of nodes: chance, decision and utility nodes. $\mathcal{A}$ is a set of arcs representing either causal influences or information influences between variables.
   - **Chance Nodes:** are represented by circles. They represent state variables $X_i \in \mathcal{C} = \{X_1,...,X_n\}$. Chance nodes reflect uncertain factors of a decision problem. A combination $x = \{x_{1i},...,x_{nj}\}$ of state variable values represents a state.
   - **Decision Nodes:** are represented by rectangles. They represent decision variables $D_j \in \mathcal{D} = \{D_1,...,D_p\}$ which depict decision options. A combination $d = \{d_{1i},...,d_{pj}\}$ of values represents a decision.
   - **Utility Nodes:** or value nodes $V_k \in \mathcal{V} = \{V_1,...,V_q\}$ are represented by diamonds. They represent local utility functions (local satisfaction degrees) $\mu_k \in \{\mu_1,...,\mu_q\}$.

   A conventional assumption that an Influence Diagram must respect is that utility nodes have no children.
2. **Numerical Components:** After specifying the structure of an Influence Diagram, uncertainty is described by means of a priori and conditional possibility distributions relative to chance nodes. Possibility distributions are defined on the scale $L = [0,1]$ and they are assumed to be normalized. In addition, decision maker should quantify value nodes, on utility scale U, to express their utilities (which may not be normalized). These components quantify different links in the DAG as follows:
   - For every chance node $X \in \mathcal{C}$, uncertainty is represented by:
     - If X is a root node, then a priori possibility degree $\pi_{ID}(x)$ will be associated for each instance $x \in \mathbb{D}_X$, such that $\max_{x\in\mathbb{D}_X} \pi_{ID}(x) = 1$.
     - If X has parents, the conditional possibility degree $\pi_{ID}(x \mid u_X)$ will be associated for each instance $x \in \mathbb{D}_X$ and $u_X \in \mathbb{D}_{Par(X)} = \times_{X_j \in Par(X)} \mathbb{D}_{X_j}$, such that $\max_{x\in\mathbb{D}_X} \pi_{ID}(x \mid u_x) = 1$, for any u_X.
   - Decision nodes are not quantified. Indeed, a value of decision node D_j is deterministic, it will be fixed by the decision maker.

Once a decision $d = \{d_{1i},...,d_{pj}\} \in \mathcal{D}$ is fixed, chance nodes of the min-based Influence Diagram form a qualitative possibilistic network induces a unique joint conditional possibility distribution relative to chance node interpretations $x = \{x_{1i},...,x_{nj}\}$, in the context of d.

$$\pi^{ID}_{min}(x \mid d) = \min_{i=1..n} \pi_{ID}(x_{il} \mid u_{X_i}). \tag{4}$$

where $x_{il} \in \mathbb{D}_{X_i}$ and $u_{X_i} \in \mathbb{D}_{Par(X_i)} = \times_{X_m \in Par(X_i), D_j \in Par(X_i)} \mathbb{D}_{X_m} \cup \mathbb{D}_{D_j}$.

- For each utility node $V_{k=1..q} \in \mathcal{V}$, ordinal values $\mu_k(u_{V_k})$ are assigned to every possible instantiations u_{V_k} of the parent variables $Par(V_k)$. Ordinal values μ_k represent satisfaction degrees associated with local instantiations of parents variables.

The global satisfaction degree $\mu^{ID}_{min}(x,d)$ relative to the global instantiation (x,d) of all variables (chance and decision nodes) can be computed as the minimum of the local satisfaction degrees:

$$\mu^{ID}_{min}(x,d) = \min_{k=1..q} \mu_k(u_{V_k}). \tag{5}$$

where $u_{V_k} \in \mathbb{D}_{Par(V_k)} = \times_{X_i \in Par(V_k), D_j \in Par(V_k)} \mathbb{D}_{X_i} \cup \mathbb{D}_{D_j}$.

A qualitative Influence Diagram is evaluated in order to identify the optimal strategy δ^*, maximizing one of the possibilistic qualitative utilities. In fact, a strategy δ assigns an instantiation d to each global instantiation x of the state variables.

3 DECOMPOSITION OF MIN-BASED POSSIBILISTIC INFLUENCE DIAGRAM

This section discusses the main contributions of this paper. Our aim is to show that a qualitative PID can be modelled by two possibility distributions, one representing agent's beliefs and the other representing the qualitative utility. So, we propose a decomposition process of min-based possibilistic Influence Diagram $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$ into two min-based possibilistic networks:

1. Agent's knowledge $\Pi K_{min} = (G_K, \pi)$. This qualitative possibilistic network should codify the same joint conditional possibility distribution π^{ID}_{min} induced by the PID.
2. Agent's preferences $\Pi P_{min} = (G_P, \mu)$. Again, this preference-based possibilistic network must codify the same qualitative utility μ^{ID}_{min} induced by the PID.

In what follows, the decomposition process of the Influence Diagram $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$ into two qualitative possibilistic networks is presented.

3.1 The Construction of a Knowledge-based Qualitative Possibilistic Network

The knowledge-based qualitative possibilistic network $\Pi K_{min} = (G_K, \pi)$ encodes agent's beliefs. It induces a unique possibility distribution π_K using Equation 3. The graphical component G_K of the new qualitative possibilistic network ΠK_{min} is defined on the set of variables $\mathcal{Y} = \mathcal{X} \cup \mathcal{D} = \{Y_1, ..., Y_{n+p}\}$ of chance and decision nodes (where $n = |\mathcal{X}|$ and $p = |\mathcal{D}|$). The construction of such network is performed in three steps:

- Each decision node D_j will be transformed into a chance node representing the total ignorance, namely:

$$\forall D_j \in \mathcal{D},\ \pi(d_{jl} \mid u_{D_j}) = 1. \tag{6}$$

  for each instance $d_{jl} \in \mathbb{D}_{D_j}$ and $u_{D_j} \in \mathbb{D}_{Par(D_j)}$
- All state nodes remain unchanged:

$$\forall X_i \in \mathcal{C},\ \pi(x_{il} \mid u_{X_i}) = \pi_{ID}(x_{il} \mid u_{X_i}) \tag{7}$$

  for each instance $x_{il} \in \mathbb{D}_{X_i}$ and $u_{X_i} \in \mathbb{D}_{Par(X_i)}$.
- All utility nodes $\{V_1, ..., V_q\}$ and their associated edges are removed.

The building of the knowledge-based possibilistic network ΠK_{min} can be summarized by Algorithm 1. The new min-based possibilistic network $\Pi K_{min} = (G_K, \pi)$ induces a unique joint possibility distribution π_K defined by the min-based chain rule (Equation 3). The following proposition ensures that the joint possibility distribution induced by the new possibilistic network ΠK_{min} encodes the same states represented by the Influence Diagram ΠID_{min}.

Data: $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$, a min-based PID.
Result: $\Pi K_{min} = (G_K, \pi)$, knowledge-based network.
begin
  foreach $D_j \in \mathcal{D}$ **do**
    Transform each decision node D_j into chance node using Equation 6.
  end
  foreach $X_i \in \mathcal{C}$ **do**
    Quantify each chance node X_i using Equation 7.
  end
  Remove utility nodes $\{V_1, ..., V_q\}$.
end

Algorithm 1: Building knowledge-based network.

Proposition 1. *Let $\Pi K_{min} = (G_K, \pi)$ be a min-based possibilistic network obtained using Algorithm 1. The joint possibility distribution π_K induced by ΠK_{min} is equal to the one induced by the Influence Diagram ΠID_{min}. Namely,*

$$\pi_K(Y_1,...,Y_{n+p}) = \pi^{ID}_{min}(X_1,...,X_n \mid D_1,...,D_p) = \min_{X_i \in C} \pi_{ID}(X_i \mid U_i) \quad (8)$$

Example 1. *Let us consider a simple decision problem represented by a min-based Influence Diagram $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$. The graphical component G_{ID} is given by Figure 1. It contains three chance nodes $\mathcal{C} = \{X_1, X_2, X_3\}$, two decision nodes $\mathcal{D} = \{D_1, D_2\}$ and two utility nodes $\mathcal{V} = \{V_1, V_2\}$. We suppose that all variables are binary.*

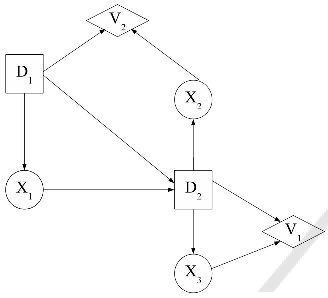

Figure 1: An example of influence diagram.

The numerical components are represented by conditional possibility distributions associated with chance nodes X_1, X_2, X_3 and qualitative utilities for the value node V_1 and V_2, in the context of their parents. Indeed, conditional possibilities are represented in Tables 1. Utilities for V_1 and V_2 are represented in Table 2. It should be noted that utilities for V_2 is not normalized.

The joint conditional possibility distribution π^{ID}_{min} induced by the Influence Diagram ΠID_{min}, using Equation 4, is given by Table 3.

Table 1: Initial possibility distributions π_{ID} on $X_1 \mid D_1$, $X_2 \mid D_2$ and $X_3 \mid D_2$.

X_1	D_1	$\pi_{ID}(X_1 \mid D_1)$	X_2	D_2	$\pi_{ID}(X_2 \mid D_2)$	X_3	D_2	$\pi_{ID}(X_3 \mid D_2)$
x_1	d_1	1	x_2	d_2	.3	x_3	d_2	1
x_1	$\neg d_1$	.4	x_2	$\neg d_2$	1	x_3	$\neg d_2$	1
$\neg x_1$	d_1	.2	$\neg x_2$	d_2	1	$\neg x_3$	d_2	.4
$\neg x_1$	$\neg d_1$	1	$\neg x_2$	$\neg d_2$	.4	$\neg x_3$	$\neg d_2$	.8

Table 2: Initial qualitative utilities μ_1 on X_3, D_2 and μ_2 on X_2, D_1.

X_3	D_2	$\mu_1(X_3, D_2)$	X_2	D_1	$\mu_2(X_2, D_1)$
x_3	d_2	.2	x_2	d_1	.5
x_3	$\neg d_2$	.3	x_2	$\neg d_1$	.9
$\neg x_3$	d_2	1	$\neg x_2$	d_1	.1
$\neg x_3$	$\neg d_2$	0	$\neg x_2$	$\neg d_1$	.4

Table 3: The joint conditional possibility distribution π^{ID}_{min} on X_1, X_2, X_3 given D_1, D_2.

X_1	X_2	X_3	D_1	D_2	π^{ID}_{min}	X_1	X_2	X_3	D_1	D_2	π^{ID}_{min}
x_1	x_2	x_3	d_1	d_2	.3	$\neg x_1$	x_2	x_3	d_1	d_2	.2
$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{d_1}$	$\mathbf{\neg d_2}$	**1**	$\neg x_1$	x_2	x_3	d_1	$\neg d_2$	.2
x_1	x_2	x_3	$\neg d_1$	d_2	.3	$\neg x_1$	x_2	x_3	$\neg d_1$	d_2	.3
x_1	x_2	x_3	$\neg d_1$	$\neg d_2$	.4	$\mathbf{\neg x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{\neg d_1}$	$\mathbf{\neg d_2}$	**1**
x_1	x_2	$\neg x_3$	d_1	d_2	.3	$\neg x_1$	x_2	$\neg x_3$	d_1	d_2	.2
x_1	x_2	$\neg x_3$	d_1	$\neg d_2$	.8	$\neg x_1$	x_2	$\neg x_3$	d_1	$\neg d_2$	.2
x_1	x_2	$\neg x_3$	$\neg d_1$	d_2	.3	$\neg x_1$	x_2	$\neg x_3$	$\neg d_1$	d_2	.3
x_1	x_2	$\neg x_3$	$\neg d_1$	$\neg d_2$	.4	$\neg x_1$	x_2	$\neg x_3$	$\neg d_1$	$\neg d_2$	.8
$\mathbf{x_1}$	$\mathbf{\neg x_2}$	$\mathbf{x_3}$	$\mathbf{d_1}$	$\mathbf{d_2}$	**1**	$\neg x_1$	$\neg x_2$	x_3	d_1	d_2	.2
x_1	$\neg x_2$	x_3	d_1	$\neg d_2$	.3	$\neg x_1$	$\neg x_2$	x_3	d_1	$\neg d_2$	.2
x_1	$\neg x_2$	x_3	$\neg d_1$	d_2	.4	$\mathbf{\neg x_1}$	$\mathbf{\neg x_2}$	$\mathbf{x_3}$	$\mathbf{\neg d_1}$	$\mathbf{d_2}$	**1**
x_1	$\neg x_2$	x_3	$\neg d_1$	$\neg d_2$	.4	$\neg x_1$	$\neg x_2$	x_3	$\neg d_1$	$\neg d_2$	.4
x_1	$\neg x_2$	$\neg x_3$	d_1	d_2	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	d_1	d_2	.2
x_1	$\neg x_2$	$\neg x_3$	d_1	$\neg d_2$	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	d_1	$\neg d_2$	.2
x_1	$\neg x_2$	$\neg x_3$	$\neg d_1$	d_2	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg d_1$	d_2	.4
x_1	$\neg x_2$	$\neg x_3$	$\neg d_1$	$\neg d_2$	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg d_1$	$\neg d_2$	.4

The global satisfaction degree $\mu^{ID}_{min}(D_1, D_2, X_1, X_2, X_3)$ generated by the Influence Diagram ΠID_{min} can be computed using Equation 5. The results are reported in Table 4.

Table 4: Global qualitative utilities $\mu^{ID}_{min}(D_1, D_2, X_1, X_2, X_3)$.

D_1	D_2	X_1	X_2	X_3	μ^{ID}_{min}	D_1	D_2	X_1	X_2	X_3	μ^{ID}_{min}
d_1	d_2	x_1	x_2	x_3	.2	$\neg d_1$	d_2	x_1	x_2	x_3	.2
d_1	d_2	x_1	x_2	$\neg x_3$	.5	$\neg d_1$	d_2	x_1	x_2	$\neg x_3$	.9
d_1	d_2	x_1	$\neg x_2$	x_3	.1	$\neg d_1$	d_2	x_1	$\neg x_2$	x_3	.2
d_1	d_2	x_1	$\neg x_2$	$\neg x_3$	.1	$\neg d_1$	d_2	x_1	$\neg x_2$	$\neg x_3$	.4
d_1	d_2	$\neg x_1$	x_2	x_3	.2	$\neg d_1$	d_2	$\neg x_1$	x_2	x_3	.2
d_1	d_2	$\neg x_1$	x_2	$\neg x_3$	.5	$\neg d_1$	d_2	$\neg x_1$	x_2	$\neg x_3$	.9
d_1	d_2	$\neg x_1$	$\neg x_2$	x_3	.1	$\neg d_1$	d_2	$\neg x_1$	$\neg x_2$	x_3	.2
d_1	d_2	$\neg x_1$	$\neg x_2$	$\neg x_3$	.1	$\neg d_1$	d_2	$\neg x_1$	$\neg x_2$	$\neg x_3$	.4
d_1	$\neg d_2$	x_1	x_2	x_3	.3	$\neg d_1$	$\neg d_2$	x_1	x_2	x_3	.3
d_1	$\neg d_2$	x_1	x_2	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	x_1	x_2	$\neg x_3$	0
d_1	$\neg d_2$	x_1	$\neg x_2$	x_3	.1	$\neg d_1$	$\neg d_2$	x_1	$\neg x_2$	x_3	.3
d_1	$\neg d_2$	x_1	$\neg x_2$	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	x_1	$\neg x_2$	$\neg x_3$	0
d_1	$\neg d_2$	$\neg x_1$	x_2	x_3	.3	$\neg d_1$	$\neg d_2$	$\neg x_1$	x_2	x_3	.3
d_1	$\neg d_2$	$\neg x_1$	x_2	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	$\neg x_1$	x_2	$\neg x_3$	0
d_1	$\neg d_2$	$\neg x_1$	$\neg x_2$	x_3	.1	$\neg d_1$	$\neg d_2$	$\neg x_1$	$\neg x_2$	x_3	.3
d_1	$\neg d_2$	$\neg x_1$	$\neg x_2$	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	$\neg x_1$	$\neg x_2$	$\neg x_3$	0

We propose to decompose the Influence Diagram ΠID_{min} given in Figure 1 into two qualitative possibilistic networks. The first min-based possibilistic network $\Pi K_{min} = (G_K, \pi)$ describes agent's knowledge, the second one $\Pi P_{min} = (G_P, \mu)$ will express its preferences.
Let us proceed to build the knowledge-based network $\Pi K_{min} = (G_K, \pi)$ using Algorithm 1. The graphical component G_K is given by Figure 2. In fact, the graphical component G_K corresponds to the Influence Diagram of Figure 1 from which we removed the utility nodes.

Using Algorithm 1, the initial possibility distribution associated with ΠK_{min} are given by Tables 5, 6 and 7.

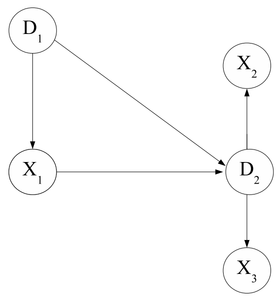

Figure 2: Knowledge-based possibilistic network.

Table 5: Initial possibility distribution ΠK_{min} on $X_1 \mid D_1$, $X_2 \mid D_2$ and $X_3 \mid D_2$.

X_1	D_1	$\pi(X_1 \mid D_1)$	X_2	D_2	$\pi(X_2 \mid D_2)$	X_3	D_2	$\pi(X_3 \mid D_2)$
x_1	d_1	1	x_2	d_2	.3	x_3	d_2	1
x_1	$\neg d_1$	.4	x_2	$\neg d_2$	1	x_3	$\neg d_2$	1
$\neg x_1$	d_1	.2	$\neg x_2$	d_2	1	$\neg x_3$	d_2	.4
$\neg x_1$	$\neg d_1$	1	$\neg x_2$	$\neg d_2$	.4	$\neg x_3$	$\neg d_2$	.8

Table 6: Initial possibility distribution ΠK_{min} on D_1.

D_1	$\pi(D_1)$
d_1	1
$\neg d_1$	1

Table 7: Initial possibility distribution ΠK_{min} on $D_2 \mid D_1 X_1$.

D_2	D_1	X_1	$\pi(D_2 \mid D_1 X_1)$	D_2	D_1	X_1	$\pi(D_2 \mid D_1 X_1)$
d_2	d_1	x_1	1	$\neg d_2$	d_1	x_1	1
d_2	d_1	$\neg x_1$	1	$\neg d_2$	d_1	$\neg x_1$	1
d_2	$\neg d_1$	x_1	1	$\neg d_2$	$\neg d_1$	x_1	1
d_2	$\neg d_1$	$\neg x_1$	1	$\neg d_2$	$\neg d_1$	$\neg x_1$	1

Table 8 contains the joint possibility distribution π_K (using Equation 3) induced by the min-based possibilistic network ΠK_{min}.

Table 8: The joint possibility distribution $\pi_K(X_1, X_2, X_3, D_1, D_2)$.

X_1	X_2	X_3	D_1	D_2	π_K	X_1	X_2	X_3	D_1	D_2	π_K
x_1	x_2	x_3	d_1	d_2	.3	$\neg x_1$	x_2	x_3	d_1	d_2	.2
x_1	x_2	x_3	d_1	$\neg d_2$	1	$\neg x_1$	x_2	x_3	d_1	$\neg d_2$	.2
x_1	x_2	x_3	$\neg d_1$	d_2	.3	$\neg x_1$	x_2	x_3	$\neg d_1$	d_2	.3
x_1	x_2	x_3	$\neg d_1$	$\neg d_2$	.4	$\neg x_1$	x_2	x_3	$\neg d_1$	$\neg d_2$	1
x_1	x_2	$\neg x_3$	d_1	d_2	.3	$\neg x_1$	x_2	$\neg x_3$	d_1	d_2	.2
x_1	x_2	$\neg x_3$	d_1	$\neg d_2$	.8	$\neg x_1$	x_2	$\neg x_3$	d_1	$\neg d_2$	.2
x_1	x_2	$\neg x_3$	$\neg d_1$	d_2	.3	$\neg x_1$	x_2	$\neg x_3$	$\neg d_1$	d_2	.3
x_1	x_2	$\neg x_3$	$\neg d_1$	$\neg d_2$	.4	$\neg x_1$	x_2	$\neg x_3$	$\neg d_1$	$\neg d_2$	.8
x_1	$\neg x_2$	x_3	d_1	d_2	1	$\neg x_1$	$\neg x_2$	x_3	d_1	d_2	.2
x_1	$\neg x_2$	x_3	d_1	$\neg d_2$	.3	$\neg x_1$	$\neg x_2$	x_3	d_1	$\neg d_2$	.2
x_1	$\neg x_2$	x_3	$\neg d_1$	d_2	.4	$\neg x_1$	$\neg x_2$	x_3	$\neg d_1$	d_2	1
x_1	$\neg x_2$	x_3	$\neg d_1$	$\neg d_2$	.4	$\neg x_1$	$\neg x_2$	x_3	$\neg d_1$	$\neg d_2$	.4
x_1	$\neg x_2$	$\neg x_3$	d_1	d_2	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	d_1	d_2	.2
x_1	$\neg x_2$	$\neg x_3$	d_1	$\neg d_2$	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	d_1	$\neg d_2$	.2
x_1	$\neg x_2$	$\neg x_3$	$\neg d_1$	d_2	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg d_1$	d_2	.4
x_1	$\neg x_2$	$\neg x_3$	$\neg d_1$	$\neg d_2$	.4	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg d_1$	$\neg d_2$	.4

It can be checked that the joint possibility distribution π_K associated to the knowledge-based possibilistic network ΠK_{min} is the same as the one induced by the possibilistic Influence Diagram ΠID_{min} (see Table 3).

3.2 Building Preference-based Qualitative Possibilistic Network

The second qualitative possibilistic network $\Pi P_{min} = (G_P, \mu)$ represents agent's preferences associated with the qualitative utility. ΠP_{min} induces a unique qualitative utility μ_P using Equation 3. This section shows that this qualitative utility is equal to the qualitative utility μ_{min}^{ID} (Equation 5) encoded by the Influence Diagram ΠID_{min}.

The graphical component G_P of the new qualitative possibilistic network ΠP_{min} is defined on the set of variables $\mathcal{Z} = \{Z_1, ..., Z_m\} \subset \mathcal{X} \cup \mathcal{D}$ of chance and decision nodes. The set of nodes $\mathcal{Z}$ represents the union of the parent variables of all utility nodes $\{V_1, ..., V_q\}$ in the Influence Diagram. Namely, $\mathcal{Z} = \{Z_1, ..., Z_m\} = Par(V_1) \cup ... \cup Par(V_q)$, where $m = |Par(V_1) \cup ... \cup Par(V_q)|$ presents the total of parent variables of all utility nodes in ΠID_{min}.

During the construction phase of the graph G_P, we need to make sure that the generated graph is a DAG structure. We should also avoid the creation of loops at the merging step of the evaluation process (Benferhat et al., 2013). So, before enumerating the decomposition process of an Influence Diagram ΠID_{min}, the notion of topological order generated by a DAG is recalled:

Definition 1. *A Directed Acyclic Graph is a linear ordering of its nodes such that for every arc from node X_i to node X_j, X_i comes before X_j in the ordering. Any DAG has at least one topological ordering.*

Construction algorithms are known for constructing a topological ordering of any DAG in linear time. The usual algorithm for topological ordering consists in finding a " start node" which have no incoming edges. Then, edges outgoing this node must be removed. This process will be repeated until all nodes will be visited.

Example 2. *The DAG $G_{ID}(\mathcal{X}, \mathcal{A})$ associated with the Influence Diagram given in Example 1 has two valid topological ordering:*

- D_1, X_1, D_2, X_2, X_3,
- D_1, X_1, D_2, X_3, X_2.

which are equivalent to:

$$D_1 \prec X_1 \prec D_2 \prec X_2 X_3.$$

We first propose a naive solution that requires a pretreatment step which consists to reduce all utility nodes into a single one. This node will inherit the parents of all value nodes. A more advanced solution preserving the initial structure will then be proposed. Hence, operating on the initial structure of the Influence Diagram induces a more compact representation.

3.2.1 Decomposition Process with a Single Utility Node

The first solution consists in reducing all utility nodes into a single one. Hence, it amounts to perform pretreatment on the initial Influence Diagram before its decomposition. Formally, the pretreatment step consists on the reduction of the number of value nodes to one, noted V_r, that will inherit the parents of all value nodes $(Par(V_1),...,Par(V_q))$ ie $Par(V_r) = Par(V_1) \cup ... \cup Par(V_q)$. The utility value associated to the new utility node V_r corresponds to the minimum of utilities, which corresponds to the global satisfaction degree, namely:

$$\mu_r(u_{V_r}) = \mu^{ID}_{min}(x,d) = \min_{k=1..q} \mu_k(u_{V_k}). \quad (9)$$

where $u_{V_r} \in \mathbb{D}_{Par(V_r)}$ and $u_{V_k} \in \mathbb{D}_{Par(V_k)}$.

Once this step is accomplished, the min-based possibilistic network $\Pi P_{min} = (G_P, \mu)$ encoding agent's preferences is built as follows:

- Select an arbitrary node, denoted $Z_k \in Par(V_r)$ to be a child of the remaining parent variables $Par(V_r) \backslash Z_k$. This selection must be in agreement with the order generated by the DAG associated with the reduced Influence Diagram. This means that the selected node Z_k must be the last in the topological ordering induced by the reduced Influence Diagram.
- Create arcs from all the remaining nodes $\{Par(V_r) - Z_k\}$ to the node Z_k.
- Each node $Z_j \neq Z_k$ will be associated a total ignorance possibility distribution, namely:

$$\forall z_{jl} \in \mathbb{D}_{Z_j},\ \mu(z_{jl}) = 1. \quad (10)$$

- The node Z_k will be quantified as follows: $\forall z_{kl} \in \mathbb{D}_{Z_k},\ \forall u_{Z_k} \in \mathbb{D}_{Par(Z_k)}$,

$$\mu(z_{kl} \mid u_{Z_k}) = \mu^{ID}_{min}(u_{V_r}). \quad (11)$$

The construction of preference-based possibilistic network ΠP_{min} can be summarized by algorithm 2.

The following proposition indicates that the min-based possibilistic network $\Pi P_{min} = (G_P, \mu)$ constructed from the previous steps, codifies the same qualitative utility encoded by the qualitative Influence Diagram $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$.

Data: $\{V_1, Par(V_1)\}, ..., \{V_q, Par(V_q)\}$, utility nodes and their parents in the qualitative Influence Diagram.
Result: $\Pi P_{min} = (G_P, \mu)$, preference-based possibilistic network.
begin
  $\mathcal{Z} \leftarrow \{Par(V_1) \cup ... \cup Par(V_q)\}$.
  Reduce all utility nodes to a single node V_r.
  Select a node $Z_k \in Par(V_r)$ to be child of the remaining parent variables according to the topological ordering induced by the reduced ID.
  Create arcs from $\{Par(V_r) \backslash Z_k\}$ to Z_k.
  Quantifying chance node Z_k using Equation 11
  foreach $Z_j \neq Z_k$ **do**
    Quantifying Z_j using Equation 10.
  end
end

Algorithm 2: Construction of preference-based possibilistic network.

Proposition 2. *Let $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$ be a min-based possibilistic Influence Diagram. Let $\Pi P_{min} = (G_P, \mu)$ be a min-based possibilistic network obtained using Algorithm 2. The joint qualitative utility μ_P induced by ΠP_{min} is equivalent to the one induced by the Influence Diagram ΠID_{min}. Namely,*

$$\mu_P(Z_1, ..., Z_m) = \mu^{ID}_{min}(X_1, ..., X_n, D_1, ..., D_p). \quad (12)$$

Example 3. *Let us continue Example 1. Applying the solution based on a single node utility, we propose to build the preference-based network $\Pi P_{min} = (G_P, \mu)$ encoding agent's preferences. The pretreatment step consists in reducing V_1 and V_2 to one utility node denoted V_r. The new utility node inherits the parents of old utility nodes V_1 and V_2, namely D_1, X_1, D_2 and X_2. The reduced Influence Diagram is given by Figure 3.*

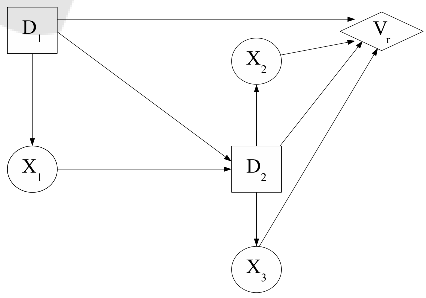

Figure 3: Min-based possibilistic Influence Diagram with single utility node.

Using Algorithm 2, the graphical component G_P will be defined on set of variables $\mathcal{Z} = Par(V_r) = \{X_2, X_3, D_1, D_2\}$. As already mentioned, an arbitrary node must be selected from $\mathcal{Z}$ to be a child of the remaining parent variables. The choice of this node must be in accordance with the topological ordering *induced by the reduced Influence Diagram. First, we give the topological ordering induced by the reduced Influence Diagram (Figure 3):*

$$D_1 \prec X_1 \prec D_2 \prec X_2X_3.$$

Then, two nodes are worn candidates for this choice: X_2 or X_3, let X_3 be this node. The graphical component G_P is given by Figure 4.

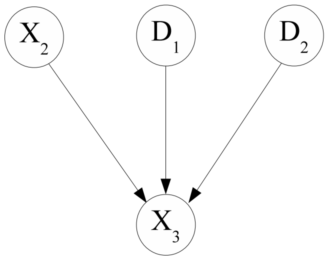

Figure 4: Preference-based possibilistic network.

The conditional possibility distribution $\mu(X_3 \mid D_1D_2X_2)$ associated to X_3 is defined using Equation 11. The results are mentioned in Table 9. Possibility distributions on other nodes $\{D_1, D_2, X_2\}$ are uniform (see Table 10).

Table 9: Initial possibility distribution ΠP_{min} on $X_3 \mid D_1D_2X_2$.

X_3	D_1	D_2	X_2	$\mu(X_3 \mid D_1D_2X_2)$	X_3	D_1	D_2	X_2	$\mu(X_3 \mid D_1D_2X_2)$
x_3	d_1	d_2	x_2	.2	$\neg x_3$	d_1	d_2	x_2	.5
x_3	d_1	d_2	$\neg x_2$	.9	$\neg x_3$	d_1	d_2	$\neg x_2$	.1
x_3	d_1	$\neg d_2$	x_2	.3	$\neg x_3$	d_1	$\neg d_2$	x_2	0
x_3	d_1	$\neg d_2$	$\neg x_2$	.1	$\neg x_3$	d_1	$\neg d_2$	$\neg x_2$	0
x_3	$\neg d_1$	d_2	x_2	.2	$\neg x_3$	$\neg d_1$	d_2	x_2	.9
x_3	$\neg d_1$	d_2	$\neg x_2$	.2	$\neg x_3$	$\neg d_1$	d_2	$\neg x_2$	.4
x_3	$\neg d_1$	$\neg d_2$	x_2	.3	$\neg x_3$	$\neg d_1$	$\neg d_2$	x_2	0
x_3	$\neg d_1$	$\neg d_2$	$\neg x_2$	.3	$\neg x_3$	$\neg d_1$	$\neg d_2$	$\neg x_2$	0

Table 10: Initial possibility distribution ΠP_{min} on D_1, D_2 and X_2.

D_1	$\mu(D_1)$	D_2	$\mu(D_2)$	X_2	$\mu(X_2)$
d_1	1	d_2	1	x_2	1
$\neg d_1$	1	$\neg d_2$	1	x_2	1

Using Equation 3, the preference-based possibilistic network ΠP_{min} induces the joint qualitative utility μ_P given by Table 11.

As illustrated by Tables 9 and 11, the conditional possibility distribution $\mu(X_3 \mid D_1D_2X_2)$ is the same as the joint qualitative utility induced by the preferences-based possibilistic network ΠP_{min}. Therefore, we conclude that the proposed solution which consists in reducing the initial Influence Diagram do not allow a compact representation of agent's preferences.

Table 11: The joint qualitative utility $\mu_P(D_1, D_2, X_1, X_2, X_3)$.

D_1	D_2	X_1	X_2	X_3	μ_P	D_1	D_2	X_1	X_2	X_3	μ_P
d_1	d_2	x_1	x_2	x_3	.2	$\neg d_1$	d_2	x_1	x_2	x_3	.2
d_1	d_2	x_1	x_2	$\neg x_3$	.5	$\neg d_1$	d_2	x_1	x_2	$\neg x_3$	.9
d_1	d_2	x_1	$\neg x_2$	x_3	.1	$\neg d_1$	d_2	x_1	$\neg x_2$	x_3	.2
d_1	d_2	x_1	$\neg x_2$	$\neg x_3$	.1	$\neg d_1$	d_2	x_1	$\neg x_2$	$\neg x_3$	.4
d_1	d_2	$\neg x_1$	x_2	x_3	.2	$\neg d_1$	d_2	$\neg x_1$	x_2	x_3	.2
d_1	d_2	$\neg x_1$	x_2	$\neg x_3$	.5	$\neg d_1$	d_2	$\neg x_1$	x_2	$\neg x_3$	.9
d_1	d_2	$\neg x_1$	$\neg x_2$	x_3	.1	$\neg d_1$	d_2	$\neg x_1$	$\neg x_2$	x_3	.2
d_1	d_2	$\neg x_1$	$\neg x_2$	$\neg x_3$	.1	$\neg d_1$	d_2	$\neg x_1$	$\neg x_2$	$\neg x_3$	.4
d_1	$\neg d_2$	x_1	x_2	x_3	.3	$\neg d_1$	$\neg d_2$	x_1	x_2	x_3	.3
d_1	$\neg d_2$	x_1	x_2	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	x_1	x_2	$\neg x_3$	0
d_1	$\neg d_2$	x_1	$\neg x_2$	x_3	.1	$\neg d_1$	$\neg d_2$	x_1	$\neg x_2$	x_3	.3
d_1	$\neg d_2$	x_1	$\neg x_2$	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	x_1	$\neg x_2$	$\neg x_3$	0
d_1	$\neg d_2$	$\neg x_1$	x_2	x_3	.3	$\neg d_1$	$\neg d_2$	$\neg x_1$	x_2	x_3	.3
d_1	$\neg d_2$	$\neg x_1$	x_2	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	$\neg x_1$	x_2	$\neg x_3$	0
d_1	$\neg d_2$	$\neg x_1$	$\neg x_2$	x_3	.1	$\neg d_1$	$\neg d_2$	$\neg x_1$	$\neg x_2$	x_3	.3
d_1	$\neg d_2$	$\neg x_1$	$\neg x_2$	$\neg x_3$	0	$\neg d_1$	$\neg d_2$	$\neg x_1$	$\neg x_2$	$\neg x_3$	0

3.2.2 Decomposition Process based on the Initial Influence Diagram

The main limitation of the first solution, presented in Section 3.2.1 and inspired from the work proposed in (Guezguez, 2012), concerns the reduction of all utility nodes into a single one that will inherit the parents of all value nodes. So, we suggest to preserve the initial structure. The solution proposed in this section is to try to have the structure of a preference-based network as close as possible to the initial structure of the Influence Diagram. Hence, as we will see, operating on the initial structure of the Influence Diagram allows a more compact representation than if we have used the reduced Influence Diagram.

The min-based possibilistic network $\Pi P_{min} = (G_P, \mu)$ encoding agent's preferences is built progressively as follows:

- For each utility node $V_k \in \mathcal{V} = \{V_1, ..., V_q\}$, define an order between parent variables. More precisely, this order is induced by the DAG associated with the initial Influence Diagram.

  Example 4. *Let us consider the Influence Diagram given in Example 1 Figure 1. The parent variables of utility node V_2 are D_1 and X_2. We recall that the Influence Diagram ΠID_{min} induces the following order:*

  $$D_1 \prec X_1 \prec D_2 \prec X_3X_2.$$

  Then, an order can be defined between parent variables D_1 and X_2 as follows:

  $$D_1 \prec X_2$$

- For each utility node $V_k \in \mathcal{V} = \{V_1, ..., V_q\}$, a set of nodes is defined, among the parent variables of V_k where each node is eligible to be a child of the remaining parent variables. The candidate nodes are those that appear in the last row of the order list generated in the previous step. Indeed, respecting the order induced between the parents of each utility node enables us to avoid the creation of loops at the merging step of the evaluation process (Benferhat et al., 2013). Each candidate node can have one of the three following status:

1. Either it has not yet been introduced in the DAG G_P under construction.
2. Or it represents a root node in the DAG part already built.
3. Or it represents a child.

- One node, denoted Z_k must be selected from the candidate set generated in the previous step. For more compact representation of the DAG G_P under construction, the selected node must satisfy as a priority the first or second property (not yet introduced or root node). If the selected node Z_k do not yet appear in the DAG G_P under construction, then we must first integrate it in G_P. In the same way, parent variables that have not yet been integrated in G_P must be created. Finally, arcs from the remaining parent variables to Z_k must be created.

  If the selected node Z_k already appears in the DAG G_P under construction as a root node, then we must only integrate the remaining parent variables that are not yet included in G_P and create arcs from the remaining parent variables to Z_k.

  Then, we proceed to compute the conditional possibility distribution $\mu(Z_k \mid U_{Z_k})$. In both situations (1 or 2), the conditional possibility distribution $\mu(Z_k \mid U_{Z_k})$ associated to the node Z_k is defined as follows:
  $\forall z_{kl} \in \mathbb{D}_{Z_k}$, $\forall u_{Z_k} \in \mathbb{D}_{Par(Z_k)}$,

$$\mu(z_{kl} \mid u_{Z_k}) = \mu_k(u_{V_k}). \quad (13)$$

  If such a node does not exist (all candidate nodes are already children), then we choose the node with a minimum number of parents in order to have more compact representation. The conditional possibility distribution $\mu(Z_k \mid U_{Z_k})$ associated to the node Z_k is defined as follows:
  $\forall z_{kl} \in \mathbb{D}_{Z_k}$, $\forall u_{Z_k} \in \mathbb{D}_{Par(Z_k)}$,

$$\mu(z_{kl} \mid u_{Z_k}) = \min[\mu(z_{kl} \mid u_{Z_k}), \mu_k(u_{V_k})]. \quad (14)$$

- For each node Z_j different from the selected node Z_k will be associated a total ignorance possibility distribution, namely:

$$\forall z_{jl} \in \mathbb{D}_{Z_j},\ \mu(z_{jl}) = 1. \quad (15)$$

The construction of preference-based possibilistic network ΠP_{min} can be summarized by Algorithm 3.

The proposed algorithm generates the qualitative min-based possibilistic network $\Pi P_{min} = (G_P, \mu)$ step by step. Indeed, for each utility node, the algorithm selects the candidate parents that can be a child of the remaining parents in the DAG G_P under construction. These candidate nodes appear in the last rank of the topological ordering generated by the ID. Among the candidates, if there exists a node that has not yet been introduced in G_P or it presents a root node, so it will be selected as the child of the remaining parent variables in the DAG G_P under construction. Otherwise, if such node does not exist then it means that all candidate nodes are already integrated in the DAG G_P and

Data: $\{V_1, Par(V_1)\}, ..., \{V_q, Par(V_q)\}$, utility nodes and their parents in the qualitative influence diagram.
Result: $\Pi P_{min} = (G_P, \mu)$, preference-based possibilistic network.
begin
  $\mathcal{Z} \leftarrow \emptyset$. /* Set of integrated variables in G_P */
  $Child \leftarrow \emptyset$.
  foreach $V_k \in \{V_1, ..., V_q\}$ **do**
    $List-order(V_k) \leftarrow \{Par(V_k)\}$ ordered in the same way that the order induced by ΠID_{min}.
    $Candidate(V_k) \leftarrow \{$ the variables with the last rank in the $List-order(V_k)\}$.
    Select a variable $Z_k \in Candidate(V_k)$ and $Z_k \notin Child$.
    if Z_k *exists* **then**
      $Child \leftarrow Child \cup \{Z_k\}$./*$Z_k$ presents child in G_P*/
      Create nodes $Par(V_k) \notin \mathcal{Z}$./*creating nodes that not appear in G_P*/
      Create arcs from $\{Par(V_k) - Z_k\}$ to Z_k./*creating arcs from the remaining parent variables to the selected node Z_k*/
      Quantifying chance node Z_k using Equation 13
    else
      Select a variable $Z_k \in Candidate(V_k)$ and $|Par(Z_k)|$ in G_P is the smallest.
      Create nodes $Par(V_k) \notin \mathcal{Z}$.
      Create arcs from $\{Par(V_k) - Z_k\}$ to Z_k.
      Quantifying chance node Z_k using Equation 14
    end
    foreach $Z_j \in Par(V_k)$ *and* $Z_j \neq Z_k$ **do**
      if $Z_j \notin Child$ **then**
        Quantifying chance node Z_j using Equation 15
      end
    end
  end
end

Algorithm 3: Preference-based possibilistic network.

they have parents (they present child). According to the selected node status (not integrated, root or child) an utility will be associated to this node. A total ignorance possibility distribution will be associated with the remaining parent variables.

It is evident that the last solution which operates on the initial ID structure (which does not require the reduction of utility nodes to a single utility one) allows a compact representation of the qualitative utility.

It should be noted that in the case of an ID with multiple utility nodes having no common parents, the preference-based qualitative possibilistic network will in fact be disconnected. Indeed, each component of the graph encodes local satisfaction degrees associated to one utility node.
The following proposition shows that the qualitative possibilistic network $\Pi P_{min} = (G_P, \mu)$, built following the previous steps, encodes the same qualitative utility encoded by the qualitative Influence Diagram $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$.

Proposition 3. *Let $\Pi ID_{min}(G_{ID}, \pi^{ID}_{min}, \mu^{ID}_{min})$ be a min-based PID. Let $\Pi P_{min} = (G_P, \mu)$ be a preferences-based possibilistic network obtained using Algorithm 3. The joint qualitative utility μ_P induced by ΠP_{min} is equal to the one induced by ΠID_{min}. Namely,*

$$\mu_P(Z_1, ..., Z_m) = \mu^{ID}_{min}(X_1, ..., X_n, D_1, ..., D_p). \quad (16)$$

4 RELATED WORKS

In possibilistic framework, few works exist on decision making. A possibilistic adaptation of the well known ID has been proposed in (Garcia and Sabbadin, 2006) (Guezguez et al., 2009) (Zhang, 2013), etc. Both knowledge and utilities are described in a same graphical structure using ordinal data. Like the probabilistic ID, the PID_s contain three types of nodes: chance, decision and utility nodes. Uncertainty is described by means of possibility distributions on chance nodes and preferences are expressed as satisfaction degrees on utility nodes. To compute optimal decisions, two methods have been proposed in literature for evaluating qualitative PID: direct and an indirect once. A direct method uses initial structures but require additional computations in order to update possibility distribution tables (Garcia and Sabbadin, 2006). Also, in (Garcia and Sabbadin, 2006), an indirect method has been proposed which consists to transform a PID into a decision tree. Recently, in (Guezguez et al., 2009) a new indirect method has been proposed to evaluate PID based on the transformation of this latter into qualitative possibilistic network. It should be noted that the proposed solution reduces utility nodes in a single one. On this new structure the inference process will be made.

Recently in (Benferhat et al., 2013), authors have proposed a new possibilistic graphical model for handling decision problems under uncertainty. The proposed solution for representing decision making under uncertainty is based on the use of min-based possibilistic networks. It suggested to encode agent's knowledge and preferences by two distinct qualitative possibilistic networks. The first one encodes a joint possibility distribution representing available knowledge and the second one encodes the qualitative utility. This new representation is in agreement with the semantic definition of a qualitative decision problem given in (Dubois et al., 1999). This new representation for decision making under uncertainty based on min-based possibilistic networks, benefits from the simplicity of possibilistic networks. Indeed, the computation of optimal decision is performed using inference process in a unified way. Unlike the solution proposed in (Guezguez et al., 2009) for computing optimal decisions, the decomposition process allows us to obtain more compact representation. In fact, the possibilistic network issued from the fusion phase (Benferhat et al., 2013) is based on more compact representation of the qualitative utility.

5 CONCLUSIONS

This paper concerns the decomposition of a Possibilistic Influence Diagram into two possibilistic networks: the first expresses agent's knowledge and the second encodes its preferences. This procedure allows a simple representation of decision problems under uncertainty. Indeed, the decomposition process described in this paper offers a natural way to express knowledge and preferences of a agent separately in unified way using only one type of nodes. And in order to perform the decomposition process, an algorithm has been proposed and has confirmed that the new model based on the possibilistic networks (Benferhat et al., 2013) for representing decision making has the capacity to encode any decision problem. The proposed algorithm ensures a more compact representation of the graph used in evaluation phase for computing optimal decisions.

As future work, we plan to extend the proposed graphical model for the representation of decision problems to deal with more complex problems involving sequential decisions. Indeed, one of the attractive benefits of Possibilistic Influence Diagrams consists on their ability of dealing sequential decisions.

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