# Efficient Large-scale Road Inspection Routing

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Abstract: Gaist Solutions Ltd. carries out large scale surveys for UK road inspection. To estimate the total distance that vehicles travel, we model routing as a Chinese Postman Problem. We propose a novel graph reduction approach that dramatically speeds up the calculation of the Chinese Postman Tour for large-scale road networks. Because the analysis of large road-network graphs is now possible, planners can explore the effects of changes to traditional inspection techniques and scheduling. Case studies of road networks from six UK cities and the county of Norfolk are tested. The graph reduction process is also analysed on ten randomly generated road networks with different characteristics, to show its ability and give advice for suitable use.

## 1 INTRODUCTION

In the UK, local governments maintain roads for public utility and safety. We can model route inspection as a Chinese Postman Problem (CPP). To estimate the total distance that vehicles travel to inspect all routes, a very large-scale CPP must be solved. Published approaches to deriving a Chinese Postman Tour (CPT) are computationally demanding and do not scale to large graphs. Graphs that represent road networks are characterised by vertices with low degree (e.g. T junctions of degree 3 or crossroads of degree 4). Also, roads in residential areas have a strong branched structure. By understanding these characteristics, we propose a graph reduction approach to improve the efficiency of routing large-scale real-world road inspection problem. Our road-inspection routes are for seven large-scale road networks monitored by Gaist Solutions Ltd. in partnership with the UK local councils of Blackpool, Southend, Manchester, Stockport, Halton, Warrington, and the rural county of Norfolk (road lengths of 515, 508, 1,315, 945, 619, 879 and 26,243 kilometres, respectively). We also evaluate our graph reduction approach using simulated networks.

Section 2 introduces road networks and their transformation to abstract graphs. Section 3 outlines CPP solutions and their scalability limitations. The graph reduction process is introduced in Section 4, then the application of the conventional CPT solution and of two variations using heuristics are introduced in Section 5. Section 6 and 7 compare our results over seven local authority road networks and ten simulated data sets. Conclusions are presented in Section 8.

## 2 REAL-WORLD ROAD NETWORKS

UK local authority road networks are designated as 1-lane, 2-lane, 3-lane and 4-lane single carriageways or 2-lane dual carriageways (with a central reservation). We represent a road network as an undirected graph $G(V,E)$. Vertices $V$ represent the data collection points: some are junctions or dead ends, but others are simply bends in the road. Edges $E$ represent the roads that link vertices.

In our inspection problems, up to 3 lanes can be monitored in one pass. Figure 1 shows 1-lane, 2-lane and 3-lane single carriageways represented by a single undirected edge. Figure 2 shows 4-lane single carriageways and dual carriageways represented as two, parallel undirected edges. A close road is represented as a loop (Figure 3), and a cul-de-sac is represented as an degree-1 vertex (Figure 4).

The road information was originally collected street by street, but has some errors or omissions. Before creating the graph representation, we pre-process and label intersections as vertices, as follows.

- All degree-2 vertices are removed, since they do not represent intersections, and thus have no im-



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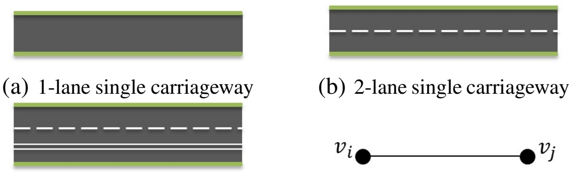
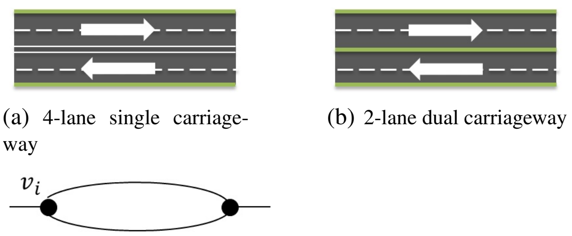


(a) 1-lane single carriageway (b) 2-lane single carriageway (c) 3-lane single carriageway (d) Graph representation

Figure 1: Graph representation of single carriageway roads.



(a) 4-lane single carriageway (b) 2-lane dual carriageway (c) Graph representation

Figure 2: Graph representation of 4-lane single carriageway and 2-lane dual carriageway.

pact on the construction or distance of inspection tours. For all degree-2 vertices, $v_k$, $e(v_i, v_k)$ and $e(v_k, v_j)$ are replaced by a single edge, $e(v_i, v_j)$, with length $l((v_i, v_j)) = l((v_i, v_k)) + l((v_k, v_j))$.

- We assume that an intersection has been omitted if the data indicates that a road terminates close to another road or road-termination point. The critical value for "close", is set at $\varepsilon = 3$ metres, based on our analysis of the original data. So, where the distance from a degree-1 vertex $v_k$ to an edge $e(v_i, v_j)$ is less than $\varepsilon$, the edge $e(v_i, v_j)$ is replaced by two new edges $e(v_i, v_k)$ and $e(v_k, v_j)$. Or, where the distance between two degree-1 vertices is smaller than some $\varepsilon$, the vertices are merged.

Removal of degree-2 vertices significantly reduces the size of the graphs, as shown in Table 1.

# 3 CPP AND THE CHALLENGE OF SCALE

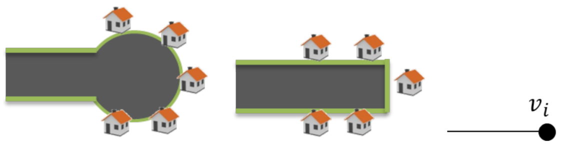
CPP was formulated by a Chinese mathematician: *A mailman has to deliver mail to every road of his service area before returning to the post office. The problem is to find the shortest walking distance for the mailman.* (Guan (1962)).

## 3.1 Edmonds' Approach

To solve a standard CPP, a CPT is derived by adding the smallest possible number of edges to construct an Eulerian graph, which must then contain an Euler tour. Edmonds and Johnson (1973) provide a widely-used CPP approach that is polynomial on the number of vertices and edges, (and does not scale well to large graphs) as follows.

(a) Close (b) Graph representation

Figure 3: Graph representation of close roads.



(a) Bulb (b)Dead-end (c)Representation

Figure 4: Graph representation of cul-de-sacs.

**Step 1:** From an undirected graph $G(V, E)$, find the shortest path between all pairs of odd-degree vertices.

Two algorithms are commonly used in this step. The most efficient implementations of Dijkstra's algorithm (Dijkstra (1959)), a single-source shortest-path algorithm, can achieve $O(m + n \log n)$, where $m$ is the number of edges (Fredman and Tarjan (1987)). Floyd (1962) and others developed an algorithm, now known as the Floyd-Warshall algorithm (FW), of complexity $O(n^3)$ where $n$ is the number of vertices.

Tested on our large-scale sparse graphs, FW systematically outperforms Dijkstra's algorithm. However, the computational efficiency of FW can itself be significantly improved by reducing the number of nodes in a graph, e.g. by applying graph reduction.

**Step 2:** Find the minimum-cost perfect matching, $M$, of odd-degree vertices.

The best-known minimum-cost perfect matching implementation (Gabow (1990)), using the blossom algorithm (Edmonds (1965); Edmonds and Johnson (1973)), is also computationally expensive ($O(n(m + n \log n))$). Kolmogorov (2009) has an implementation which achieves $O(n^2 m)$.

Again, the complexity of the algorithm is dependent on the number of edges and vertices in the graph.

Table 1: Number of vertices before and after removal of 2-degree vertices for seven UK local authority road networks.

| Vertices | Blackpool | Southend | Manchester | Stockport | Halton | Warrington | Norfolk |
|---|---|---|---|---|---|---|---|
| Total | 26302 | 22864 | 45408 | 44470 | 29610 | 24518 | 549345 |
| less degree-2s | 5103 | 4571 | 13337 | 8665 | 4796 | 7471 | 44912 |

**Step 3:** Add extra edges to connect all matched pairs of vertices through the shortest path in $G$, to create an Eulerian graph.

**Step 4:** Find the CPT: an Euler tour in the Eulerian graph. Then, the length, $l_{CPT}$, of the identified Chinese Postman Tour (CPT) is:

$$l_{CPT} = \sum_{e \in E} l(e) + l(M)$$

The best known approach to finding an Euler tour is Hierholzer's algorithm (Hierholzer and Wiener (1873)), with time complexity $O(m)$. Another widely applied approach, by Fleury (1883), achieves $O(m^2)$.

## 3.2 Other Approaches

Apart from Edmonds' CPP solution, Laporte (1997) introduces methods of transforming an arc routing problem into an equivalent TSP. This approach is also used by Irnich (2008) to solve a large-scale real-world postman problem. Heuristics for the TSP can then be used to solve the transformed CPP. For an extensive bibliography of CPP variations and solvers, see (Corberán and Laporte (2015)).

## 3.3 Road Inspection and the CPP Model

An advanced inspection vehicle is able to monitor up to three lanes of a typical UK road in one traversal. Hence, to achieve the road inspection task, each edge in the abstract graph has to be visited at least once. The total travel distance is the length of Chinese Postman Tour (CPT) (Guan (1962); Thimbleby (2003)).

Representing a road network as an undirected graph ignores the necessarily directed nature of 4-lane single carriageways and dual carriageways (Figure 2). However, we generate an Eulerian graph from a given road network, so according to Lemma 3.1 we can always find a CPT that passes complementary directions of parallel edges, and is thus legal for 4-lane single and dual carriageways.

**Lemma 3.1.** *If a graph $G(V,E)$ is eulerian and edge $\{v,w\}$ appears twice in E, then there is an Euler tour of G where $\{v,w\}$ is travelled in both directions $\{v,w\}$ and $\{w,v\}$.*

*Proof.* Suppose that there is an Euler tour $v-w-x_1-x_1-\ldots-x_n-v-w-y_1-y_2-\ldots-y_n-v$, where edge $(v,w)$ is travelled in the same direction both times. Then $v-w-v-y_n-y_{n-1}-\ldots-y_1-w-x_1-x_2-\ldots x_n-v$ is another Euler tour where edge $(v,w)$ is travelled in both directions. □

# 4 REDUCTION OF LARGE GRAPHS

As noted above, the greatest contribution to computational complexity of the algorithms used to derive a CPT is the number of vertices and edges. Our graph reduction applies graph contraction techniques as used in graph minor theory (Chartrand and Oellermann (1993); Lovász (2006)). Edge contraction is a fundamental operation in graph minors which deletes an edge from a graph $G$ and merges the two end points. Here, we propose a novel graph reduction method to decrease the size of the calculations whilst maintaining the necessary characteristics of the original graph to reconstruct a CPT. After the data preparation described in Section 2, each road network is represented as a finite undirected graph which contains parallel edges and self-loops. All degree-2 vertices have been removed.

## 4.1 Reduction Process

Let $V_{even}$ and $V_{odd}$ describe the even-degree vertex set and odd-degree vertex set, respectively, of our graph $G(V,E)$. $l(v_i,v_j)$ represents the length of the shortest edge between vertices $v_i$ and $v_j$. If there is no direct connection between $v_i$ and $v_j$, $l(v_i,v_j)=\infty$. For all vertices $v_i$, $l(v_i,v_i)=0$. The shortest path between $v_i$ and $v_j$ in the graph $G$ is represented as $p(v_i,v_j)$.

We delete vertices systematically, but record the length and location of removed edges incident to degree-1 vertices in a structure, $E^*$. Figure 5 shows how this works on a stylised representation of a road network graph. We make the following observations.

1. Deleting a self-loop from a graph does not change the parity of a vertex's degree.
2. The shortest path $p(v_i,v_j)$ between vertex $v_i$ and vertex $v_j$ does not include any self-loops.
3. The paths $P$ of the minimum cost matching $M(V_{odd})$ include all the edges connected to

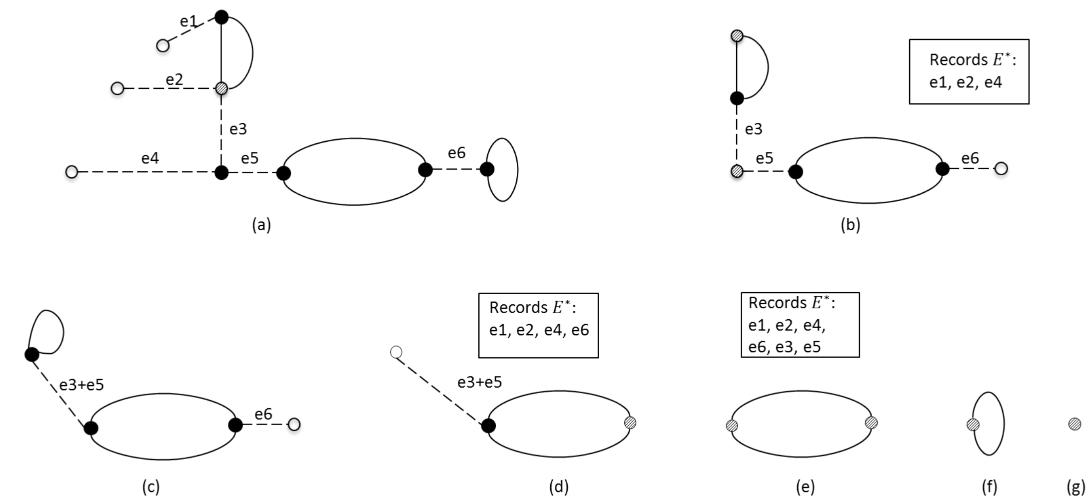



Figure 5: Systematic reduction of an undirected graph. (a) is the graph after removal or degree-2 vertices, with the matching $M_{odd}$ shown in dashed lines. (b) shows the graph after removal of degree-1 vertices, with their originally connected edges recorded in $E^*$. (c)-(g) show the result of repeating these steps – here, the result is a null graph. White nodes have degree 1; striped nodes have even degree and black nodes have odd degree.

degree-1 vertices: if you reach a dead end, then you have to leave the way you entered.

4. If a shortest path between vertex $v_i$ and $v_j$ is $p(v_i,v_j) = (v_i,v_{i+1},v_{i+2},...,v_j)$, then the shortest path between $v_{i+1}$ and $v_j$ is $p(v_{i+1},v_j) = (v_{i+1},v_{i+2},...,v_j)$.
5. When deleting a degree-1 vertex and its adjacent edge, the total number of odd degree vertices is either unchanged or reduced by 2.

From these observations, we deduce:

1: Deleting a self-loop $(v_i,v_i)$ from a graph $G$ will not change the paths in the minimum perfect matching $M(V_{odd})$ of an undirected graph.

2: There is a path set $P$ of the matching $M(V_{odd})$ of the original graph $G$ (dashed lines in Figure 5(a)) that equals the deleted edges $E^*$ incident to degree-1 vertices (as shown by the $E^*$ in Figure 5(b)), plus a path set $P'$ in the new matching $M'(V_{odd})$ of the simplified graph $G'$(dashed lines in Figure 5(b)), such that, $P = E^* \cup P'$.

### 4.2 Further Reduction to Remove Even-degree Vertices

The graph reduction results in a simplified graph $G'$, plus a record of all deleted edges that were incident to degree-1 vertices, $E^*$ in the reduction process. We observe that the FW algorithm used to find the least distance between pairs of odd-degree vertices has complexity exponential in the number of vertices, and graph $G'$ still contains many even-degree vertices which do not affect the shortest path calculation. Thus we further reduce the graph by deleting these even-degree vertices.

Our preferred approach to even-degree vertex deletion detects and deletes all even-degree vertices in $G'$ without affecting the shortest connection and distance between other vertices. Deleting vertices with degree higher than three may increase the total number of edges, but always reduces the total number of vertices. The complexity of deleting each even-degree vertex $v_i \in V_{even}$ is $((m(v_i)(m(v_i) - 1)) / 2)$, where $m(v_i)$ is the number of edges incident to vertex $v_i$.

## 5 CPT DERIVATION

Having achieved a reduced graph, we apply Edmonds' conventional CPT derivation (see Section 3.1), and also two alternative approaches that replace resource-intensive algorithms with heuristics.

### 5.1 Applying Edmonds' CPT Derivation

Our graph reduction approach makes it computationally feasible to use the conventional CPT derivation algorithms (see Section 3.1) on large networks. Because the graph reduction reduces the number of vertices considered, its effects are principally in step 1, the use of FW to derive the shortest path between all pairs of odd-degree vertices.

In step 2, we use the blossom algorithm to find the minimum-cost perfect matching, $M(V'_{odd})$ of graph

Table 2: Labelling of the combinations of the three graph forms and three solvers applied, used in subsequent figures and tables. For details see text. FW = Floyd-Warshall Algorighm (step 1). Blossom, greedy and BFS are used for matching (step 2). GR = Graph reconstruction (step 3). HA = Hierholzer Algorithm to find the CPT (step 4).

| | Steps 1-4 | | |
|---|---|---|---|
| Graph | FW, *blossom*, GR, HA | FW, *Greedy*, GR, HA | *BFS*, GR, HA |
| graph with 2-degree vertices removed | m1 | m2 | m3 |
| … and reduction applied | m4 | m5 | m6 |
| … and all even degree vertices removed | m7 | m8 | m9 |

$G'$. The length of the minimum perfect matchings is represented as $l_{M(V'_{odd})}$. The length $l_{CPT}$ of the CPT of the original graph $G$ is:

$$l_{CPT} = \sum_{e \in E} l(e) + \sum_{e \in E^*} l(e) + l_{M(V'_{odd})} \quad (1)$$

To construct an Eulerian graph from the original graph $G$ (step 3), using Deduction 2, we only need to re-insert the edges recorded in $E^*$ and $M(V'_{odd})$ to the original graph $G$. Thence, we use Hierholzer's algorithm to find the Euler tour (step 4). To generate a CPT in a real world situation, when Hierholzer's algorithm meets a vertex connected to 4-lane single carriageway or dual carriageway edges, priority is given to the edge whose underlying direction is away from this vertex.

## 5.2 Heuristic Alternatives for CPT

We explore potential improvements in computational efficiency of CPT derivation for our large, sparse graphs, by introducing simple heuristic alternatives to the blossom algorithm for minimum-cost matching in step 2, above.

**Greedy method (*greedy*)** is a heuristic approach to constructing a matching where shortest distances between pairs of vertices are known. The greedy algorithm 1, which is run after FW, is based on those by (Kurtzberg (1962)) and (Reingold and Tarjan (1981)).

**Algorithm 1:** Greedy heuristic for matching.

```
VL is a list of all the vertices, v_i in a given graph
while VL contains at least two vertices do
    Choose the pair of vertices with shortest distance
    (v_i, v_j) ∈ VL
    Add (v_i, v_j) to the matching M
    delete v_i, v_j from VL
end while
Return M
```

**Breadth first search (*BFS*)** is a basic search: each unmatched odd-degree vertex $v_i$ in graph $G$ becomes the root of a BFS to find the next unmatched odd-degree vertex $v_j$. The two vertices $v_j$ and $v_i$ are a matching pair and the path from $v_i$ to $v_j$ in the BFS tree is the matching path. The BFS terminates when there are no unmatched odd-degree vertices left. BFS does not need to run FW first, which should significantly improve the computational efficiency of the CPT derivation.

# 6 RESULTS ON REAL-WORLD DATA

Our experiments run each of the variant CPT derivations on three versions of each road network graph: with removal of degree-2 vertices (Section 2), then also with graph reduction (Section 4.1), and finally also with removal of all even-degree vertices (Section 4.2). Table 2 introduces labels $m1 \ldots m9$ for these combinations. The experiments compare the computation time for each combination, on a standard desktop PC: Intel i7-3770 CPU at 3.40 GHz with 24GB memory under the Windows 7 operation system.

Figure 6 plots the CPU time taken for each experiment. For each road network, the graph reduction and graph reconstruction times are negligible, and do not show up on this scale. In all network cases, the graph reduction (m4 – m9) shows significantly lower overall CPU time. The major contributions come from the time saving in the process of finding the shortest path (using FW) and the minimum cost matching. This result demonstrates the importance of our graph reduction approach for sparse graphs of this scale.

The results show that, having achieved computational feasibility through graph reductions, the exact method, using the blossom algorithm for matching, outperforms the heuristic approaches. This result is useful since the blossom approach guarantees to find the optimal CPT, whereas the heuristic approaches typically find a close but suboptimal solution.

Table 3 gives numerical results for m1, m4 and m7 (the blossom-based solvers) on the three versions of the graphs. The results again emphasise the ef-

Table 3: CPU time taken to calculate a CPT for each road network using the blossom algorithm matching. The lower right panel gives total CPU time for each road network and each experiment. Column *n* is the number of vertices (cf. Table 1); *m* the number of edges in the graphs after data cleaning. RT is CPU time for graph reduction. FW is CPU time for the FW algorithm. MT is CPU time for the blossom algorithm. CT is CPU time to construct the final graph. CPT is CPU time to extract the final inspection route using the Hierholzer algorithm.

| | **Blackpool** (B) | | | | | | | **Southend** (SO) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n | m | | | CPU (s) | | | n | m | | | CPU (s) | | |
| | | | RT | FW | MT | CT | CPT | | | RT | FW | MT | CT | CPT |
| m1 | 5103 | 7124 | 0 | 143.32 | 28.97 | 0.016 | 0.187 | 4571 | 5738 | 0 | 189.84 | 31.14 | 0.001 | 0.202 |
| m4 | 3398 | 5419 | 0.14 | 45.54 | 11.11 | 0.016 | 0.234 | 2016 | 3162 | 0.14 | 16.05 | 4.26 | 0.001 | 0.203 |
| m7 | 2772 | 10803 | 0.51 | 28.84 | 10.34 | 0.016 | 0.016 | 1746 | 5922 | 0.28 | 10.51 | 4.25 | 0.001 | 0.202 |
| | **Halton** (H) | | | | | | | **Stockport** (ST) | | | | | | |
| | n | m | | | CPU (s) | | | n | m | | | CPU (s) | | |
| | | | RT | FW | MT | CT | CPT | | | RT | FW | MT | CT | CPT |
| m1 | 4796 | 5430 | 0 | 134.38 | 39.44 | 0.002 | 0.187 | 8665 | 10482 | 0 | 910.62 | 114.21 | 0.000 | 0.671 |
| m4 | 1211 | 1852 | 0.15 | 2.96 | 1.97 | 0.001 | 0.218 | 3277 | 5063 | 0.45 | 55.91 | 12.46 | 0.006 | 0.826 |
| m7 | 1148 | 1970 | 0.16 | 2.53 | 1.96 | 0.002 | 0.218 | 3004 | 5662 | 0.53 | 40.86 | 11.79 | 0.003 | 0.858 |
| | **Warrington** (W) | | | | | | | **Manchester** (M) | | | | | | |
| | n | m | | | CPU (s) | | | | | | | CPU (s) | | |
| | | | RT | FW | MT | CT | CPT | n | m | RT | FW | MT | CT | CPT |
| m1 | 7471 | 8556 | 0 | 343.18 | 95.32 | 0.001 | 0.499 | 13337 | 16500 | 0 | 4438.04 | 355.83 | 0.013 | 1.58 |
| m4 | 2136 | 3218 | 0.29 | 10.31 | 5.84 | 0.000 | 0.561 | 5842 | 8949 | 1.02 | 369.71 | 47.75 | 0.015 | 1.79 |
| m7 | 2108 | 3265 | 0.31 | 9.78 | 5.83 | 0.001 | 0.561 | 5486 | 9622 | 1.12 | 306.29 | 46.71 | 0.015 | 1.67 |
| | **Norfolk** (N) | | | | | | | Summary | | | | | | |
| | n | m | | | CPU (s) | | | | | | Total CPU(s) | | | |
| | | | RT | FW | MT | CT | CPT | B | SO | H | ST | W | M | N |
| m1 | 44912 | 53482 | 0 | - | - | - | - | 172 | 221 | 174 | 1025 | 439 | 4795 | - |
| m4 | 15647 | 23904 | 18.27 | 4977.19 | 5499.49 | 0.171 | 34.523 | 57 | 21 | 5 | 70 | 17 | 420 | 10530 |
| m7 | 14796 | 26487 | 19.66 | 4227.01 | 4759.01 | 0.182 | 34.881 | 49 | 15 | 5 | 54 | 16 | 356 | 9041 |

Table 4: CPT length of seven road networks.

| | **Blackpool** | **Southend** | **Halton** |
|---|---|---|---|
| **l(CPT)** | 671.60km | 676.02km | 917.16km |
| | **Stockport** | **Warrington** | **Manchster** |
| **l(CPT)** | 1369.60km | 1300.92km | 1839.09km |
| | **Norfolk** | | |
| **l(CPT)** | 18268.84km | | |

fect of graph reduction in reducing CPU time. These methods calculate the optimal CPT of the original graph, and the distance that vehicles should travel for road inspection. The CPT length for each network is shown in Table 4.

## 7 SIMULATED DATA EXPERIMENTS

To explore the effect of graph reduction further, we conducted a set of experiments on randomly generated graphs, using the blossom-based solvers (as in m1, m4 and m7).

The random graphs are generated, using an algorithm proposed by Blitzstein and Diaconis (2011), with a fixed number (1000) of vertices, and with specific vertex-degree distributions. As shown in Table 5, we also record average vertex degree of the generated graphs before and after the initial removal of degree-2 vertices. Three parameter settings of random graphs (g1, g2, g3) are generated with similar vertex-degree distribution to the graphs representing the real-world road networks before any data processing. A further five parameter settings of random graphs (g4 – g8) have similar vertex-degree distribution to the graphs of real-world road networks after removal of degree-2 vertices. There are also two parameter settings of random graphs that are dominated by degree-4 (g9) and degree-5 (g10) vertices. In the random generation, all edge lengths are set to one.

For each graph, we run the blossom-based solvers with progressively greater reductions (as for m1, m4 and m7, above). For each experiment, we report the average of 30 runs, a number selected to give acceptable total run time but a suitably-low statistical error.

Table 6 presents details of the CPU time taken for the overall CPT identification process, and then for the three levels of graph reduction, on each graph.

Each graph reduction makes a big contribution to the CPU time reduction. The results for the first group of graphs, those generated to match the vertex degree distribution of the complete road network graphs,

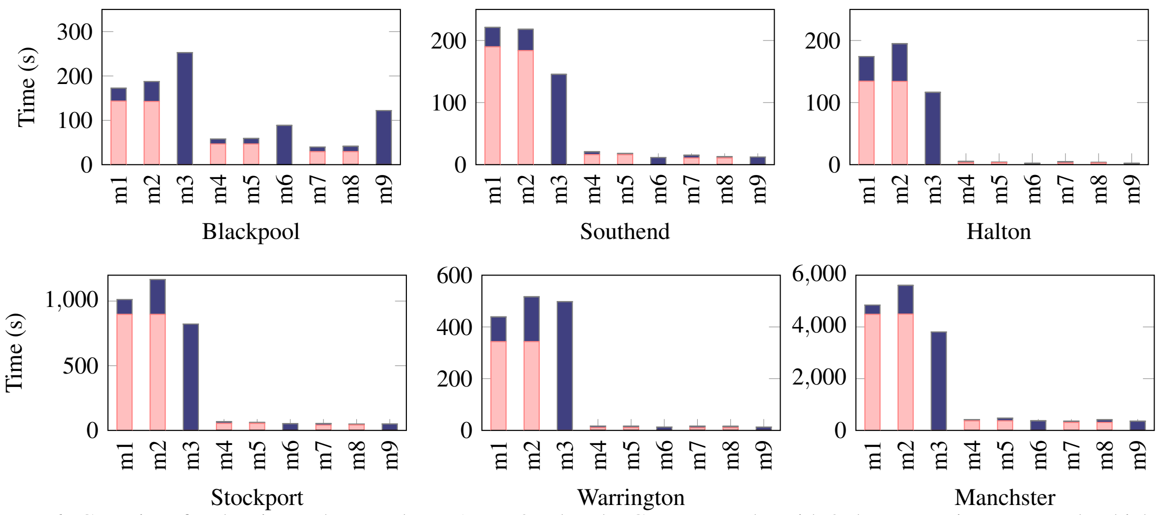



Figure 6: CPU time for the six road networks. m1 to m3 solve the CPP on graphs with 2-degree vertices removed, which is the standard approach widely used for network representation. m4 to m6 solve the CPP on graphs after our graph reduction process, and m7 to m9 solve the CPP on further reduced graphs with all even degree vertices removed. See table 2 definitions.

Table 5: Vertex degree distributions for generated graphs. All generated graphs have 1000 vertices and edges of length 1 only.

| | **parameters** | | | | | | |
|---|---|---|---|---|---|---|---|
| | **vertex degree** | | | | | **average degree** | |
| **Graph structures** | 1 | 2 | 3 | 4 | 5 | initial | no d-2 |
| degree distribution match: | | | | | | | |
| **g1** | 15% | 70% | 12% | 3% | - | 2.03 | 2.1 |
| **g2** | 10% | 70% | 15% | 5% | - | 2.15 | 2.5 |
| **g3** | 5% | 80% | 10% | 5% | - | 2.15 | 2.75 |
| degree distribution with no degree-2 vertices: | | | | | | | |
| **g4** | 35% | - | 60% | 5% | - | 2.35 | 2.35 |
| **g5** | 35% | - | 50% | 15% | - | 2.45 | 2.45 |
| **g6** | 35% | - | 40% | 25% | - | 2.55 | 2.55 |
| **g7** | 35% | - | 40% | 10% | 15% | 2.7 | 2.7 |
| **g8** | 35% | - | 40% | 5% | 20% | 2.75 | 2.75 |
| dominated by high-degree : | | | | | | | |
| **g9** | 20% | - | 15% | 60% | 5% | 3.3 | 3.3 |
| **g10** | 20% | - | 15% | 5% | 60% | 3.85 | 3.85 |



show the importance of removing degree-2 vertices. A further large time saving occurs when the graph reduction process (Section 4) is applied. Comparing the improvement in CPU times between the full graph reduction (last column of Table 6) and only removing degree-2 vertices (middle column total), the first group of graphs show CPU time savings of 95.5%, 90.3% and 78.5%, respectively.

The graphs in group g4 – g8 (and g9, g10) are generated without degree-2 vertices. For these graphs, the graph reduction steps also show a very large CPU time reduction.

For the graphs dominated by higher-degree vertices, g9 and g10, the results again show that graph reduction leads to time reductions. However, for g9 which is dominated by degree-4 vertices (60% of all vertices), the further reduction step, to remove all even vertices, is by far the most computationally expensive step (three times the CPU time for just removing degree-one vertices), making the reduction more expensive than solving the unreduced graph. In this case, the last column of Table 6 shows that there is a very large CPU time overhead for deleting the significant number of degree-4 vertices and any other higher even-degree vertices. Hence, only applying the reduction process in section 4.1, is the most efficient approach in this situation. In contrast, the graphs dominated by degree-five vertices, g10, show a pattern that is consistent with the graphs that are generated to have similar degree to the real road network graphs.

Table 6: CPU time for CPT tour extraction on generated random graphs g1 – g10, showing the effect of graph reductions. Lowest overall CPU times for each graph type are in emboldened.

| | **CPU(ms)** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Graph structure** | **FW, blossom GR, HA** | **removing degree 2** | | **and reduction applied** | | **and removing all even degree** | |
| | | reduction | total | reduction | total | reduction | total |
| **g1** | 19691.82 | 10.66 | 577.56 | 11.61 | 29.49 | 12.07 | **25.91** |
| **g2** | 19616.67 | 10.21 | 568.72 | 11.36 | 86.71 | 12.93 | **54.86** |
| **g3** | 19522.89 | 11.30 | 183.39 | 11.75 | 60.01 | 15.18 | **39.37** |
| **g4** | 20464.62 | - | - | 6.34 | 1831.37 | 7.76 | **1436.3** |
| **g5** | 20312.60 | - | - | 6.22 | 2086.82 | 14.64 | **1062.11** |
| **g6** | 20582.34 | - | - | 5.55 | 2528.90 | 863.95 | **1666.97** |
| **g7** | 20433.91 | - | - | 4.88 | 2780.43 | 13.07 | **1414.97** |
| **g8** | 20359.70 | - | - | 4.87 | 2810.04 | 10.99 | **1622.76** |
| **g9** | 20339.90 | - | - | 3.88 | **8468.91** | 248918.2 | 249369.4 |
| **g10** | 20407.10 | - | - | 3.68 | 9209.61 | 17.08 | **4992.87** |

# 8 CONCLUSIONS

To estimate the total distance that vehicles need to travel for road inspection tasks, we model this problem as a CPP. We present a systematic method for processing raw data from road surveys, applicable road networks in general, and show further abstract graph representations. We propose a graph reduction technique that dramatically speeds up the calculation of CPT of very large-scale road networks.

Our graph reduction technique is notably helpful on sparse graph like real-world road networks; even the graph of a UK county road network is amenable to CPT route calculation on a standard PC. For road networks in residential areas, the reduction helps to manage the many branch roads, close road and cul-de-sacs which lead to complex graph structures.

Our approach is applicable to any road-tour derivation, such as large-scale door-to-door delivery or monitoring services and rubbish collection; the efficient computational approach can facilitate experimentation with innovative policies and techniques for route planning.

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