Control of Uncertain Robot Manipulators using Integral Backstepping and Time Delay Estimation

Yassine Kali[1], Maarouf Saad[2], Khalid Benjelloun[1] and Mohammed Benbrahim[3]
[1]*Ecole Mohammadia d'Ingenieurs, Mohammed V University, Rabat, Morocco*
[2]*Ecole de Technologie Superieure, Quebec University, Montreal, Canada*
[3]*Sciences Faculty, SMBA University, Fez, Morocco*

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Abstract: In this paper, a novel controller is proposed and applied for high accuracy tracking trajectory in the workspace of robot manipulators in presence of uncertainties and external disturbances. Most of nonlinear controllers are based on the mathematical model of robot manipulator, but a lot of robotic systems do not have exact model. This novel approach which consists on designing an Integral Backstepping with Time Delay Control (IBTDC) can estimate uncertainties and keep high tracking performance. The proposed controller is able to stabilize the robot system, and also to drive the trajectory tracking errors to converge in finite time. Furthermore, experimental results are given to illustrate the effectiveness of the proposed method applied to the 7-DOF ANAT robot arm.

1 INTRODUCTION

Control of robot manipulators, has received wide attention and is a topic of great research interest. These research works have focused on tracking control problems in the joint and task space. In literature, we can find many nonlinear techniques such as Sliding Mode Control (SMC) (Utkin et al., 1999), Feedback Linearization (Park and Cho, 2007), Backstepping (H.-J.Shieh and C.-H.Hsu, 2008). However, the robot manipulators are uncertain Multiple-Input Multiple-Output (MIMO). They suffer from the plant uncertainties due to uncertain parameters, load variations and external disturbances, which may seriously degrade the performance of the tracking control and/or deteriorate the controlled system.

The backstepping approach is a recursive Lyapunov procedure, proposed in the beginning of 1990*s*. This approach was introduced first in (P.V. Kokotovic, 1995). The basic idea of this technique is to design a controller by selecting appropriate stabilization functions for some state variables chosen as virtual controls (Slotine and Li, 1991; Lewis et al., 1993). This allows, in addition to the control objective for which the technique is developed (tracking and/or stabilization), to ensure, at all times, the stability of the controlled system. Therefore, Backstepping provides robust and high-accuracy solutions. However, one main restriction remains. Nonlinear backstepping is sensitive to uncertainties and external disturbances, otherwise, the control may easily cause unacceptable practical complications.

To cope with the aforementioned problem, some works proposed adaptive backstepping (H.-J.Shieh and C.-H.Hsu, 2008; Zhou and Wen, 2008) which provide an adaptation of the control to be sufficiently robust to eliminate the effect of uncertain nonlinear dynamics and unexpected disturbances but this overparametrization may cause inequality of the number of parameter estimates and the number of unknown parameters. Other works proposed a combination of backstepping and intelligent control techniques (neural-network or fuzzy logic) (Jagannathan and Lewis, 1998; Weisheng et al., 2015; Yoo and Ham, 2000; Su et al., 2015). These controllers have the merit to estimate uncertain dynamics and unexpected disturbances but they introduce fuzzy rules in case of fuzzy logic or a large number of parameters in case of neural-network that may make implementation impossible.

A possible solution to consider is a combination of Integral Backstepping (Tan et al., 2000; Skjetne and Fossen, 2004) and Time Delay Control (TDC)

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(Youcef-Toumi and Ito, 1990; Hsia and Gao, 1990). TDC can estimate unknown dynamics and external disturbances simply and effectively using time-delayed information provided from control inputs signals and the states variables without a prior exact knowledge of robot model. When the TDC is applied, a so-called TDE error appear that can be reduced using integral backstepping, else, the system will lead with a large steady state error. As a consequence it can be expected that the system can be stabilized even in presence of uncertainties and external disturbances.

The paper is organized in five sections. In Section 2, the dynamics of serial n-link rigid robot manipulators is presented with sufficient property and assumptions. In Section 3, the design of the controller called integral backstepping with time delay control is described with stability analysis using Lyapunov function. In Section 4, experimental results of the proposed method applied to the 7-DOF ANAT robot are presented and comparison with nonlinear backstepping and Sliding Mode with Time Delay Control (SMTDC) is discussed. Finally, the conclusion is drawn in section 5.

2 PROBLEM FORMULATION

The dynamic model expressed in joint space coordinates according to the Lagrange theory of n-joint robot manipulator is given by:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+F(\dot{q})-\tau_d=\tau \tag{1}$$

where $q\in\mathbb{R}^n$, $\dot{q}\in\mathbb{R}^n$ and $\ddot{q}\in\mathbb{R}^n$ are the joint positions, velocities and accelerations vectors, respectively, $M(q)\in\mathbb{R}^{n\times n}$ is the symmetric positive-definite inertia matrix, $C(q,\dot{q})\dot{q}\in\mathbb{R}^n$ is the centrifugal and Coriolis vector, $G(q)\in\mathbb{R}^n$ is the gravitational vector, $F(\dot{q})\in\mathbb{R}^n$ is the viscous/static friction torque at the joints vector, $\tau_d\in\mathbb{R}^n$ denotes disturbance vector and $\tau\in\mathbb{R}^n$ is the torque input vector. The inertia matrix can be written into two parts, without loss of any generality:

$$M(q)=M_0(q)+\Delta M(q) \tag{2}$$

where $M_0(q)$ is the nominal part while $\Delta M(q)$ denotes the uncertain part. Then, we can rewrite the model given in Eq.1 as:

$$M_0(q)\ddot{q}+H(q,\dot{q},\ddot{q})=\tau \tag{3}$$

where

$$H(q,\dot{q},\ddot{q})=\Delta M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)+F(\dot{q})-\tau_d$$

For ease of control design, let's denote $M_0=M_0(q)$ and $H=H(q,\dot{q},\ddot{q})$.

The objective in this paper is to design a robust controller able to ensure that the joint position q tracks a desired trajectory q_d with high accuracy even if the dynamics is uncertain and in presence of external disturbances. To this end, we will design the controller and carry out its stability analysis based on the following property and assumptions:

- **Property 1.** The nominal part of inertia matrix $M_0(q)$ is positive-definite symmetrical and bounded such that:

  $$m_1 I\leq M_0(q)\leq m_2 I$$

  where m_1 and m_2 are two known positive constants (Spong et al., 2005).
- **Assumption 1.** The joint position states and its first time derivative are measurable.
- **Assumption 2.** H is a globally lipschitz function.

3 IBTDC

3.1 Controller Design

For the development of this method, we consider the robot system given by Eq.3. As said before, the control objective is to track a desired trajectory with high accuracy even in the presence of nonlinear unknown dynamics and external disturbances by designing a robust control. The closed-loop system is represented in Fig.1.

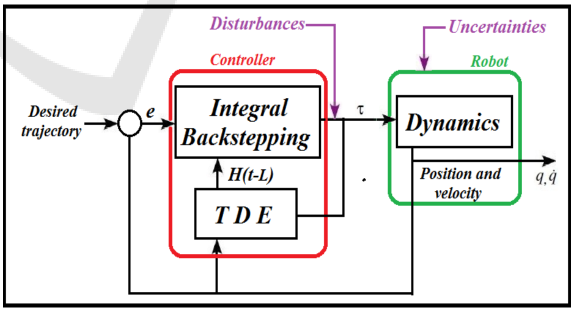

Figure 1: Block diagram of the proposed controller.

For the development of IBTDC, we define the position tracking error e:

$$e=q-q_d \tag{4}$$

where $q_d\in\mathbb{R}^n$ is the desired position trajectory vector. Now, let's select the regulated variable ε_1 as follows:

$$\varepsilon_1=e+\lambda\int_0^t e\,dt \tag{5}$$

where $\lambda = diag(\lambda_{ii})$ for $i = 1, \cdots, n$ is a positive constant gain matrix. Its time derivative is:

$$\begin{aligned} \dot{\varepsilon}_1 &= \dot{e} + \lambda e \\ &= \dot{q} - \dot{q}_d + \lambda e \end{aligned} \quad (6)$$

Choosing $\dot{q}$ as virtual control variable, we select the following stabilizing function as:

$$\alpha_1 = \dot{q}_d - \lambda e - K_1 \varepsilon_1 \quad (7)$$

where $K_1 = diag(k_{1i})$ for $i = 1, \cdots, n$ is a positive constant diagonal gain matrix that are in direct relation with the convergence rate. Defining now the error ε_2 obtained by the difference between the stabilizing function and virtual control in Eq.6 as:

$$\begin{aligned} \varepsilon_2 &= \dot{q} - \alpha_1 \\ &= \dot{q} - \dot{q}_d + \lambda e + K_1 \varepsilon_1 \\ &= \dot{\varepsilon}_1 + K_1 \varepsilon_1 \end{aligned} \quad (8)$$

Thus in terms of Eq.8, we can rewrite Eq.6 as follows:

$$\dot{\varepsilon}_1 = \varepsilon_2 - K_1 \varepsilon_1 \quad (9)$$

Therefore, differentiating Eq.8 with respect to time leads to:

$$\begin{aligned} \dot{\varepsilon}_2 &= \ddot{q} - \dot{\alpha}_1 \\ &= \ddot{q} - \ddot{q}_d + \lambda \dot{e} + K_1 \dot{\varepsilon}_1 \\ &= M_0^{-1}[\tau - H] - \ddot{q}_d + \lambda \dot{e} + K_1 \dot{\varepsilon}_1 \end{aligned} \quad (10)$$

According to the Lyapunov stability analysis, we choose:

$$\dot{\varepsilon}_2 = -K_2 \varepsilon_2 - \varepsilon_1 \quad (11)$$

where $K_2 = diag(k_{2i})$ for $i = 1, \cdots, n$ is a positive constant diagonal gain matrix. Then, we obtain:

$$\tau = M_0 u + H \quad (12)$$

where $u = \ddot{q}_d - \lambda \dot{e} - K_1 \dot{\varepsilon}_1 - K_2 \varepsilon_2 - \varepsilon_1$.

The control performance is affected since H is uncertain. Then, if **Assumption 2** given in Section 2 is verified we can estimate $H(t)$ using a TDE (Youcef-Toumi and Ito, 1990; Hsia and Gao, 1990) as:

$$\begin{aligned} \hat{H}(t) &\cong H(t-L) \\ &= \tau(t-L) - M_0(t-L)\ddot{q}(t-L) \end{aligned} \quad (13)$$

where L is the estimation time delay. Clearly the accuracy of $\hat{H}(t)$ improves as L decreases. In practice, the smallest estimation time delay L is chosen to be the sampling period which means that the unknown dynamics are identified every L times. Then, the proposed integral backstepping with time delay control is obtained as:

$$\begin{aligned} \tau &= M_0 u + \hat{H} \\ &= \tau(t-L) - M_0(t-L)\ddot{q}(t-L) + M_0 u \end{aligned} \quad (14)$$

As $\ddot{q}(t-L)$ may not be at our disposal, we use:

$$\ddot{q}(t-L) = \frac{1}{L^2}\left(q(t) - 2q(t-L) + q(t-2L)\right).$$

3.2 Stability Analysis

To prove the stability of the overall system, the following Lyapunov function candidate is used:

$$V = \frac{1}{2}\left(\varepsilon_1^T \varepsilon_1 + \varepsilon_2^T \varepsilon_2\right) \quad (15)$$

Taking time derivative gives:

$$\begin{aligned} \dot{V} &= \varepsilon_1^T \dot{\varepsilon}_1 + \varepsilon_2^T \dot{\varepsilon}_2 \\ &= \varepsilon_1^T (\varepsilon_2 - K_1 \varepsilon_1) \\ &\quad + \varepsilon_2^T \left(M_0^{-1}[\tau - H] - \ddot{q}_d + \lambda \dot{e} + K_1 \dot{\varepsilon}_1\right) \\ &= -\varepsilon_1^T K_1 \varepsilon_1 + \varepsilon_1^T \varepsilon_2 \\ &\quad + \varepsilon_2^T \left(M_0^{-1}[\tau - H] - \ddot{q}_d + \lambda \dot{e} + K_1 \dot{\varepsilon}_1\right) \end{aligned} \quad (16)$$

Using **Property 1** given in Section 2 and replacing the IBTDC given in Eq.14 in the Lyapunov function derivative given in Eq. 16, we obtain:

$$\begin{aligned} \dot{V} &= -\varepsilon_1^T K_1 \varepsilon_1 + \varepsilon_1^T \varepsilon_2 - \varepsilon_2^T \varepsilon_1 \\ &\quad + \varepsilon_2^T \left(M_0^{-1}[\hat{H} - H] - K_2 \varepsilon_2\right) \\ &= -\varepsilon_1^T K_1 \varepsilon_1 + \varepsilon_2^T \left(M_0^{-1} \Delta H - K_2 \varepsilon_2\right) \\ &\le -\varepsilon_1^T K_1 \varepsilon_1 - \varepsilon_2^T K_2 \varepsilon_2 + \varepsilon_2^T \frac{1}{m_1} I_n \Delta H \\ &\le \sum_{i=1}^{n} -k_{1i}\varepsilon_{1i}^2 - |\varepsilon_{2i}| \left(k_{2i}|\varepsilon_{2i}| - \frac{1}{m_1}|\Delta H_i|\right) \end{aligned} \quad (17)$$

where $\Delta H = \hat{H} - H$ is the TDE error and $I_n \in \mathbb{R}^{n \times n}$ denotes the identity matrix. To ensure $\dot{V}$ is a negative-definite function, the following condition must be fulfilled:

$$k_{2i} > \frac{1}{m_1}|\Delta H_i||\varepsilon_{2i}|^{-1} \quad (18)$$

Otherwise, using **Assumption 2**, we have:

$$\begin{aligned} |\Delta H_i| &= |\hat{H}_i - H_i| \\ &= |H_i(t-L) - H_i(t)| \\ &\le l_i|(t-L) - (t)| \\ &\le l_i L \end{aligned} \quad (19)$$

where $l_i > 0$ is the Lipschitz constant. Then, the condition given in Eq. (18) becomes:

$$k_{2i} > \frac{l_i L}{m_1}|\varepsilon_{2i}|^{-1} \quad (20)$$

Therefore, the Lyapunov function derivative is negative definite $\dot{V} < 0$, i.e, the error e and its derivatives go to zero, hence the stability of the closed loop system is proven.

4 CASE STUDY

To illustrate the effectiveness of the proposed controller, the IBTDC is implemented on the ANAT robot shown in Fig.2 using Simulink with Real-Time Workshop (RTW). ANAT robot is a 7-DOF hyper redundant articulated nimble adaptable trunk: the first joint is prismatic (joint 1), followed by three redundant rotational joints (joints 2, 3, and 4) and finally three rotational joints (joints 5, 6, and 7), the end-effector is mounted on the last joint (Fareh et al., 2012). For simplicity, three joints are locked during the experimental tests (joints 1, 6 and 7).

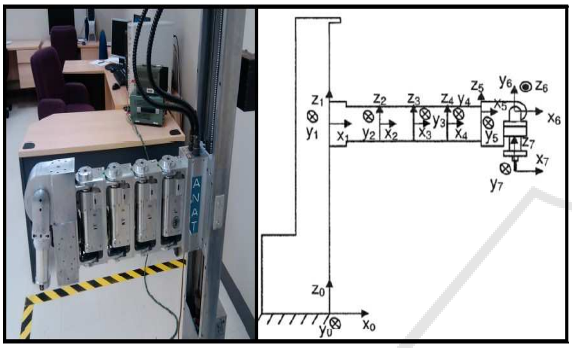

Figure 2: ANAT robot arm and D-H frames.

For the ANAT robot, modified Denavit-Hartenberg (D-H) convention (Dombre and Khalil, 2007) is used for selecting frames of references as shown in Fig.2. The D-H parameters of the ANAT robot are given in Tab.1.

Table 1: D-H parameters of 7-DOF ANAT robot.

Joints	α_{i-1}	a_{i-1}	d_i	q_i
1	0	0	q_1	0
2	0	L_1	0	q_2
3	0	L_0	0	q_3
4	0	L_0	0	q_4
5	0	L_0	L_2	q_5
6	$\pi/2$	L_3	0	q_6
7	$-\pi/2$	0	$-L_4$	q_7

The initial position of the end-effector in task space is $x(0) = [0.6764\ 0\ \ -0.196]$ while the initial joint positions and joint velocities are 0 rad and 0 rad/sec, respectively. The objective here is to follow a desired triangle defined in XY plane of the task space of the ANAT robot. For our robot, assuming that the desired trajectory is away from singular configuration, the desired accelerations in joint space and the desired accelerations and velocities in task space are linked by:

$$\ddot{q}_d = J^+\ddot{x}_d - J^+\dot{J}J^+\dot{x}_d \tag{21}$$

where J denotes the Jacobian matrix, $J^+ = J^T(JJ^T)^{-1}$ denotes the generalized inverse, $\ddot{q}_d$ is the desired joint acceleration vector, $\ddot{x}_d$ is the desired workspace acceleration vector, $\dot{x}_d$ is the desired workspace velocity vector. The desired joint velocity $\dot{q}_d$ is obtained from desired joint acceleration using an integrator while the desired joint position q_d is obtained from desired joint velocity using another integrator.

During the experiment, we placed a load of $2.25kg$ on the 5^{th} joint at $t = 10s$. In addition, a disturbances was added to the torque input representing 10% of maximum value of the torque as:

$$\tau_d = \begin{bmatrix} 0 \\ 0.15e^{-5(t-5)^2}sin(2\pi t) \\ 0.15e^{-5(t-5)^2}sin(3\pi t) \\ 0.03e^{-13(t-13)^2}sin(4\pi t) \\ 0.025e^{-18(t-18)^2}sin(5\pi t) \\ 0 \\ 0 \end{bmatrix}$$

In Section 3, the development of IBTDC for uncertain robot manipulators is given in Eq.14 where the controller gains are tuned to achieve the optimal performance such as the stability is guaranteed and the condition given in Eq (20) is verified: $\lambda = 5 * I_7$, $K_1 = 5 * I_7$ and $K_2 = 7 * I_7$. The estimation time delay is selected as the smallest sampling period $L = T_s = 0.03s$.

4.1 Experimental Results

The experimental results are shown in Fig.3 to Fig.7: workspace tracking, joint space tracking, joint space tracking error and control torque input obtained by using the proposed integral backstepping with time delay controller.

It is obvious that the proposed controller ensures good tracking trajectory even in presence of uncer-

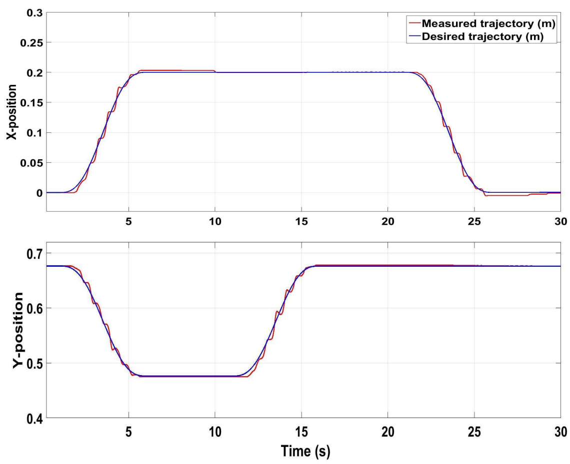

Figure 3: Workspace tracking trajectory.

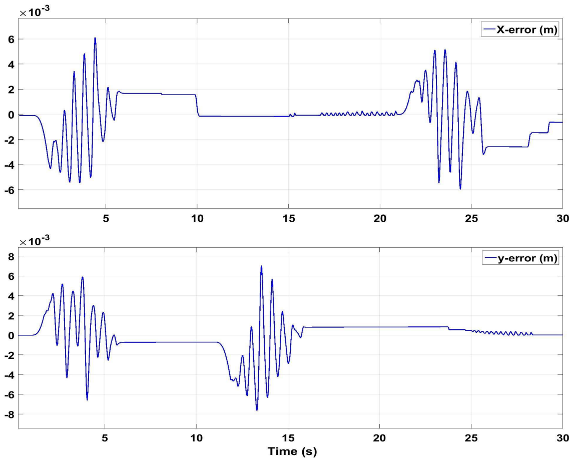

Figure 4: Workspace tracking error.

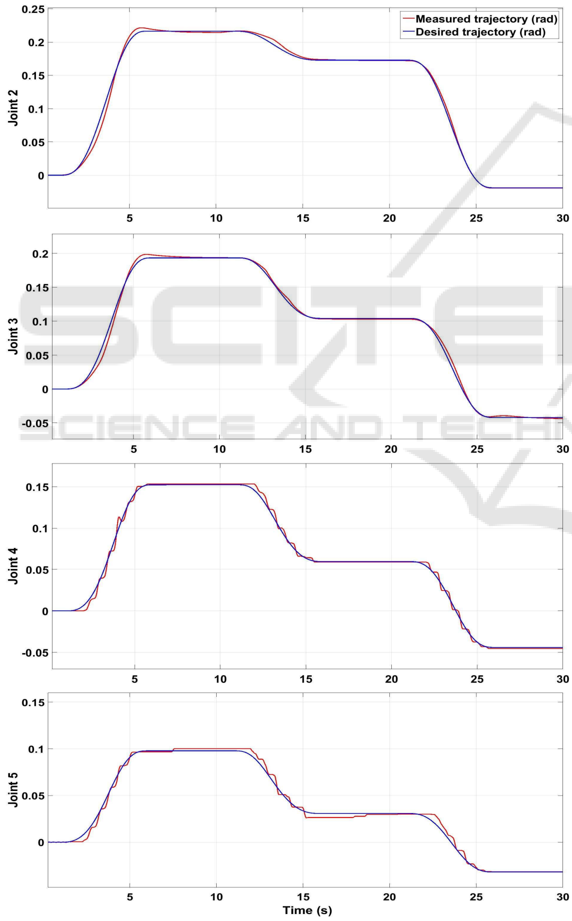

Figure 5: Joint space tracking trajectory.

tainties as shown in Fig.5 and confirmed by the small tracking errors as depicted in Fig.6. In addition, using the direct kinematics, we can notice that the controller ensures also a good tracking in task space as shown in Fig.3 and Fig.4. However, using integral backstepping cause a small overshoot. From Fig.7, we observe

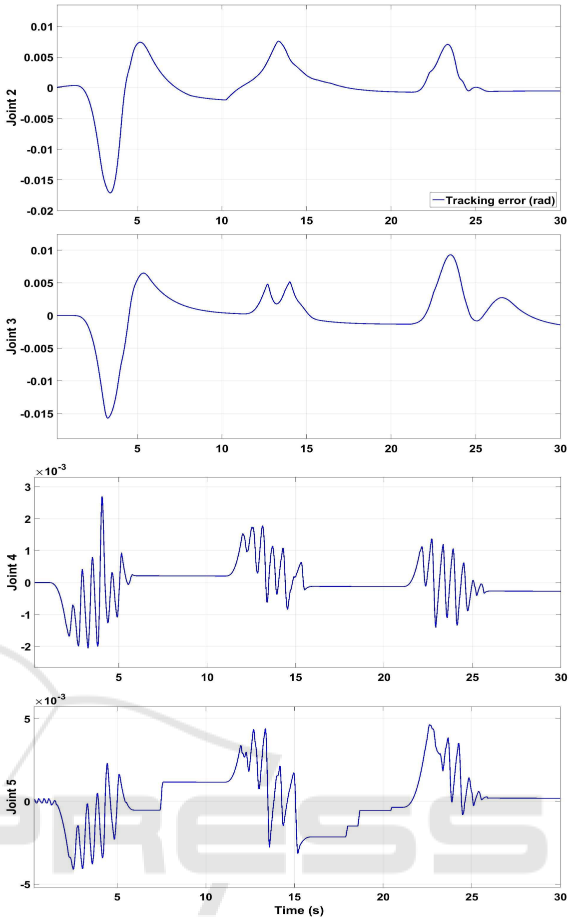

Figure 6: Joint space tracking error.

that the control inputs evolves continuously with acceptable values for the motors of the ANAT robot.

To show the effectiveness of the proposed IBTDC, it is compared with the nonlinear backstepping (Zhou and Wen, 2008) and the SMTDC (**?**) in terms of energy and stability by using the root-mean-squared (RMS) errors and torque inputs as:

$$\|\tau\|_{RMS} = \sqrt{\frac{1}{N}\sum_{i}^{N}\|\tau(k)\|^2}, \ \|e\|_{RMS} = \sqrt{\frac{1}{N}\sum_{i}^{N}\|e(k)\|^2}$$

where N denotes the number of sampling steps of the experimentation. the quantitative analysis is presented in Tab.2

Table 2: Controllers comparison.

Controller	$\|\tau\|_{RMS}$	$\|e\|_{RMS}$
Backstepping	1.72	$4.9 * 10^{-2}$
SMTDC	1.31	$2.75 * 10^{-2}$
IBTDC	1.253	$9.8 * 10^{-3}$

From the above comparison, we can notice that IBTDC stabilizes the system even in presence of un-

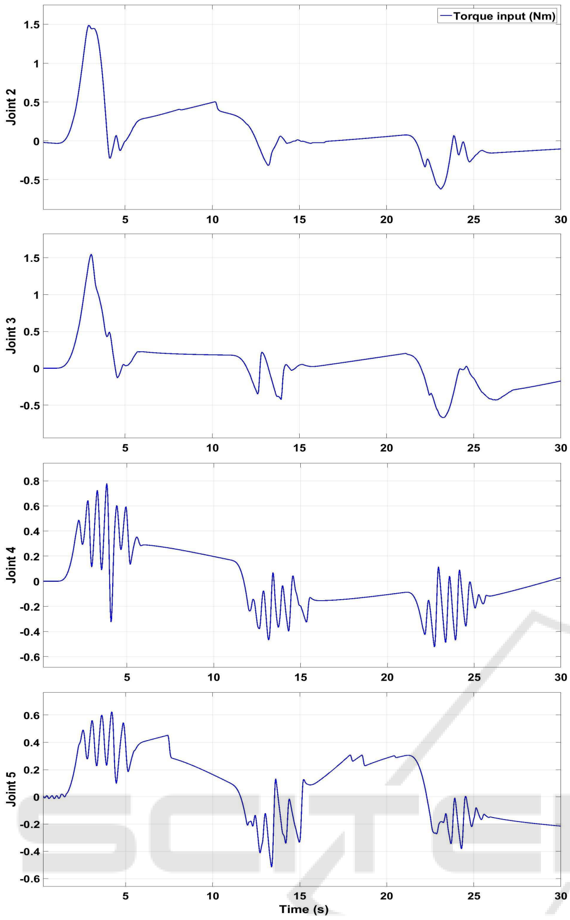

Figure 7: Control torque input.

certainties and external disturbances in finite time with the best tracking and less energy.

5 CONCLUSIONS

For a class of uncertain n-link robot systems, an integral backstepping with time delay controller which is a combination of integral backstepping and TDC, is presented. TDC is used to estimate uncertain nonlinear dynamics and to cancel the effect of external disturbances while integral backstepping is used to eliminate the TDE error. Experimental results on the 7-DOF ANAT robot showed the merit of IBTDC, particularly regarding the uncertain dynamics, external disturbances and finite time convergence. Otherwise, using integral backstepping may cause a large overshoot known as windup phenomenon. Further research should be pursued in the direction to overcome this phenomenon. We will also implement the proposed controller on other nonlinear systems.

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