Flow Map of Products Transported among Warehouses and Supermarkets

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Abstract: Representing large amounts of data using flow maps involves dealing with the reduction of visual cluttering. This article presents a method for generating flow maps and visualizing products being transported from warehouse to supermarkets in a major retail company in Portugal. Our approach uses a swarm-based system to reduce visual clutter, bundling edges in an organic fashion and improving clarity. Additionally, the Dorling cartograms technique is applied to reduce overlapping of graphical elements that render locations in geographic space. Finally, different design decisions enable a multi-perspective visualization of the same dataset.

1 INTRODUCTION

Flow maps are a technique used to show the movement of objects from one location to another, such as people migration, the amount of goods being traded, amounts of products being transported from warehouses to supermarkets, etc. Flow maps say little or nothing about the pathway, but include the information about what is flowing (moving, migrating, etc.), the direction of flow, and how much is being transferred. In most cases, the data is represented using line width, line color and spatial properties. Flow maps are advantageous in what regards to visual clarity and ease of visual communication. This is achieved by merging edges that share similar destinations, or in some cases by tracing them through a similar path. However, this technique often fails when applied on large amounts of flow data. The visualization might become cluttered, making the map difficult to read, and difficult to distinguish the grouped individual streams.

In this work we describe a method for the generation of flow maps that is able to depict large amounts of transitions from one location to another (further expressed as *Origin-Destination* or simply *OD*). This method uses a customized swarming system to trace edges in an intuitive and organic fashion, and to reduce visual clutter. Our method employs graphic design decisions to promote clarity of visual communication in a high density environment. In order to improve clarity of representation of geographic locations, a technique, known as Dorling cartograms (Dorling, 1996), was applied. With this technique, overlapped points were separated retaining some degree of spatial relationship. Finally, our technique supports mixed types of points – geo-referenced data points and those that have no fixed position in space.

This work tackles the issue of depicting large amounts of products being transported from warehouses to hyper and supermarkets of a major retail company in Portugal. Our dataset has approximately 15 to 90 millions of warehouse-to-supermarket transitions per day over a time span of 6 months. The locations consist of approximately 60 warehouses, major part of which are located outside Portugal, 1039 supermarkets in Portugal and 230 supermarkets outside the Portugal, including the geo localization of 680 supermarkets.

In the following sections our approach is described in more details. Section 3 presents the underlying idea and the method in detail. Section 4 describes an application of this technique on the given dataset. Finally, section 5 presents a comparison of the results obtained by our approach and the existing one.

2 RELATED WORK

Direct visualization of large amounts of Origin-Destination transitions can generate high degrees of visual clutter. In these cases a reduction strategy

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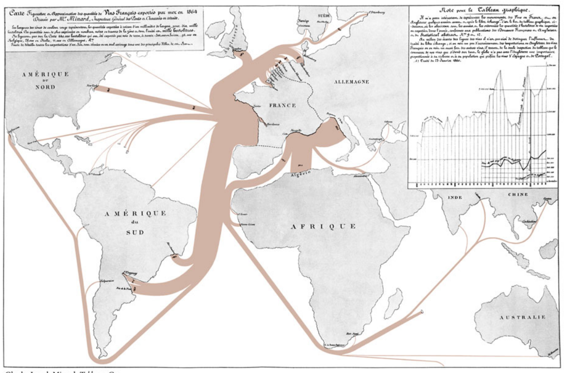

Charles Joseph Minard, *Tableaux Graphiques et Cartes Figuratives de M. Minard*, 1845–1869, a portfolio of his work held by the Bibliothèque de l'École Nationale des Ponts et Chaussées, Paris.

Figure 1: Export of French wine by Charles Minard, 1864.

known as edge bundling can be applied. This is characterized not only by graph simplification, but also by the revelation of the principal streams of flow. Holten introduced edge bundling for compound graphs (Holten, 2006). His work consisted in routing edges through a hierarchical layout using B-Splines. Nowadays, there are several variations of edge bundling such as force-directed edge bundling (Holten and van Wijk, 2009), or sophisticated kernel density estimation strategies (Hurter et al., 2012). Generally, edge bundling consists of drawing similar edges on the same path, i.e. edges that are related in geometry and direction are routed along the same path.

In the geographic context an Origin-Destination representation, as a rule, refers to the flow visualization (also known as flow maps), which is deeply rooted in the history of information visualization. Early examples, such as wine exports from France, produced by Minard (Tufte, 1983, page: 25), represent quantity as well as direction of wine exports encoded by the thickness of the corresponding edges, which disjoin from the parent edge (see Figure 1). The work of Phan et al. (2005) presents an automated approach to generate flow maps using a hierarchical clustering algorithm, given a series of nodes and flow data. Generally, in geographic context, a flow map depicts quantities of any type of objects that move from one location to another – e.g. migrations, transportation of goods, etc. The advantage of flow maps is that they reduce visual clutter by merging edges. However, when representing large amounts of data this technique presents a series of problems, such as poor perception regarding the directionality of flow, high degrees of visual clutter, overlapping of graphical elements that represent locations.

Another important characteristic of this work is the focus on nature-inspired approaches. The underlying idea is based on self-organizing system, and more precisely on the phenomenon of emergence in such systems. As the term indicates self-organization is a process in complex systems, in which the structure or organization appears without any explicit interference from outside. Self-organizing processes often result in the occurrence of emergent phenomena. More precisely, when the complex structure or behavior appear due to the interaction of a collective of individuals, which were not programmed for that (Di Marzo Serugendo et al., 2011). In the field of data visualization, there are techniques of graphical representation that are based on such systems. For instance *Geoboids* (Macgill and Openshaw, 1998) employs a method to reveal patterns in spatial data through the use of customized flocking system. In this system, each, so called geoboid explores geographic space in accordance with the simple rules of interaction with other geoboids and the data found nearby. The visualization, which emerges from this simple process, shows areas containing interesting information. Another series of works by Vande Moere exploit self-organization and emergence in information visualization. He introduces the idea of *infoticle*, which designates a particle that responds to data values and static forces in a particle system (Vande Moere et al., 2004). The visual output portrays the Internet file usage of a medium-sized company over time, conveying the patterns of file downloads. Another nature-inspired approach is the *information flocking visualization* (Moere, 2004). In this work, Vande Moere uses an artificial flocking system, originally proposed by Reynolds (1987), where the forces of attraction and repulsion are modified proportionally to the similarity between the data objects that each boid encodes. The emergent patterns analyzed in a higher-level, where each composition portrays short-term and long-term data tendencies in time-varying dataset, conveying meaningful changes over time.

3 FLOW MAP FLOCKING MODEL

In order to reflect the flowing nature of the data we resort to a flocking system. The underlying model to construct our flow map shares common characteristics with the work of Polisciuc et al. (2015). The visualization itself can be seen as a directed graph composed by nodes and edges. The system consists of artificial agents (further referred to as *boids*) each one tracing an individual Origin-Destination edge. A boid is characterized by its position in space, direction and speed. During the simulation each boid leaves per-

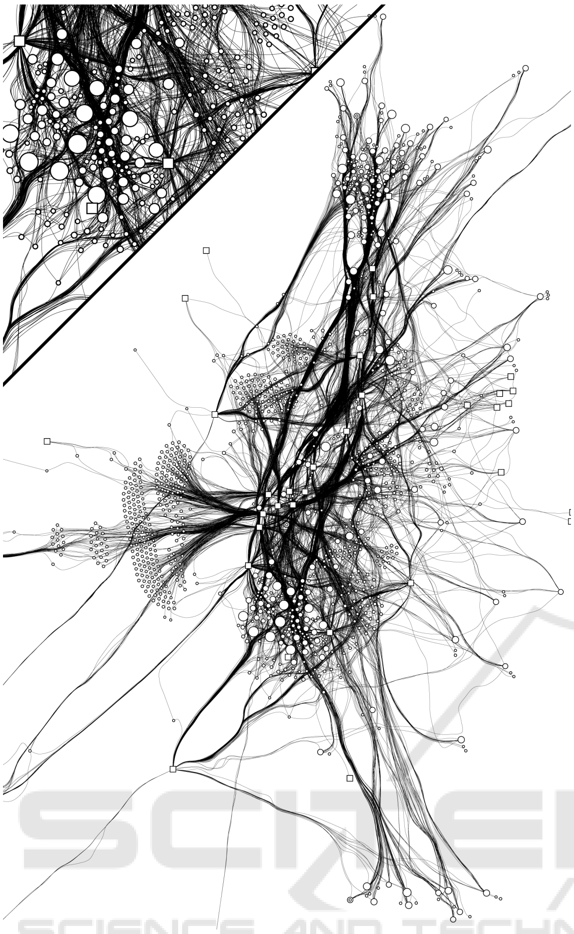

Figure 2: Visual output from the system after 5 full cycles, image at the bottom. Detail of the computed traces in black, image at the top. The rectangular and circular nodes represent origin and destination of the flow, respectively.

sistent traces, further referenced as ghosts, which inherit the location at each simulation instance, direction and the edge being encoded. Since the process is asynchronous, i.e. each trace is computed separately, the boids in the system interact only with the ghosts instead of other boids. While interacting with the ghosts, each boid follows simple rules: *attract* to friendly ghosts; *repel* from the unfriendly ones; and *avoid* static points, which are the nodes of the graph. In order to determine the relationship between the boids and the ghosts we used a pairwise similarity measures between edges, including geometric properties and weight of edges. The output of the system is shown in Figure 2.

In this section we describe our flocking system; our similarity metrics; implementation and optimization considerations; and the graphical variables used in this flow map.

3.1 Model

As previously mentioned, our model consists of a set of boids characterized by location in space, direction, speed and the field of vision. The boids move in space reacting to the presence and characteristics of other neighboring boids. A pairwise interaction between boids and ghosts determines their behavior. If the agents encode similar edges, they are considered *friendly*. If the agents encode dissimilar edges, they are considered *unfriendly*. Otherwise, they ignore each other. The degree of similarity, which is described in the following sub section, affects the force of attraction or repulsion between agents and ghosts. Therefore, friendly agents advance together as a group and unfriendly agents repel from each other avoiding collisions.

Each trace is computed individually, starting at the origin node and finishing at the destination node. The computation of each trace only starts when the previous one has finished, and so forth for all the traces. In each iteration the boid's paths are updated according to the current state of the system. More precisely, during the execution cycle each boid interacts with the ghosts left by other boids and never with their own ghosts. The process repeats until the visual result is acceptable and the user decides to stop it. During the computation of each trace the acceleration vector and the speed of the boid B with position $\vec{p_B}$ is determined by the characteristics of each ghost G within the field of vision VF_B. The acceleration vector is computed as following: i) compute a vector relative to the ghosts; ii) compute a vector relative to the static points; iii) compute a normalized vector pointing towards the destination (see section 3.3 for implementation details). Having all the three vectors computed, they are weighted, and then added to the acceleration vector; the acceleration is added to the speed vector; and the speed is limited to the predefined maximum and is added to the current location. The maximum defined speed reflects on the visual output resulting in high and low curvature of edges for speed limited to 1 and 5 (in relative spatial units), respectively.

3.2 Edge Similarity

Edge similarity (further referred to as similarity score or simply score) is calculated on the input graph taking into account geometric properties of edges. The output from pairwise calculation is then stored in nxn matrix, where n is the number of edges. The score consists of the following three components:

Angle. The boids implement different behaviors and the angle between $\vec{OD}$ vectors dictates whether boids are friendly or not. The angle $\alpha \in [0,\pi]$ that the two vectors make is linearly mapped to the range $[-1,1]$. The sign translates directly into the direction of the force during the interaction between boids.

Distance. Determining the minimum Euclidean distance between two line segments is a typical problem in areas dealing with geometric data. We used an algorithm proposed by Lumelsky (1985), which takes as input the coordinates of the end points of the two segments and outputs the distance. This algorithm is efficient, since the distance is not computed until the endpoints do not satisfy certain condition. This is, the endpoints are not the closest points and the segments do not intersect. In order to translate the distance to the range $[0,1]$ the following function was used: $k/(d+k)$, where d is the distance and k is a constant, in our case empirically defined as 10.

Length difference. The similarity between edges is proportional to the absolute difference between their lengths. In other words, the edges that have equal lengths are considered similar and vice versa. In the end, the values are normalized in order to equalize scaling. Formally, let l and p be the lengths of the pairs of edges, we calculate $1-(|l-p|/max(l,p))$.

In order to calculate the final score as a measure of similarity, we apply the following function: $(distanceScore + lengthScore) \times angleScore$. As can be seen, the angle score is prioritized, again because of the behavior. In other words, if two edges are parallel and pointing towards the same direction, the length and the distance scores come into calculation. In this case, if the lengths are similar and the distance between segments is small the boids that encode these edges have high attraction forces and route the traces through similar paths.

3.3 Implementation and Optimization

This sub-section describes each step of the computation of the acceleration vector of a boid in detail provided in pseudo-code. First, a vector for a boid B relative to the *ghosts* is computed as following:

```
Integer c = 0
for each G in VF:
   Float s = similarity value for G

   if s >= 0:
     Vector bg = vector from B towards G
   end if

   if s < 0:
     Vector bg = perpendicular to direction of B
     // left or right perpendicular depends
     // on the position of G in relation of B
   end if
   normalize bg
   multiply bg by s
   add bg to v
   c = c + |s|
end for each

divide v by c
return v
```

Second, a vector for a boid B relative to the *static points* is computed as following:

```
Integer c = 0
for each static point SP:
  Float dd = radius of SP
  Float d = distance from B to SP

  if d < dd:
    Vector spb = vector from SP to B
    normalize spb
    multiply by (dd - d)
    add spb to v
    c = c + 1
  end if
end for each

divide v by c
return v
```

Finally, when the boid approximates its destination, all the forces, except the destination force, are ignored and the speed is limited to 1. This restriction ensures that each boid reaches its destination.

In order to reduce the computation cost we applied a hierarchical structure of spatial data – more precisely – a *quadtree*. This type of structures is based on the principle of recursive decomposition and smart subdivision of spatial data (Samet, 1984). This hierarchical representation is useful because it focus on interesting subsets of data, and is efficient in execution times and performance of getting and setting operations. We used the quadtree to store and access ghosts of boids.

In our implementation, the *get* operation is straight-forward. At execution time, we access ghosts that are inside the region that is defined from the field of vision of the boid being computed. Then we only iterate over the elements that are within this field. The *set* operation is conditional and is performed as following: first, check if in a certain range there are no existing ghosts; if this condition is true, then update their positions and the direction vectors; otherwise, add a newly created ghost to the quadtree.

Finally, each ghost is updated taking into account the data value they encode. The idea is the following: the ghosts that encode large values have greater impact over the ones that encode smaller values. Therefore, making the traces that have less impact follow the ones that have bigger impact. The following pseudo-code exemplifies the function that updates the existing ghosts:

```
Float dv = normalized data value of G0
for each G in VF of G0:
  // update position
  Vector gg0 = vector from G to G0
  normalize gg0
  divide gg0 by count G in VF of G0
  multiply gg0 by dv
  add gg0 to position of G

  // update direction vector
  Vector v = direction vector of G0
  multiply v by dv
  add direction vector of G to v
  normalize v
  set v to direction vector of G
end for each
```

4 APPLICATION

In this section we describe a visualization application of our method for the flow map. We apply this technique to visualize the flow of products among warehouses and supermarkets. In other words, the visualization depicts movements of stocks of products inside a major retail company in Portugal.

4.1 Data Description

Our dataset consists of approximately 15 to 90 millions of warehouse-to-supermarket transitions of product per day over a time span of 6 months. Each transition has the following attributes: product id, quantity of products in transit (further referenced as *stock in transit* or *SIT*), quantity of products delivered (further referenced as *stock on hand* or *SOH*), warehouse id, supermarket id, and the date of transition. The locations consist of approximately 60 warehouses, some of which are located outside the Portugal, 1039 supermarkets in Portugal and 230 supermarkets outside the Portugal, including the geo localization of 680 supermarkets (see also (Polisciuc et al., 2015; Maçãs et al., 2015a,b)).

In order to get a graph representation of the data we proceeded to the calculation phase. First, the data were aggregated by days. Each day sums-up SIT quantities by aggregating pairs of warehouse-supermarket locations, which constitute the edges and the nodes of the graph. Finally, the total of SOH quantities per supermarket is calculated. Therefore, we get a weighted directed graph whose edges are directed from warehouse to supermarket nodes and weighted by SIT quantities. The nodes that represent supermarkets have assigned SOH quantity, while the nodes that represent warehouse have none.

4.2 Flow of Products Visualization

This visualization depicts amounts of products that have been transported and the amounts of products that have been delivered during one day. It is important to describe the process to get a readable layout of mixed graph. There are two challenges to visualize this particular graph: not all the locations have a geographical positions; but, the ones that do have, can overlap. The first issue was solved by fixing the nodes that have a geographical location, and by using force-directed graph layout algorithm (Fruchterman and Reingold, 1991) to compute the location of other ones. This algorithm is efficient in what concerns about the graph topology, since it considers clusters of nodes and not individual nodes. The second issue was solved by applying Dorling cartogram technique over the precomputed layout. The beauty of this algorithm is that it preserves original relative positioning of geo-referenced elements. This enables the map of Portugal to be recognizable, making the locations distinguishable and increasing visual clarity (see Figure 3).

In the visualization each trace represents the quantity of products in transit from warehouse to supermarket. These quantities are encoded by two means – *color* and *line thickness*. The two graphical elements are complimentary and make use of different mappings. The color uses a linear mapping. The values are mapped to a pale yellow-purple-dark blue gradient, being pale yellow and dark blue for lowest and highest values, respectively. The thickness variable, on the other hand, uses an exponential scale, emphasizing high values. Finally, we use an arrow, which is rendered at the end of a trace and scaled proportionally to the thickness of the line, in order to represent the direction of the flow (see Figure 3). The orientation of the arrow is determined in three steps. First, the length l and the with w of the arrow is computed. The w and l are equal to the 2x and 3x thickness, respectively. Second, the last vertex v of a trace that is located at the minimum distance from location of the node c plus its radius and the l is determined. Third, the arrow orientation is defined by the vector pointing from v to c.

In order to represent different types of nodes we use different graphical elements – *squares* and *circles* to represent warehouses and supermarkets, respectively. The nodes are colored according to country they represent. Since our data contains a high number of countries, the color could become ineffec-

Figure 3: For the sake of simplicity this image shows only transitions from one warehouse. The whole graph can be seen in the Figure 7.

tive. In this case, we calculate the total number of locations, aggregating by country, sort the countries in descendant order, and consider only the first two countries, which are Portugal and Spain, and treat the others as an unique instance. The colors used was green, red and blue for Portugal, Spain and others,

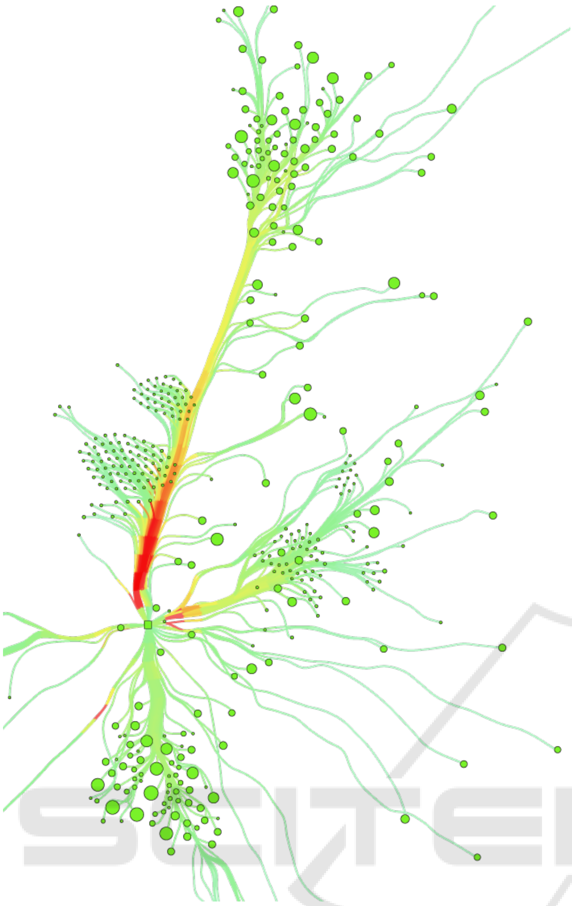

Figure 4: A visualization of edge overlapping. The number of overlapped edges is represented with the color from pale green to intensive red. This image shows only the transitions from a selected warehouse.

respectively. Finally, the nodes that have geographic reference were rendered with the graphical shape of an X in the center of the node (see Figure 3).

Due to the high degree of traces overlapping, it is hard to identify main streams. For that reason, we proceed with a color-temperature visualization of the flow map. Using the same graph we apply different graphical elements. In this visualization the traces are colored according to the total number of overlapping elements. More precisely, we compute the number of overlapped segments that build-up the traces located within a defined range. Then at the render instance this value is mapped to a color scale from *intensive red* to a *pale green*, where red and green mean high and low degrees of overlaps. This representation is useful to get another perspective of such complex visualization (see Figure 4).

5 RESULTS

The visualizations shown in this section depicts our data. All the visualizations that we generated use the technique described in section 4, except in the line thickness. To facilitate the comparison in this small scale we reduced visual complexity: lines with constant thickness; no arrows at the end of the traces; nodes colored in white.

The very first comparison reveals the efficiency of visual clutter reduction. As can be observed, the force directed edge bundling (FDEB) method (see Figure 5, image in the middle) generates less visual clutter in comparison with our approach. When using swarms, main streams of flow are visually distinct from each other leaving enough space for the ones with less impact. In addition our approach considers edge weight, therefore, resulting in meaningful representation of data being visualized (see Figure 5).

In our approach each boid attempts to avoid the boids with opposite directions, as such the traces are never routed through the same trail. This enables separation of streams that encode opposite directions. In contrast, this is not the case when using the FDEB. This type of algorithms does not take into account the directionality of streams, which is an emergent characteristic of the swarm-based approach. Finally, since the boids in the system attempt to avoid static points, the nodes are clearly visible and do not visually interfere with the lines (see Figure 5, image on the left).

As previously mentioned, our method is sensible to the parameters of maximum speed and the field of vision of boids. The bigger the field of vision the less bundled the edges. Also, main streams tend to aggregate more edges comparing to the lower values. Maximum speed, on the other hand, translate into the "waviness" of traces. Using bigger values boids tends to draw traces in a more organic manner. However, there is a loss in detail of trace and overall visualization. Figure 6 (image in the middle) displays the visual output using maximum speed 5 and radius of vision field equal to 50 degrees.

Finally, our approach gives diverse perspectives over the same dataset. Depending on the sorting order the visualization can emphasize streams that represent low or high values (see Figure 6, image on the left). Additionally, our approach enables a visualization of the density of streams by applying an appropriate color scheme described in section 4.

Figure 5: Comparison between the techniques straight lines (left), FDEB generated using 5 cycles, 50 iterations and stiffness 10 (image in the middle), our approach using SIT quantities in calculations (right).

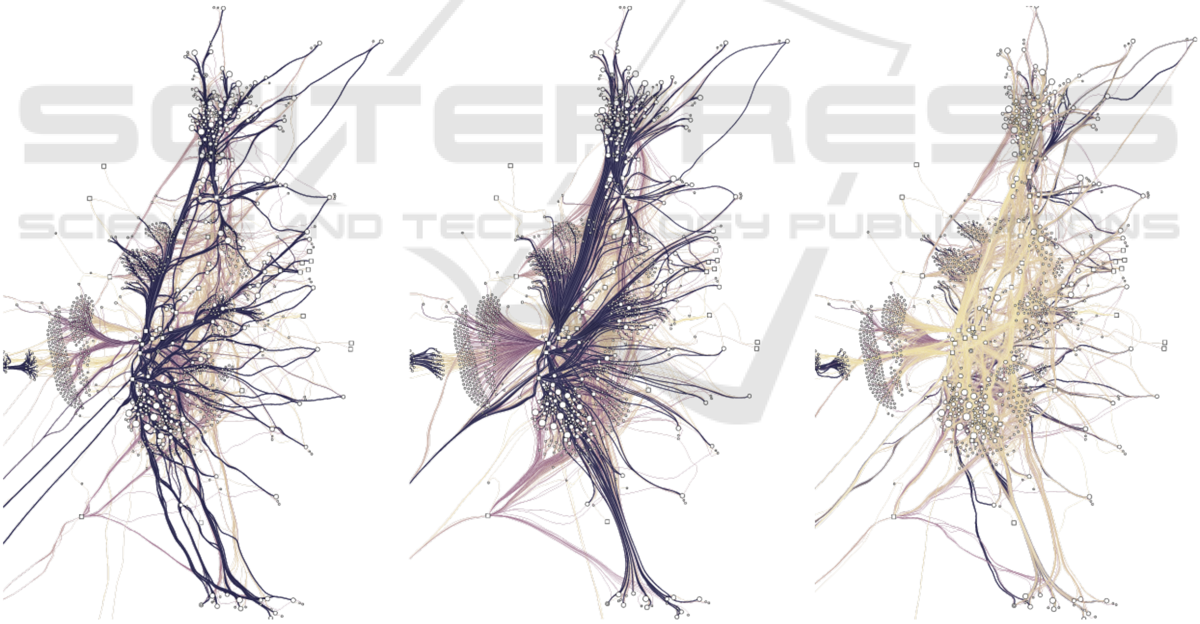

Figure 6: Flow map generated by our method without considering SIT quantities (left), parameters of maximum speed set to 5 and vision field set to 50 degrees (image in the middle), inverse sorting order (right).

6 CONCLUSIONS

As previously mentioned, applying the flow map on a large amount of data is challenging, since it involves dealing with high degrees of visual clutter. In this article we presented a method for generating flow maps that overcomes the cluttering issue in visualization. Our approach relies on a nature-inspired algorithm resulting in emergent visual patterns. Ultimately, resulting in edge bundling to reduce visual clutter and to promote visual clarity in the representation. In addition, we explored different graphical languages applied on the generated graph to give diverse perspectives over the same dataset.

Our method consists of a set of boids that trace a path to represent each edge in the graph. Each single boid follows simple rules through the interaction with the neighboring trails. The similarity between the edges, which belongs to the range [-2, 2], determines whether boids are friendly or not. This makes the boids to attract or repel from each other. Furthermore, in cases when the similarity is zero, the boids ignore each other. In addition, the boids that represent more products have higher impact on other members of the system. Finally, every boid attempts to avoid static points, which are the nodes of the graph.

We described two types of graphical representation. We presented the main visualization, which depicts transitions of products from warehouses to supermarkets. The total amounts of products being transported are represented with color and line thickness. The directionality of movement is indicated by an arrow at the end of each trace. The nodes use color to show different countries, while the shape of each node indicate either its is a warehouse or a supermarket. Finally, the fixed nodes are marked with an "X" in the center of the node. Additionally, the sorting order of the edges reflects the emphasis on low or high values. Then, we presented a graphical approach to distinguish main streams of flow. This is achieved by coloring the edges by their degree of overlapping. In this case, the red and green colors represent high and low number of overlapped traces, giving a visual representation of the complexity of the graph.

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APPENDIX

Figure 7: Final render of whole graph with the focus on the continental part of Portugal.