A Method to Improve the Precision of Interferometric Phase-recognization under Open-loop PZT Drive

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Keywords: Phase Recognization, Four Steps Algorithm, Ellipse Fitting, Lagrange Interpolation, Open-loop PZT Drive.

Abstract: For the monochromatic light interferometry under open-loop PZT drive, a high precision method for phase recognization that satisfies the four steps phase-shift algorithm is proposed. The total idea of the phase recognization method is as follows. Firstly, two pixels with suitable phase-difference are selected from the interference field and the interference equations of the two pixels' gray values are established in one driven cycle of PZT. Secondly, the parameters of interference equations can be obtained by using ellipse fitting algorithm. Thirdly, the point-to-point step length of PZT drive and sequence phases can be determined through reverse calculation of sequence phases. Finally, in order to calculate initial phase of every pixels four interference grayscale images that meet the four steps phase-shift algorithm are designed and calculated through Lagrange parabolic interpolation. The experimental results have shown that this method decreases the requirement for hardware, environment and needs less interference grayscale images than traditional methods. The method can meet the high precision demands of surface topography measurement and has high processing speed.

1 INTRODUCTION

In the interference measurement of surface topography by monochromatic light or quasi monochromatic light, the calculation accuracy of interferometric phase determines directly the accuracy of the measurement results. Therefore, the phase information should be accurately extracted before unwrapping operation for the interferometric phase. At present, there are many ways to extract interferometric phase. For example, three-step method (Wyant et al., 1984), four-step method (Wyant, 1982）, five-step method (Hariharan et al., 1987), FFT algorithm (Wang and Da, 2012), wavelet transform (Cui et al., 2012), phase retrieval method used for wavelength-scanning (Liu et al., 2014) or wavelength-tuning Interferometer (Kato and Yamaguchi, 2000) and so on. The essence of these algorithms is to obtain the initial phases of interference sequence of every pixel by eliminating the influence of amplitude and offset parameters. The difference among three-step method, four-step method and five-step method is the anti-noise-interference ability. The FFT algorithm requires the interference sequence for a complete cycle and always needs a large amount of data processing. As a result, it has low efficiency. Futhermore, these methods require PZT be closed-loop controlled, which increases the difficulty and cost of driver element (Deng, 2014). Because tiny vibration during measurement process will lead to the driving step length change and has a great influence on the final measurement results, the measurement system has also very high requirements for measuring environment.

An algorithm for calculating the interference phase through arbitrary driving step lengths was presented (Hao et al., 2009). The requirement of the approach is as follows. Firstly, find out the maximum and the minimum gray values of a point in the interference field from the sequence interference grayscale images. Then the sequence phases are calculated. Finally, the sequence phases of every point in the interference field are computed. The method doesn't require controlling the driving step length of PZT, but it costs more time on image acquisition and needs more interference images than other methods. When the number of interference images is small, the real maximum and minimum of gray values cannot be found out. As a result, this method cannot be used.

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This paper has two core problems need to be solved. The first is how to obtain the initial phase information of every pixel in the interference field based on sequence grayscale images under the open-loop drive conditions of PZT. The second is to ensure phase information has high recognition precision and recognition precision is not effected by environment.

Based on four-step method, this paper proposed a high accuracy phase recognition method that doesn't require PZT to be equal driving step length. The main idea is to acquire exactly the interference sequence phases or step lengths of PZT by ellipse fitting algorithm according to gray correlation information of every pixel in the interference field based on the interference equations. By extracting sequence phases of interference grayscale images and Lagrange parabolic interpolation algorithm (Liu et al,2010), four interference grayscale images with phase difference π/2 which meet requirement for calculation of four-step phase-shift method are calculated and constructed. Finally, through interference grayscale images, phase information of every pixel in the interference field can be obtained by using four-step method.

2 CALCULATION METHOD OF PZT DRIVING STEP LENGTH

In theory, gray values g_i of every pixel in the interference field satisfies the expression:

$$g_i = A\cos(\theta_0 + \theta_i) + C \tag{1}$$

Where i is the No. of driving sequence points $i=1,\cdots,N$. θ_i is the driving sequence phases. A, C, θ_0 are the amplitude, offset and initial phase of every pixel.

The relation between θ_i and driving displacement Δ_i satisfies equation (2):

$$\theta_i = 4\pi\Delta_i / \lambda \tag{2}$$

Therefore, the driving step length of PZT is proportional to the interferometric phase. The key to determine the driving step length is obtain driving phase of every point in interference sequence θ_i. The calculation of θ_i depends on the calculation of A and C when the gray values are known. In this section we discuss how to calculate A and C.

13 sequence phases of a pixel is shown in Fig.1. The figure shows clearly that the driving step lengths don't satisfy the equal step lengths condition, because the sequence gray curve should be an approximate the trigonometric function curve if the driving step lengths are equal. This means the gray values with large random noise.

2.1 Establishment of Interference Equations

By taking it into account that every point in the interference field has the same driving step length or sequence phases at any time of PZT drive and by choosing two points arbitrarily, sequence gray values equations can be established as follows:

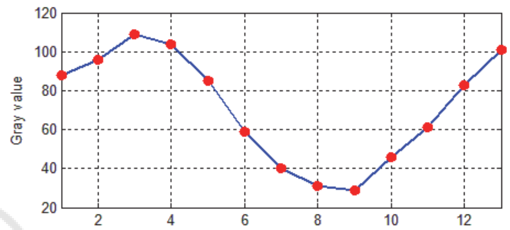


Figure 1: Sequence gray values of single pixel.

$$\begin{cases} g_{1i} = A_1\cos(\theta_{10} + \theta_{1i}) + C_1 \\ g_{2i} = A_{21}\cos(\theta_{20} + \theta_{2i}) + C_2 \end{cases} \tag{3}$$

If g_{1i}, g_{2i} are respectively the transverse and longitudinal coordinates, the theoretical trajectory of sequence points should be an ellipse. The centers of the ellipse are respectively C_1, C_2. The parameters of the ellipse are determined by alternating amplitude A_1, A_2 and initial phase difference (θ_{20}-θ_{10}) between two pixels. The trajectory is a positive ellipse when the phase difference is π/2.

Parameters C_1, C_2, A_1, A_2 and (θ_{20}-θ_{10}) can be obtained through ellipse fitting. The fitting accuracy of the parameters is mainly affected by the noise of pixel gray values and the oval shape.

2.2 Method to Improve the Fitting Precision of the Interference Equations

Because the fitting accuracy of ellipse parameters is influenced by the noise of gray values of pixels and shape of the ellipse, the pixel points used for fitting should be selected properly and de-noise processing for the gray values of pixel points should be conducted.

The gray values of interference sequence satisfy orthogonal relation and fitted ellipse has highest accuracy when the phase difference between two pixels is $\pi/2$. Therefore, the phase difference between two pixels used for ellipse fitting should meet requirement of $\pi/2$ as far as possible. The adopted scheme is as follows.

Firstly, the sequence gray values of every pixel should be calculated as follows in specified area (where has high SNR):

$$s_{mn} = \frac{\sum_{i=1}^{N/2} g_{mn}(i) - \sum_{i=N/2+1}^{N} g_{mn}(i)}{\sum_{i=1}^{N} |g_{mn}(i)|} \quad (4)$$

Where s_{mn} is in [-1,1]. In order to get two points whose phase difference is $\pi/2$, the maximum and the minimum in absolute values of s_{mn} are selected as the fitted objects. It can be proved in theory that the phase difference between two pixel points is close to $\pi/2$, which is conducive to enhancing the accuracy of ellipse fitting.

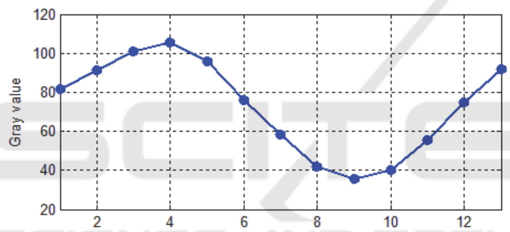


Figure 2: Neighborhood average sequence gray values.

Secondly, for the two selected pixel points, de-noise processing can be conducted by neighborhood average operation for gray values in order to eliminate the random noise.

The sequence gray average values of neighborhood 100 points of a pixel is shown in Fig.2. Compared to Fig.1, the random noise of sequence gray values has been greatly suppressed by observing the variation trend of every point's slope on the curve or the degree of curve smoothing. Fig.2 has shown that the open-loop PZT drive has disadvantage of unequal linear step lengths.

2.3 Solving Method of Ellipse Fitting

The ellipse fitting is mainly to obtain the regression coefficients by linear regression of elliptic equation in a rectangular coordinate system. Then the parameters of interference equations can be calculated according to the relation between the regression coefficients and these parameters.

The elliptic rectangular coordinate equation of interference equations is shown as below:

$$g_{1i}^2 + a g_{1i} g_{2i} + b g_{2i}^2 + c g_{1i} + d g_{2i} + d = 0 \quad (5)$$

Where the parameters a, b, c, d, e can be obtained by regression calculation based on the linear least squares fitting. Through the relation between the parameter equation and coordinate equation, equation (6) can be derived:

$$\begin{cases} \phi = \theta_{20} - \theta_{10} = \pm\cos^{-1}(-a/\sqrt{4b}) \\ C_2 = (ac - 2d)/(4b - a^2) \\ C_1 = -(c + aC_2)/2 \\ A_1 = \sqrt{\frac{C_1(ac-2d)}{4} + \frac{c^2}{4} - e} / \sin\phi \\ A_2 = A_1/\sqrt{b} \end{cases} \quad (6)$$

Where the symbol of phase difference φ is determined by the rotation direction of sequence gray values in the ellipse. The symbol is negative when the direction is counterclockwise, otherwise it is positive. Vector cross product is adopted as a method for judging symbol. The judgment equation is as follows:

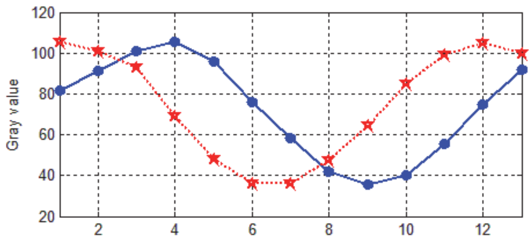
$$(d_x, d_y, d_z) = (g_{11} - c_1, g_{21} - c_2, 0) \times (g_{12} - c_1, g_{22} - c_2, 0) \quad (7)$$

If $d_z > 0$, the symbol is negative, otherwise it is positive.

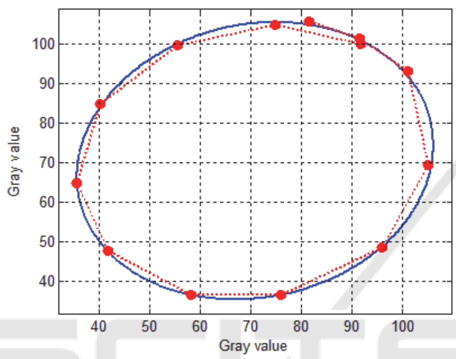
13 sequence gray values of two pixels after de-noise processing and zero mean and ellipse fitting result are shown respectively in Fig.3 (a) and (b). Because the ellipse fitting accuracy of the data points in integral period is higher than that of non-integral period in the same conditions. Therefore, 13 points are used for ellipse fitting.

2.4 Inverse Calculation of Driving Phases and Step Lengths

According to the result of ellipse fitting, the phase calculation of two sequence gray values under certain conditions may be started after getting the parameters of equation (6). The conditions mentioned above are as below: (a) the head and tail points are not used in calculation. (b) Inverse calculation of phases can be conducted when both the difference between gray values of the current point and the front point $g_i - g_{i-1}$ and the difference between gray values of the current point and back point $g_{i+1} - g_i$ are positive or negative. Otherwise calculation is not conducted.



(a)



(b)

Figure 3: (a) Sequence gray values of two pixels after de-noise processing and zero mean; (b) ellipse fitting for sequence gray values of two pixels.

The calculation equations are as follows:

$$\begin{cases} \phi_{1i} = \theta_{10} + \theta_{1i} = \pm\cos^{-1}\left(\dfrac{g_{1i} - C_1}{A_1}\right) \\ \phi_{2i} = \theta_{20} + \theta_{2i} = \pm\cos^{-1}\left(\dfrac{g_{1i} - C_2}{A_2}\right) \end{cases} \tag{8}$$

Where the symbol of phase is determined by the differential symbol of current point. If $g_i - g_{i-1} > 0$, the symbol of phase is negative, otherwise is positive.

The sequence phases and synthetic phases of sequence gray values are calculated in Table 1.

Besides the head and tail points, the points that don't participate in inverse calculation include the 4th, 9th points on the dashed line and the 6th, 12th points on the solid line in Fig.3.

Based on the fitting phase φ= 1.4543, the method for calculating the synthetic phases of the 11 points is as follows: if the phases of two corresponding points in two sequences both exist, the synthetic phase $\phi_i = (\phi_{1i} + \phi_{2i} - \phi)/2$. If the phase only exist in sequence 1, $\varphi_i = \varphi_{1i}$. If the phase only exist in sequence 2, $\varphi_i = \varphi_{2i} - \varphi$. By rounding with 2π, the calculated phase is between ($-\pi$, π). The synthetic phases are shown as the final row in Table 1.

10 actual point by point driving phases are obtained by unwrapping and differential processing for the 11 synthetic phases. The actual driving step lengths of PZT can be got by equation (2). Fig.4 shows that actual step lengths of every point are different from equal step lengths in theory.

3 CONSTRUCTION OF FOUR INTERFERENCE GRAYSCALE IMAGES

The four interference grayscale images with phase difference 90 º can be constructed based on the actual driving phases and step lengths. After ellipse fitting, the calculation of actual phase is effective from the 2th interference image of sampling sequence. Therefore, the 2th interference grayscale image is taken as an initial phase grayscale image g_0 and the rest 3 interference grayscale images with phase difference $\pi/2$ are constructed. The construction method is as follows.

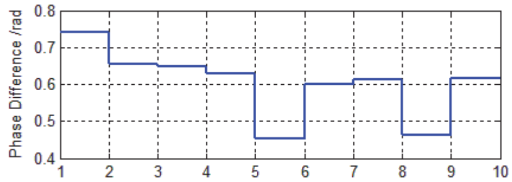
Firstly, unwrapping calculation for the sequence synthetic phases in Table 1 is conducted and the starting point is offset to zero by a simple calculation. The unwrapped phases before and after

Table 1: Sequence phases and synthetic phases.

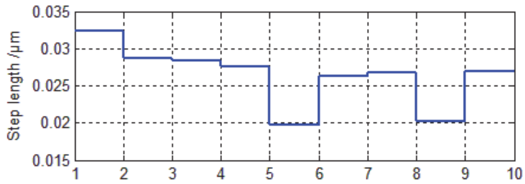
Sequence number	2	3	4	5	6	7	8	9	10	11	12
Sequence phases 1 (rad)	1.032	0.717	*X*	0.892	1.442	1.892	2.387	*X*	-2.445	-1.968	-1.472
Sequence phases 2 (rad)	0.688	0.970	1.594	2.151	*X*	-2.578	-2.170	-1.707	-1.198	-0.753	*X*
Synthetic phases (rad)	-0.899	-0.601	0.139	0.795	1.442	2.071	2.523	3.122	-2.549	-2.087	-1.472

shift are shown in Fig. 5.

Then three points whose phases close to $\pi/2$, π and $3\pi/2$ are selected and the differences ε_1, ε_2 and ε_3 between the three points and its target point are calculated respectively.



(a)



(b)

Figure 4: (a) The real driving phases of PZT; (b) the real driving step length of PZT.

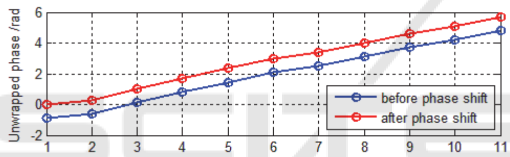


Figure 5: The results of synthetic phases after unwrapping operation.

For example, the three points in Fig. 5 whose phases close to $\pi/2$, π and $3\pi/2$ are the 4th, 6th and 9th point. Radian deviations which take the three points as the center respectively are (-0.5324, 0.1231, 0.7698), (-0.8010, -0.1711, 0.2806) and (-0.6912, -0.0789, 0.3828).

Finally, by using Lagrange parabolic interpolation algorithm the grayscale images of target phase are reconstructed. For example, the equation to reconstruct the grayscale image of $\pi/2$ is as shown in equation (9):

$$g_{\pi/2} = k_1 g_2 + k_2 g_3 + k_3 g_4 \tag{9}$$

Where g_2, g_3 and g_4 represent respectively gray values of the 2rd, 3th and 4th grayscale image.

The reconstruction coefficients k_1, k_2 and k_3 can be calculated by equation (10):

$$\begin{cases} k_1 = \varepsilon_2\varepsilon_3 / [(\varepsilon_1 - \varepsilon_2)(\varepsilon_1 - \varepsilon_3)] \\ k_2 = \varepsilon_1\varepsilon_3 / [(\varepsilon_2 - \varepsilon_1)(\varepsilon_2 - \varepsilon_3)] \\ k_3 = \varepsilon_1\varepsilon_2 / [(\varepsilon_3 - \varepsilon_1)(\varepsilon_3 - \varepsilon_2)] \end{cases} \tag{10}$$

The g_π and $g_{3\pi/2}$ can be got by the same way. The theoretical calculating data shows that the truncation errors of gray values of the three reconstructed grayscale images, which caused by Lagrange interpolation, are less than 0.21%, 0.24% and 0.13% respectively.

4 EXPERIMENTAL RESULTS

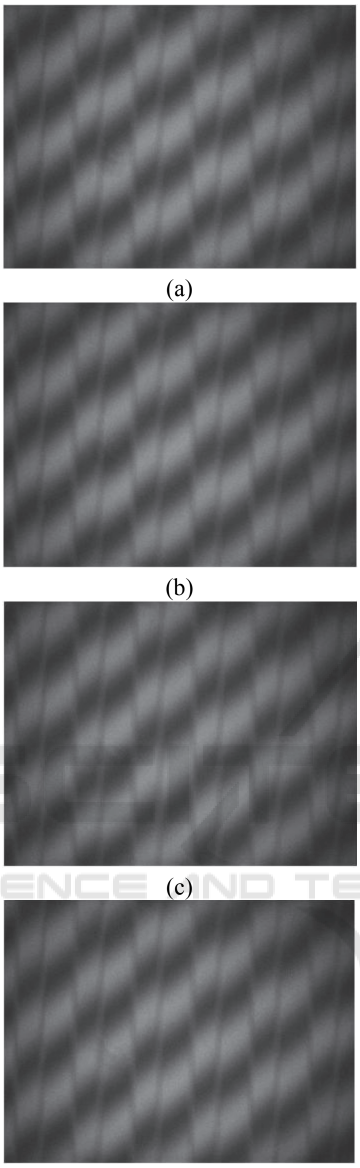
Taking a square wave specimen with multiple grooves as measured surface, multi-wavelength interference grayscales images are sampled and phases of every sequence interference grayscale image is extracted by above methods. Four grayscale image after sequence interference grayscale images which wavelength is 530nm are reconstructed and with phase difference $\pi/2$ are shown in Figure 6.

At last, phase of every point is calculated from the reconstructed grayscale images. The equations for calculation are as follows:

$$\theta_0 = \tan^{-1}\left(\frac{g_{3\pi/2} - g_{\pi/2}}{g_0 - g_\pi}\right) \tag{11}$$

The calculation results of phases of a row in the interference grayscale image are shown in Fig.7. The trend of obtained phases reflects the topography change of square wave specimen with multiple grooves well. Phases are extracted sequentially from the sequence interference grayscale images of 550nm and 640nm wavelength in the same way. For two-wavelength measurement, if the difference of wavelength is small, the measuring range will enlarge quickly, but the measuring accuracy will be very low (Houairi and Cassaing, 2009). So the phase differences between two near wavelength (530nm, 550nm) are applied to recognize the measurement result in a large scale. The phase differences of two far wavelengths (550nm, 640nm) are used to reduce the error of measurement result. At last the phase of single wavelength (550nm) is used to calculate final measurement result and the relative heights of every point in the interference grayscale image, namely the surface topography information, are got (Warnasooriya and. Kim, 2007).

According to above method, the roughness *Ra* of the square wave specimen with multiple grooves is 0.4390μm by calculation. Based on the highest national roughness standard of China, *Ra* of the square wave specimen with multiple grooves is calibrated by China National Institute Metrology is 0.44μm. The expand uncertainty of calibration results is U_{95}=5%. Therefore the relative measurement error δ of the above method is 0.23%.



(a)

(b)

(c)

(d)

Figure 6: Reconstructed four interference grayscale images with $\pi/2$ phase-difference. (a)0º; (b)$\pi/2$; (c)π; (d)$3\pi/2$.

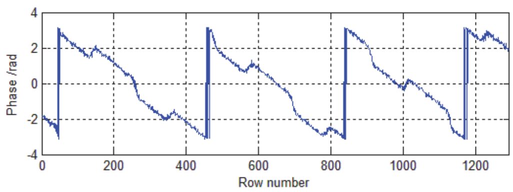


Figure 7: The phase results of a row of interference grayscale image.

5 CONCLUSIONS

The method requires only 12-14 sequence interference grayscale images to complete the whole operation and the amount of processing data is smaller than that of the random driving step lengths algorithm. It does not require strictly equal driving step lengths and simplifies control for PZT in traditional four step method. Through the incidence relation between sequence gray values of two pixels, the solution of driving step lengths is transformed to ellipse fitting problem in the mathematics. The actual driving step lengths and driving phases under open-loop PZT drive are obtained accurately, which provides conditions for constructing the four sequence interference grayscale images with high precision. The method has a guiding significance to related technologies of phase recognition. The relative error of reconstruction of gray values can be controlled within 0.5% easily by using the Lagrange parabolic interpolation algorithm. Therefore, the PZT in the method is open-loop drive. This method has good resistance to local environmental vibration disturbance and the influence of nonlinear driving error of PZT is small. The method reduces the difficulty and cost of driving and close-loop control, and lays a foundation for multi-wavelength switching measurement.

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