Selective Maintenance for Failure-prone Multi-state Systems When the Durations of Missions and Scheduled Breaks Are Stochastic

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Abstract: This paper addresses the selective maintenance optimization problem for a multi-component and multi-state system (MSS). The system performs several missions with breaks between each two consecutive missions. At the end of a mission, the reliability of the system is defined as the probability that the system satisfies the required demand level during the next mission. This probability is evaluated using the z-transform method. To improve the system's reliability, its components are maintained during breaks. To each component, a list of maintenance actions is available from minimal repair to overhaul through imperfect maintenance actions. Durations of missions and breaks are considered not constant but rather stochastic. These durations are therefore modeled as random variables with appropriate probability distributions. The selective maintenance optimization problem proposed is modeled as a non-linear and stochastic program. The fundamental constructs and the relevant parameters of this decision-making problem are solely investigated and discussed. An illustrative example is provided to demonstrate the added value of solving this selective maintenance problem as a stochastic optimization program.

1 INTRODUCTION

Selective maintenance is dedicated especially to multi-component systems operating an alternate sequence of missions and scheduled breaks. To successfully execute the next mission, maintenance activities are performed on the system components during the scheduled breaks. However, due the limited duration of a scheduled break, in addition to the possible budget and maintenance resources constraints, only a set of components may indeed be selected for maintenance. To meet the minimum predetermined performance level required to operate the next mission, it is therefore mandatory to select an optimal set of components to maintain as well as the type of maintenance actions to be performed on these components.

Selective maintenance was first introduced by (Rice et al., 1998) and applied for a series-parallel system where each subsystem is composed of independent and identical components. The lifetime of each system's component is assumed to follow an exponential distribution and the replacement of failed component is the only one maintenance alternative to improve system's reliability. To overcome the drawback hypothesis of independent and identically distributed components in (Rice et al., 1998), (Cassady et al., 2001b) proposed a more generalized selective maintenance modeling framework for systems whose reliability block diagram may be a combination of series, parallel and bridge structures. (Cassady et al., 2001a) considered the selective maintenance problem for a series-parallel system where components' lifetimes are Weibull distributed. Three maintenance actions are then considered, namely the minimal repair, the corrective replacement of failed components and the preventive replacement of functioning components. To solve the resulting optimization problem, an exhaustive enumeration method is used. In (Rajagopalan and Cassady, 2006), the authors proposed four improved enumeration procedures to reduce the computational time in (Cassady et al., 2001b). To deal with selective maintenance problem for large sized systems, (Khatab et al., 2007) proposed two heuristic-based methods. The authors in (Lust et al., 2009) proposed also an exact method based on the branch-and-bound procedure and a Tabu search algorithm. More recently, imperfect maintenance actions are considered in the selective maintenance setting.

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(Zhu et al., 2011) considered the age reduction coefficient approach of (Malik, 1979) to model imperfect preventive maintenance. (Panday et al., 2013) also studied the selective maintenance problem for binary systems under imperfect PM. They considered, the hybrid hazard rate model introduced by (Lin et al., 2000) to model imperfect maintenance actions. The authors in (Djelloul et al., 2015) considered selective maintenance problem in the case the system operates missions of random duration.

The selective maintenance studied in the above mentioned works merely rely on binary state systems having only two operating states, namely functioning and failure states. However, many industrial systems are designed to operate their tasks according to a range of performance levels varying from perfect functioning to complete failure. Such systems are known as multi-state systems (MSS). Dealing with selective maintenance of MSS, only few works appeared in the literature. The first work is reported in (Chen et al., 1999). Each system's component as well as the system itself may be in one of the $(K+1)$ possible states. Replacement of failed components is the only available maintenance option. A maintenance optimization problem is then derived to minimize the total maintenance cost while providing a given required system reliability level for the next mission. To solve this problem, a procedure based on the short path method is proposed. In (Liu and Huang, 2010), the authors studied the selective maintenance problem for a MSS where components are characterized by two operating states (functioning and failure states) while the system performs several states with different output performance levels. Several maintenance options are also considered from minimal repair to replacement through imperfect maintenance. A genetic algorithm is used to solve the resulting selective maintenance optimization problem. To overcome the restrictive hypothesis of binary components in (Liu and Huang, 2010), (Pandey et al., 2013) studied selective maintenance for a series-parallel MSS where components are characterized by more than one performance level (i.e. multi-state components). The transition rates between states of a component are assumed constants, i.e. components are modeled as continuous-time Markov chain. In (Khatab and Ait-Kadi, 2008), the authors generalized the selective maintenance optimization problem to consider more than one mission. To improve the reliability of the system, preventive maintenance actions are performed during breaks. The selective maintenance problem consists on finding an optimal sequence of maintenance actions the cost of which minimizes the total maintenance cost while providing the desired system reliability level for each mission. The resulting optimization problem is solved using the extended great deluge algorithm.

Dealing with the MSS selective maintenance problem, all the above mentioned papers assumed that the duration of the break as well as that of the next mission are both known and constant. However, this assumption may no longer be valid in many real-world situations where it is usually difficult to evaluate the exact duration of a mission. Indeed, such duration may unfortunately be impacted by the occurrence of random events which lead the system either to abort the mission or at most to continue operating the mission but with more additional time. Similarly, the occurrence of random events may conduct the decision maker to shorten or even to extend the break duration. As a consequence, it is more realistic and practical to consider that mission as well as break durations are not precisely known but rather random and should therefore be governed by appropriate probability distributions.

The present paper addresses the selective maintenance problem for a MSS systems when the duration of the next mission and that of the break are stochastic and modeled as random variables. Right after each mission, the system becomes available for maintenance during a limited duration of the break, to meet the required reliability level for the execution of the next mission. Due to the limited maintenance resources, not all components are likely to be maintained. The selective maintenance decision problem to be solved consists first in selecting a subset of components and then choosing the level of maintenance to be performed on each of the selected components. The objective function may consist of maximizing the successful completion of the next mission while taking into account maintenance budget and time allotted to the break, or of minimizing the total maintenance cost subject to the required reliability level and the time allotted to the break, or of minimizing the total maintenance time subject to the required reliability level and the maintenance budget. The present paper considers the second objective function. The stochastic selective maintenance problem is then formulated and solved.

The remainder of this paper is organized as follows. Section 2 describes the investigated system and defines its reliability. Section 2 shows how the $z-$transform is used to estimate MSS reliability. Section 3 presents the imperfect maintenance model and defines time and cost for each maintenance action. Section 4 discusses the reliability computation to operate missions of random durations. The stochastic selective maintenance optimization model is developed and discussed in Section 5. A numerical example is provided to illustrate the benefit of considering stochastic durations of missions and breaks. Finally, some conclusions are drawn in Section 6.

2 SYSTEM DESCRIPTION AND RELIABILITY COMPUTATION

The selective maintenance problem addressed in the present work concerns a multi-state system (MSS) S composed of n failure-prone components. Each component $C_i (i = 1,\dots,n)$ is characterized by two performance rates $g_{i1} = 0$ and $g_{i2} \neq 0$. The later is the nominal output performance when component C_i is functioning, while the former corresponds to the output performance when C_i fails. In this paper, the performance of a system's component is defined by its productivity or capacity. The entire system is therefore characterized by a range of $K = 2^n$ performance levels from complete failure up to perfect functioning. Let G_k be the output performance level of the k^{th} system's state and $\Pr\{G(t) = G_k\} = q_k(t)(k = 1,\dots,K)$ with $G(t)$ being the output performance of the system at time t. Then, the output performance distribution (OPD) of the system can be completely determined by the following two sets $\mathbf{G}$ and $\mathbf{q}$:

$$\mathbf{G} = \{G_k : 1 \leqslant k \leqslant K\}, \text{ and} \quad (1)$$

$$\mathbf{q} = \{q_k(t) : 1 \leqslant k \leqslant K\}. \quad (2)$$

The reliability $R(t,W)$ of the MSS is defined as its ability to satisfy the required performance level (*demand*) W at a given time t. Since $G(t)$ represents the output performance of the system at time t, this reliability is then defined as (Xue and Yang, 1995):

$$R(t,W) = \Pr\{G(t) \geqslant W\}. \quad (3)$$

According to Equations (1) and (2), the MSS reliability is equivalently defined as the probability that the system resides during the time interval $[0,t]$ in states where the output performance level is at least equal to the required demand W. Therefore, the MSS reliability is given as:

$$\begin{aligned} R(t,W) &= \Pr\{G(t) \geqslant W\} \\ &= \sum_{G_k \geqslant W} q_k(t). \end{aligned} \quad (4)$$

In the present work, MSS reliability computation is performed on the basis of the universal z-transform techniques developed in (Ushakov, 1986). In the literature, the universal z-transform is also called universal moment generating function (UMGF) and denoted as u-function. For more details, te reader may refers to (Levitin and Lisnianski, 2001; Lisnianski and Levitin, 2003; Levitin, 2005). The UMGF corresponding to the MSS S is given by the polynomial $U(t,z)$ such that:

$$U(t,z) = \sum_{k=1}^{K} q_k(t) \cdot z^{G_k}. \quad (5)$$

From the above equation, the MSS reliability can then be computed as:

$$\begin{aligned} R(t,W) &= \Pr\{G(t) \geqslant W\} \\ &= \sum_{k=1}^{K} q_k(t) \cdot \Phi(z^{G_k - W}), \end{aligned} \quad (6)$$

where Φ is a function defined as:

$$\Phi(z^{G_k - W}) = \begin{cases} 1 \text{ if } G_k \geqslant W, \\ 0 \text{ otherwise.} \end{cases}$$

To evaluate the MSS reliability, two basic operators are defined. The UMGF corresponding to the entire system reliability is then obtained by using simple algebraic operations on individual UMGF of systems' components. These operators allow to take into account how components are connected in series or in parallel. To illustrate, let us consider the simple case of a MSS composed of two components C_1 and C_2 characterized, respectively, by the UMGF $u_1(t,z)$ and $u_2(t,z)$ such that:

$$\begin{aligned} u_1(t,z) &= q_{11}(t) \cdot z^{g_{11}} + q_{12}(t) \cdot z^{g_{12}} \\ &= q_{11}(t) \cdot z^0 + q_{12}(t) \cdot z^{g_{12}}, \text{ and} \quad (7) \\ u_2(t,z) &= q_{21}(t) \cdot z^{g_{21}} + q_{22}(t) \cdot z^{g_{22}} \\ &= q_{21}(t) \cdot z^0 + q_{22}(t) \cdot z^{g_{22}}. \quad (8) \end{aligned}$$

In the above equations, parameters $g_{i1} = 0$ and g_{i2} represent the output performance levels of component C_i, while $q_{i1}(t)$ and $q_{i2}(t)$ are the instantaneous probabilities corresponding, respectively, to the output performance levels g_{i1} and g_{i2}. In this paper, performance of a components and that of the system itself is defined by productivity. Therefore, if components C_1 and C_2 are connected in parallel, the resulting MSS productivity is the sum of its components' productivity. In this case, the UMGF of the MSS is given as:

$$U(t,z) = \sum_{i=1}^{2} \sum_{j=1}^{2} q_{1i}(t) \cdot q_{2j}(t) \cdot z^{g_{1i} + g_{2j}}. \quad (9)$$

However, the total productivity of components C_1 and C_2 connected in series corresponds to the minimum of all components capacities. In this case, the UMGF $U(t,z)$ corresponding to the MSS is given as:

$$U(t,z) = \sum_{i=1}^{2} \sum_{j=1}^{2} q_{1i}(t) \cdot q_{2j}(t) \cdot z^{\min(g_{1i}, g_{2j})}. \quad (10)$$

3 IMPERFECT MAINTENANCE MODEL AND RELATED COST AND TIME

To perform maintenance activities, a list of L_i maintenance options (levels) $\{1,\ldots,l_i,\ldots,L_i\}$ is available for each component C_i. Among these maintenance options, there are two particular values $l_i = 1$ and $l_i = L_i$. The former corresponds to the minimal repair maintenance action that when performed brings the component to an *as bad as old* conditions, while the later corresponds to the overhaul after which the component becomes *as as good as new*. Values of l_i where $1 < l_i < L_i$ represent imperfect maintenance actions such that when performed they bring the component's condition between the *as good as new* and *as bad as old* conditions. In the present paper, the age reduction coefficient of Malik (Malik, 1979) is used to model the imperfect maintenance options. According to this model, when an imperfect maintenance action is performed on a component it reduces its age from, say t, to $\theta \times t$ where θ is the age reduction coefficient ($0 \leq \theta \leq 1$). Accordingly, the system becomes *as good as new* (overhaul)if its age is reset to zero ($\theta = 0$) while it becomes *as bad as old* (minimal repair) if the age reduction coefficient $\theta = 1$.

The maintenance cost $MC(l_i)$ induced by a maintenance action of level l_i when performed on component C_i is a preventive maintenance cost $MC^p(l_i)$ if C_i is functioning at the end of the current mission, or a corrective maintenance cost $MC^c(l_i)$ if C_i is failed at the end of the current mission. Similarly, the maintenance time $MT(l_i)$ consumed by a maintenance action of level l_i executed on component C_i is equal to $MT^p(l_i)$ if C_i is still functioning at the end of the current mission, or $MT^c(l_i)$ otherwise. The particular values of maintenance cost $MC(l_i)$ and maintenance time $MC(l_i)$ are defined for $l_i = 1$ (minimal repair) and $l_i = L_i$ (overhaul). For a failed component $MC^c(1) = MRC_i$, $MC^c(L_i) = OC_i^c$, $MT^c(1) = MRT_i$, and $MT^c(L_i) = OT_i^c$. In the case where the component is functioning at the end of the current mission, the cost $MC^p(L_i)$ and time $MT^p(L_i)$ induced by preventive overhauling component C_i are defined, respectively, as $MC^p(L_i) = OC_i^p$ and $MT^p(L_i) = OT_i^p$. Maintenance cost $MC^p(1)$ and maintenance time $MT^p(1)$ are not eligible since minimal repair is assumed to be admissible as a maintenance option only for a failed component.

4 PROBABILITY OF NEXT MISSION SUCCESS

Let us assume that the system have just finished a mission and then, turned off during the scheduled break of finite length and becomes available for possible maintenance activities. The system is thereafter used to execute the next mission of a given duration. When the system is set for maintenance, a component can be either in a functioning state or in a failed state. Hence, two state variables X_i and Y_i are used to describe the status of component C_i, respectively, at the beginning and at the end of a mission:

$$Y_i = \begin{cases} 1, \text{ if } C_i \text{ is functioning at the end of} \\ \text{the current mission,} \\ 0, \text{ otherwise.} \end{cases} \quad (11)$$

$$X_i = \begin{cases} 1, \text{ if } C_i \text{ is functioning at the beginning} \\ \text{of the next mission,} \\ 0, \text{ otherwise.} \end{cases} \quad (12)$$

In what follows we let A_i be the age of the component at the beginning of the next mission and B_i be the age of C_i at the end of the current mission. Furthermore, duration D of the scheduled break is a random variable governed by a probability density function (pdf) $f_D(t)$ and a cumulative distribution function (cdf) $F_D(t)$. The duration of the next mission is also stochastic and represented by a random variable O whose pdf and cdf are denoted by $f_O(t)$ and $F_O(t)$, respectively.

According to the age reduction imperfect PM model, if a maintenance action with an eligible level l_i is performed on component C_i at the end of the current mission, the effective age A_i of C_i at the beginning of the next mission becomes then:

$$A_i = \theta_{l_i} \cdot B_i. \quad (13)$$

Let R_i^c be the conditional probability that component C_i successfully operates the next mission given that its initial age is A_i at the beginning of the next mission. If T_i denotes the random variable of the lifetime of component C_i, then the conditional reliability R_i^c is evaluated as:

$$R_i^c = \Pr(T_i > A_i + O|_{T_i > A_i}). \quad (14)$$

Taking into account Equation (13), and the fact that O is a random variable governed by the cdf $F_O(t)$ defined on the support $[O_{\min}, O_{\max}]$, then the conditional reliability R_i^c is evaluated to:

$$R_i^c = \frac{\int_{O_{\min}}^{O_{\max}} R_i(\theta_{l_i} \cdot B_i + u) \cdot dF_O(t)}{R_i(\theta_{l_i} \cdot B_i)} \quad (15)$$

In the above equation $R_i(t)$ refers to the unconditional survival function of component C_i. According to Equation (15), the UMGF $u_1(O,z)$ corresponding to component C_i is written as:

$$u_1(O,z) = (1-R_i^c)\cdot(1-X_i)\cdot z^{g_{i1}} + R_i^c \cdot X_i \cdot z^{g_{i2}} \quad (16)$$

Let us denote $q_{i1}(O) = (1-R_i^c)\cdot(1-X_i)$ and $q_{i2}(O) = R_i^c \cdot X_i$. Let us also denote W_0 the required demand level to be satisfied during the next mission with stochastic duration O. Using results of Section 2, the UMGF $U(O,z)$ of the entire system is evaluated as:

$$U(O,z) = \sum_{k=1}^{K} q_k(O)\cdot z^{G_k}. \quad (17)$$

where $q_k(O)$ stands for the probability that the system resides in state $k(k=1,\ldots,K)$ at the end of the next mission, and G_k is the system's output performance in that state. Following the development of Section 2, the probability $R(O,W_0)$ that the system successfully operate the next mission is given as:

$$R(O,W_0) = \sum_{k=1}^{K} q_k(O)\cdot \Phi(z^{G_k - W_0}). \quad (18)$$

5 THE STOCHASTIC SELECTIVE MAINTENANCE OPTIMIZATION PROBLEM

Assume that the system has just operated the current mission and system's components may undergo maintenance activities. However, not all components may possibly be maintained due to the limitation on both maintenance budget and time. Consequently, a selective maintenance problem must be solved. The objective consists then on minimizing the total maintenance cost taking into account, on one hand, the required minimal system's reliability to successfully completing the next mission, and the limited duration of the break, on the other hand. The probability of completing the next mission is obtained from the reliability $R(O,W_0)$ given by Equation (18). To evaluate the total cost induced by maintenance actions and the corresponding total time consumed from the break duration, we define the following decision variable $s_i(l_i)$:

$$s_i(l_i) = \begin{cases} 1, \text{ if } C_i \text{ is selected for maintenance and} \\ \text{maintenance level } l_i \text{ is performed,} \\ 0, \text{ otherwise.} \end{cases} \quad (19)$$

The total cost of maintenance during the break is denoted by TMC and computed as:

$$TMC = PMC + CMC, \quad (20)$$

where PMC and CMC denote the total cost induced by, respectively, preventive and corrective maintenance actions performed during the break. The total cost of preventive maintenance actions is evaluated as:

$$PMC = \sum_{i=1}^{n}\sum_{l_i=2}^{L_i} MC^p(l_i)\cdot Y_i \cdot s_i(l_i), \quad (21)$$

where a PM action of level $l_i > 1$ is allowed to be performed on component C_i only if C_i is in working state at the end of the current mission, i.e. $Y_i = 1$. By analogy, the total cost induced by corrective maintenance actions is evaluated as:

$$CMC = \sum_{i=1}^{n}\sum_{l_i=1}^{L_i} MC^c(l_i)\cdot(1-Y_i)\cdot s_i(l_i), \quad (22)$$

where $(1-Y_i)$ states that corrective maintenance actions are available only for failed components.

The total time required to perform maintenance actions during the break is also composed of preventive and corrective maintenance times denoted, respectively, by PMT and CMT. These quantities are evaluated to:

$$PMT = \sum_{i=1}^{n}\sum_{l_i=2}^{L_i} MT^p(l_i)\cdot Y_i \cdot s_i(l_i), \quad (23)$$

$$CMT = \sum_{i=1}^{n}\sum_{l_i=1}^{L_i} MT^c(l_i)\cdot(1-Y_i)\cdot s_i(l_i). \quad (24)$$

Hence, the total time spent in maintenance during the break is denoted by TMT and computed as:

$$TMT = PMT + CMT. \quad (25)$$

The stochastic selective maintenance optimization problem is then formulated as follows:

$$\text{Min}\begin{pmatrix} \sum_{i=1}^{n}\sum_{l_i=1}^{L_i} MC^c(l_i)\cdot(1-Y_i)\cdot s_i(l_i) \\ +\sum_{i=1}^{n}\sum_{l_i=2}^{L_i} MC^p(l_i)\cdot Y_i\cdot s_i(l_i)\end{pmatrix} \quad (26)$$

Subject to:

$$R(O,W_0) \geq R_0, \quad (27)$$

$$\Pr(D \geq TMT) \geq \tau_s, \quad (28)$$

$$\sum_{l_i=1}^{L_i}(1-Y_i)\cdot s_i(l_i) + \sum_{l_i=2}^{L_i} Y_i\cdot s_i(l_i) \leq 1, \quad (29)$$

$$s_i(1) \leq 1 - Y_i, \quad (30)$$

$$X_i = Y_i + \sum_{l_i=1}^{L_i}(1-Y_i)\cdot s_i(l_i), \quad (31)$$

$$A_i = [\theta_{l_i}\cdot s_i(l_i) + (1-s_i(l_i))]\cdot B_i, \quad (32)$$

$$s_i(l_i) \in \{0,1\};\ i=1,\ldots,n;\ l_i = 1,\ldots,L_i.$$

In the above optimization model, Equations (27) and (28) are, respectively,the required reliability level and the maintenance time constraints. The constraint (28) is the newly introduced constraint requiring particular treatment. In fact, the probability of executing a selective maintenance plan should at least be equal to a required service ratio τ_s. This constraint ensures with the probability at least equal to τ_s that the selected components will be maintained each with the corresponding selected maintenance level. The risk corresponding to the inability to perform a selected maintenance plan is then evaluated to $1-\tau_s$. This results from the fact that the duration of the break is considered stochastic rather than constant. For each component C_i, Equations (29) states that only one maintenance level can be selected if the component is to be maintained. The constraint (30) states that minimal repair is eligible only on a failed component. The constraint (31) allows to update the operating state of components. For a given system's configuration, this stochastic optimization problem can be solved using the usual stochastic optimization techniques. The following section presents an illustrative examples and discusses how the stochasticity of the mission and break durations impact the maintenance level selection decisions. In this experiment, durations and costs are respectively given in time and monetary units.

6 NUMERICAL EXAMPLE

This experiment investigates the selective maintenance problem for a series-parallel MSS whith stochastic durations of missions and breaks. The system is composed of two series subsystems. The first subsystem is composed of 3 parallel components C_i $(i = 1,2,3)$, and the second also contains 3 components C_i $(i = 4,5,6)$ arranged in parallel. The failure time of the system's component C_i follows a Weibull distribution whose shape and scale parameters are respectively given by β_i and η_i $(i = 1,\ldots,n)$. Values of these parameters are shown in Table (1). This table shows also components' performance rates g_{i2}, the value of B_i corresponding to the age of components C_i at the end of the current mission, in addition to the value of the state variables Y_i corresponding to the its operating state (functioning or failed). According to this table, only components C_1 and C_4 survived the current mission ($Y_1 = Y_4 = 1$) while the other components are in failed state.

A same list of $L_i = 6$ $(i = 1,\ldots,6)$ possible maintenance levels is available for all system's components. Age reduction coefficients corresponding to these maintenance levels are given in Table (2). Corrective maintenance costs and times are given, respectively, in Tables (3) and (4), while preventive maintenance costs and times are shown in Tables (5) and (6), respectively.

Table 1: Components' parameters.

$C_{ij})$	β_i	η_i	g_{i2}	Y_i	B_i
C_1	1.5	75	55	1	35
C_2	2.4	114	80	0	24
C_3	1.6	84	120	0	45
C_4	2.4	102	70	1	36
C_5	2.5	78	95	0	44
C_6	2.0	84	80	0	28

Table 2: Maintenance levels and their respective age reduction coefficient values.

l_i	1	2	3	4	5	6
θ_{l_i}	1	0.7	0.5	0.3	0.2	0

Table 3: Minimal repair and corrective maintenance costs.

$\frac{C_i}{l_i}$	C_1	C_2	C_3	C_4	C_5	C_6
1	5	6	6	6	5	5
2	2.81	0.32	3.48	1.88	5.29	1.74
3	5.88	1.73	6.28	4.41	7.99	4.58
4	9.56	5.25	9.27	7.73	10.49	8.65
5	11.59	8.15	10.81	9.65	11.69	11.14
6	16	17	14	14	14	17

For the analysis below, we assume that the duration of the break is deterministic and fixed to $D = 10$, while the duration O of the next mission is stochastic and governed by a truncated normal distribution $\mathcal{N}(14,2.5)$ on the support [10,25] (see Figure 1). We also set the required minimum level of the system's reliability R_0 to execute the next mission to $R_0 = 70\%$ with a required demand $W_0 = 150$. If no maintenance is performed on the systems' components, the probability of the system successfully completing the next mission is null. Indeed, only

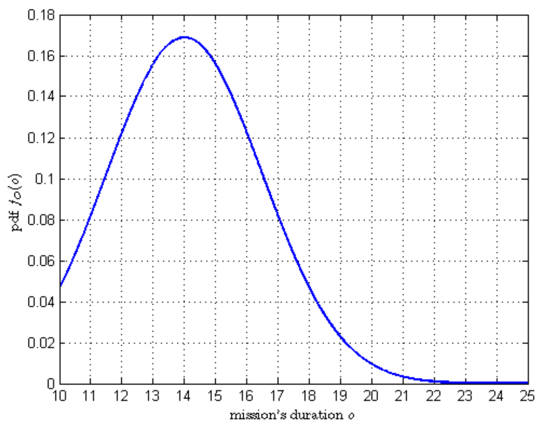

Figure 1: The pdf corresponding to the duration of the next mission.

Table 4: Minimal repair and corrective maintenance times.

$\frac{C_i}{l_i}$	C_1	C_2	C_3	C_4	C_5	C_6
1	2	2	3	3	3	2
2	0.7	0.08	1.24	0.67	1.89	0.41
3	1.47	0.41	2.24	1.57	2.85	1.08
4	2.39	1.23	3.31	2.76	3.75	2.04
5	2.9	1.92	3.86	3.45	4.17	2.62
6	4	4	5	5	5	4

Table 5: Preventive maintenance costs.

$\frac{C_i}{l_i}$	C_1	C_2	C_3	C_4	C_5	C_6
1	–	–	–	–	–	–
2	2.46	0.28	2.98	1.61	4.53	1.43
3	5.14	1.53	5.38	3.78	6.85	3.77
4	8.36	4.63	7.94	6.62	8.99	7.13
5	10.14	7.19	9.27	8.27	10.02	9.18
6	14	15	12	12	12	14

Table 6: Preventive maintenance times.

$\frac{C_i}{l_i}$	C_1	C_2	C_3	C_4	C_5	C_6
1	–	–	–	–	–	–
2	0.35	0.04	0.25	0.13	0.38	0.2
3	0.73	0.2	0.4	0.31	0.57	0.54
4	1.19	0.62	0.66	0.55	0.75	1.02
5	1.45	0.96	0.77	0.69	0.83	1.31
6	2	2	1	1	1	2

components C_1 and C_4 are functionning at the end of the current mission mission. Therefore, the total out put performance of the the system is evaluated to $min(g_{12}, g_{42}) = min(55, 70) = 55$ which is less than the required minimum demand level W_0. To improve this reliability, the selective maintenance problem is therefore solved.

Given the required minimum demand W_0 with the reliability level R_0, and the limited time D of the break, if the selective maintenance problem in solved by assuming the deterministic duration of the next mission (i.e. the next mission duration is set to 14.12 which is the average value of O), in this case the optimal selective maintenance plan suggested is as follows. Components C_1 and C_4 are selected to undergo preventive maintenance actions of levels, respectively, 6 and 2,i.e. a preventive overhaul is performed on C_1 and an imperfect maintenance is executed on C_2. Furthermore, components C_3 and C_6 are selected to undergo corrective maintenance actions of levels, respectively, 5 and 2. This maintenance plan induces a total cost $TMC = 30.86$ and requires a total time $TMT = 5.95$. The resulting system's reliability is evaluated to 70.20%. Applying this maintenance plan in the case where the duration of the next mission is stochastic leads to a system's reliability equal to 61.95% which is indeed less than the required minimum reliability level R_0. In the stochastic case, this selective maintenance plan is unable to allow the system's reliability to reach the required minimum level for the next mission. Thus, if for some reason, the mission duration is extended, there will be then a high risk of not completing the mission with the required reliability level. However, if this same selective maintenance problem is solved by considering the duration of the next mission to be stochastic, the following selective maintenance plan is obtained according to which components C_1 and C_4 are selected to undergo preventive maintenance actions of levels, respectively, 6 and 3. In addition, components C_3 and $C_5$are selected to receive corrective maintenance of levels 5 and 4. The resulting system's reliability is evaluated to 70.41%. The total cost induced by this selective maintenance plan is $TMC = 40.29$ and the corresponding total maintenance time is $TMT = 8.02$.

Let us now consider the additional constraint represented by the stochastic limited break duration D. This duration is also modeled as a random variable with pdf and cdf, respectively, denoted by $f_D(d)$ and $F_D(d)$. They are defined on a support $[D_{min}, D_{max}]$ meaning that the break takes a duration d which lies between D_{min} and D_{max}. In the present example, we assume that D follows a uniform distribution $\mathcal{U}(6, 14)$; its corresponding average value is $E(D) = 10$. It follows that the probability to successfully performing the selective maintenance plan is given by $\Pr(TMT \leq D) = 1 - F_D(TMT)$ and evaluated to 74.75%. Accordingly, the selective maintenance plan is then a feasible solution of the stochastic selective optimization problem only if the service ratio level τ_s is fixed to a value less than or equal to 74.75% ($\tau_s \leq 74.75\%$).

7 CONCLUSION

This paper addressed the selective maintenance optimization problem for multi-component and multi-state system. For each component of the system, a list of maintenance actions is available from minimal repair to overhaul through imperfect maintenance actions. Each maintenance actions is characterized by a reliability improvement level. The system performs several missions separated by scheduled breaks during which maintenance activity can takes place. Durations of both breaks and missions are considered as random variables governed by an appropriate probability distributions. These distributions are integrated in the selective maintenance problem resulting in a non-linear stochastic optimization program. A numerical example is studeied to demonstrate how the stochasticity of missionss durations impacts the selective maintenance decisions.

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