Closed - Loop Control of Plate Temperature using Inverse Problem

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Abstract: In this paper the temperature at one side of a plate is used to control in closed loop the temperature on the opposite side of the plate. To solve this problem, Laplace transform is used to obtain the quadrupole model of the direct heat equation and the analytical solution for the transfer function for the inverse problem. The resulting hyperbolic functions are approximated by Taylor expansions to facilitate the real-time closed loop temperature control formulation. Simulation results illustrate the advantages and permit to identify the limitations of using inverse problem to closed loop control temperature of a plate.

1 INTRODUCTION

In a metal plate the temperature distribution is characterized by the fast decay with regards to frequency. The goal of the paper consists in applying input temperature at one side in order to modify the temperature on the other side of the plate; in closed loop control this is approached using the inverse problem solution, known to lead to an ill-posed problem (Maillet, et al., 2000), (Beck et al., 1985). There are many methods to address this ill-posed problem and an investigation is required to find out a suitable one for each application. In this paper is searched a suitable solution for closed loop control of a plate temperature. The books (Maillet, et al., 2000), (Beck, et al., 1985) and (Necsulescu, 2009) presented a variety of solutions for solving inverse heat transfer problems in case of temperature monitoring for plates. Feng et al in 2010 solved the problem of heat conduction over a finite slab to estimate temperature and heat flux on the front surface of a plate from the back surface measurement, (Feng, et al., 2010) and (Feng, et al., 2010). Feng et al. in 2011 solved the same problem using a 1-Dimensional (1D) modal expansion (Feng, et al., 2010). Fan et al. obtained temperature distribution on one side of a flat plate by solving the inverse problem based on the temperature measurement on the other side of the plate, using the modified 1D correction and the finite volume methods, (Fan, et al., 2009). Monde developed an analytical method to solve inverse heat conduction problem using Laplace transform technique (Monde, 2000). Piazzi and Visioli investigated dynamic inversion using transfer functions (Piazzi, Visioli, 2001).

In this paper the 1D heat conduction equation is formulated in the Laplace domain to determine the hyperbolic transfer functions relating input and output temperature of a thin plate for both direct and inverse problems. In this case, hyperbolical functions depend on square root of complex variable s and this does not facilitate real-time applications. Closed loop control problem differs from the known monitoring problems and from open-loop control problems (Necsulescu, Jarrah, 2016). For real-time applications, this is approached using finite Taylor expansions of the hyperbolic functions that permit to obtain transfer functions that approximate hyperbolic functions for a given frequency domain.

Temperature control of metal plates is investigated for the case of heating one side to bring the temperature on the other side at a desired value. Earlier attempts to solve the ill-posed inverse problem of indirect temperature estimation referred to the study of overheating of the outer shell of a rocket entering the atmosphere using temperature measurement from inside (Beck, et al., 1985). Closed-loop control of plate temperature is applicable to achieving accurate temperature output of heating plates and to inside tanks temperature control using outside heating.

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2 SYSTEM MODEL AND CLOSED-LOOP CONTROLLER

The 1D heat conduction equation in complex domain is:

$$\frac{d^2\theta(z,s)}{dz^2} = \frac{s}{\alpha}\theta(z,s) \quad (1)$$

where θ is the temperature and z is the 1D position variable 0 < z < L for a plate of thickness L. α is thermal diffusivity.

Boundary conditions suitable for this case are the following:

$$\theta_1(0,s) = A\frac{\omega}{S^2+\omega^2}, \qquad \theta_2(L,s) = \text{free} \quad (2)$$

$$\emptyset_1(0,s) = \text{free}, \qquad \emptyset_2(L,s) = 0 \quad (3)$$

where $\emptyset$ is the heat flux and $\theta_1(0,s)$ is the Laplace transform of $\theta_1(0,t) = A\sin\omega t$.

The equations 2 and 3 define the thermal quadrupole ends, $\theta_1$and $\emptyset_1$ for input and $\theta_2$and $\emptyset_2$for output, [1].

These boundary conditions were chosen for the investigation of temperature control with sinusoidal input $\theta_1(0,t) = A\sin\omega t$ resulting in the temperature output $\theta_2(L,t)$on the opposite side of the plate of thickness L. The heat flux $\emptyset_1(0,t)$ results from the imposed$\theta_1(0,t)$, while heat flux $\emptyset_2(L,t)$ corresponds to isolated side of the plate [9].

The solution of this equation is [2, 3]:

$$\theta(z,s) = A_1\cosh(Kz) + A_2\sinh(Kz) \quad (4)$$

The heat flux is given by

$$\emptyset(z,s) = -Ks\frac{d\theta}{ds} \quad (5)$$

where

$$K = \sqrt{\frac{s}{\alpha}} \quad (6)$$

Applying boundary conditions, equations (2) and (3), to equation 4, gives the following:

$$A_1 = A\frac{\omega}{S^2+\omega^2}, \qquad A_2 = -A\frac{\omega}{S^2+\omega^2}\tanh(KL) \quad (7)$$

For the aboveA_1andA_2, the solutions become;

$$\theta(z,s) = A\frac{\omega}{S^2+\omega^2}[\cosh(Kz) - \tanh(KL)\sinh(Kz)] \quad (8)$$

$$\emptyset(z,s) = -KsA\frac{\omega}{S^2+\omega^2}[\cosh(Kz) - \tanh(KL)\sinh(Kz)] \quad (9)$$

The dynamics of boundary temperatures $\theta_1$and θ_2:

$$\theta_1 = \theta(0,s) = A\frac{\omega}{S^2+\omega^2} \quad (10)$$

$$\theta_2 = \theta(L,s) = A\frac{w}{S^2+\omega^2}[\cosh(KL) - \tanh(KL)\sinh(KL)] = A\frac{\omega}{S^2+\omega^2}[1/\cosh(KL)] \quad (11)$$

The transfer function of the direct problem linking θ_2 to $\theta_1$is

$$G_1 = \frac{\theta_2}{\theta_1} = \left[\frac{1}{\cosh(KL)}\right] = \text{sech}(KL) \quad (12)$$

The transfer function for the inverse problem [1-3] is

$$G_2 = \frac{1}{G_1} = \cosh(KL) \quad (13)$$

The closed loop control block diagram is shown in Figure 1 for unity feedback and proportional control constant k.

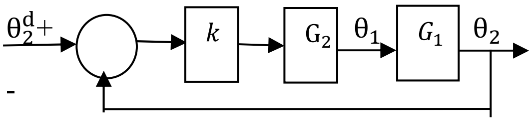


Figure 1: Closed loop controller block diagram.

In this approach for closed loop control of plate temperature, the proposed linear controller, kG_2 resembles the polynomial and model predictive controllers. The merit of the proposal consists in a novel controller design, where in the transfer function approximation of the hyperbolic functions, the order of the polynomials is chosen such that the response is for the desired range of frequencies. This permits to avoid arriving to an ill-posed problem by limiting the inverse problem to lower frequency domain adequate for temperature control. This solution differs from the known regularization approach of ill-posed problems and is particularly suitable for real-time applications [1, 2].

MATLAB™ and Simulink™ are used for simulating the above system.

In this formulation, the hyperbolic functions G_1 and G2 contain square root of s

$$x = KL = \sqrt{\frac{s}{\alpha}}L \quad (14)$$

The design of the closed loop temperature control of the plate requires the derivation of transfer functions.

Taylor series expansion provides equations in s, given that it results in even indexed terms only. For G_1, Taylor series expansion is

$$G_1 = \text{sech}(x) = \sum_{n=0}^{\infty}\left(\frac{E_n}{n!}\right)x^n \text{for} |x| < \pi/2 \quad (15)$$

where Euler numbers E_n are zero for odd-indexed numbers, while even indexed numbers are

$E_0=1$
$E_2=-1$
$E_4=5$
$E_6=-61$
$E_8=1385$
$E_{10}=-50521$
$E_{12}=2702765$
$E_{14}=-199360981$
$E_{16}=19391512145$
$E_{18}=-2404879675441$ etc.

For $G_2 = \cosh(x)$, also an even function, results

$$G_2 = \cosh(x) = \sum_{n=0}^{\infty} \left(\frac{1}{(2n)!}\right) x^{2n} \qquad (16)$$

In order to avoid the limitation of sech(x) to $|x|<\pi/2$, the alternative form sech(x)=1/cosh(x) is used, given that cosh(x) has no domain limitation. The above Taylor series expansions of $1/G_1=\cosh(x)$ and $G2=\cosh(x)$ contain only even-indexed terms, and give integer number powers polynomials in s for simulation. For computation, the infinite series are truncated to finite number of terms, chosen subject to acceptable approximation error. This polynomial approximation is particularly useful in real-time control of the plate temperature.

For the Simulink simulation, Taylor expansion of G_1, direct problem transfer function will be limited to N terms, and G_2, inverse problem transfer function is limited to M terms. For the transfer function G_1*G2, N and M are chosen such that N>M. The simulated item is a thin Aluminum plate has the thickness L = 0.03 [m] and thermal diffusivity α= 9.715e-5 [m^2/sec], such that:

$$x= KL = \sqrt{\frac{s}{\alpha}}L=\sqrt{\frac{s}{9.715*10-5}}\,0.03 \qquad (17)$$

For the simulations were chosen M=4 and N=6, i.e. N >M:

$$G_2(s) = 1 + 4.632s + 3.576s^2 + 1.104s^3 \qquad (18)$$

$$G_1(s) = \frac{1}{\cosh(KL)} = 1/(1 + 4.632s + 3.576s^2 + 1.104s^3 + 0.1827s^4 + 0.0188s^5) \qquad (19)$$

Closed loop equivalent transfer function is

$$\frac{\theta_2}{\theta_2^d} = \frac{kG1*G2}{1+kG1*G2} \qquad (20)$$

Actual implementation of the closed loop control using the inverse problem can be achieved using the time domain differential operator equivalent of $G_2(s)$

$$G_2= 1+ 4.632\, d/dt + 3.57\, d^2/dt^2+ 1.104 d^3/dt^3 \qquad (21)$$

Obviously, in actual implementation, direct problem is replaced by the physical relationship of $\theta_2(L,t)$ function of $\theta_1(0,t)$.

3 RESULTS AND DISCUSSION

Simulations were carried out for closed loop control for different values of input frequency and for the desired sinusoidal temperature amplitude of 20^0 above the original temperature.

Simulations were carried for the direct problem G_1 for N=6 while for inverse problem G_2 for M= 4. The input was $\theta_1(0,t)$ =20sin(ωt). Simulation results for ω = 0.1, 1, 5, 10 and 20 rad/sec are shown in Figure 2 for k=1 and Figure 3 for k=10.

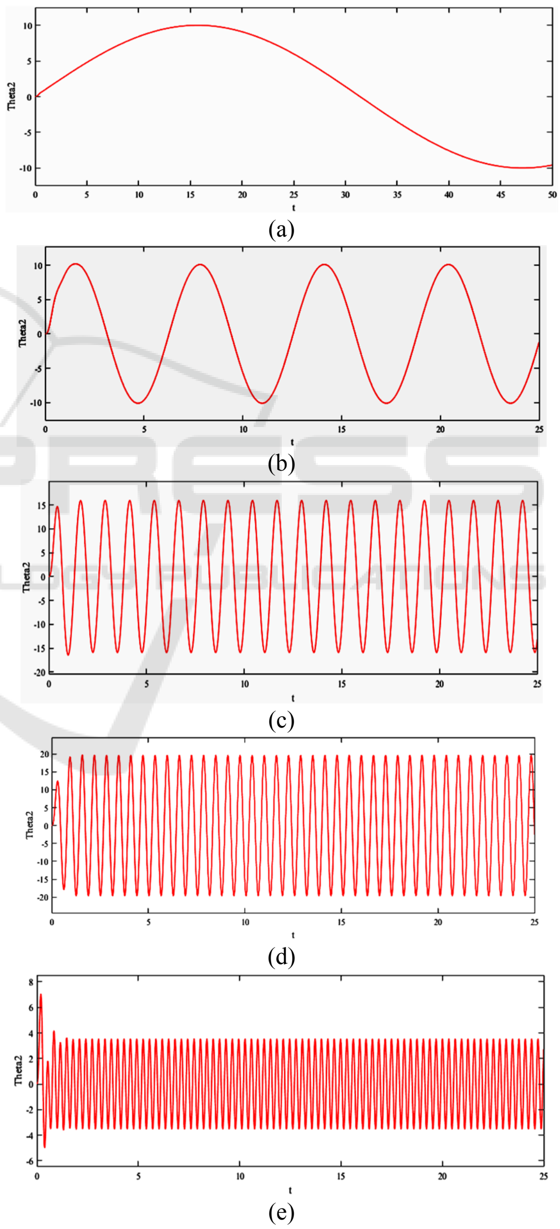


Figure 2: Closed loop control response for M=4, N=6 k=1 and ω = (a) 0.1 Hz, (b) 1 Hz, (c) 5 Hz, (d) 10 Hz, (e) 20 rad/sec.

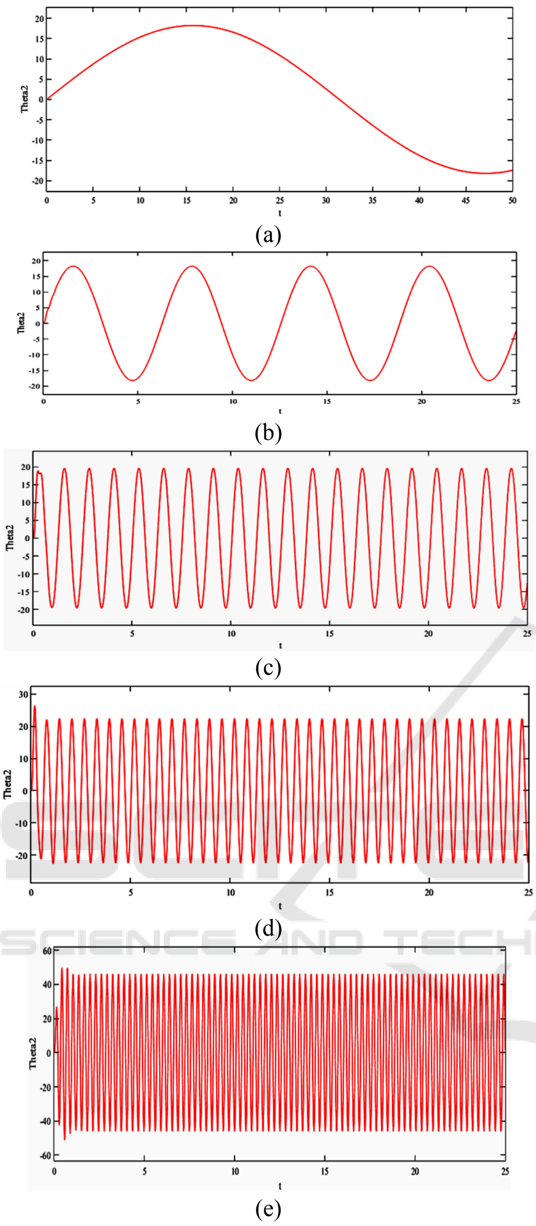


Figure 3: Closed loop control response θ2for M=4, N=6 k=10 and ω = (a) 0.1 Hz, (b) 1 Hz, (c) 5 Hz, (d) 10 Hz, (e) 20 rad/sec.

The simulation results in Figure 2 for k=1 and in Figure 3for k=10 represent the outputs of the closed loop control θ_2 with N=6, M=4 terms for ω = 0.1, 1, 5, 10 and 20 rad/sec. The output temperature $\theta_2$results in Figure 2 and 3, for lower frequencies of 0.1 and 1 rad/sec, compared to desired one, 20 sinωt, are very close. The results for output temperature θ_2, for higher frequencies of 5 and 10 rad/sec, compared to desired one, 20 sinωt, and of the command temperature θ_1, are significantly different. This can be explained by the very high amplitudes of the output of the inverse problem, shown in Figure 4, which lead eventually to an ill-posed inverse problem at higher frequencies, particularly with regard to parameters L and α uncertainty.

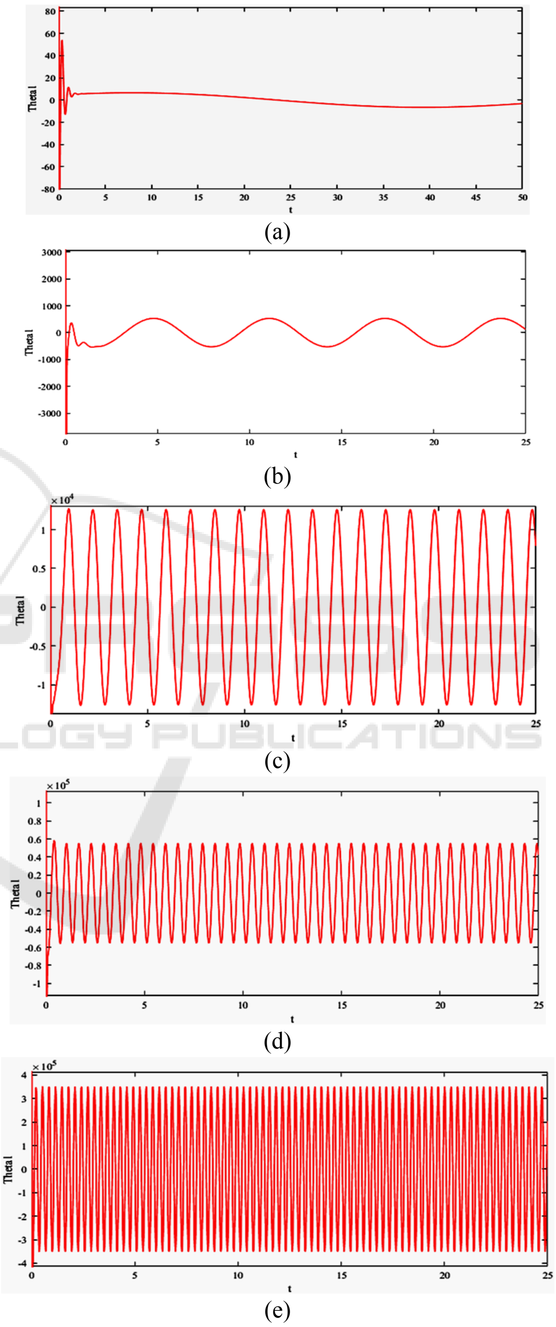
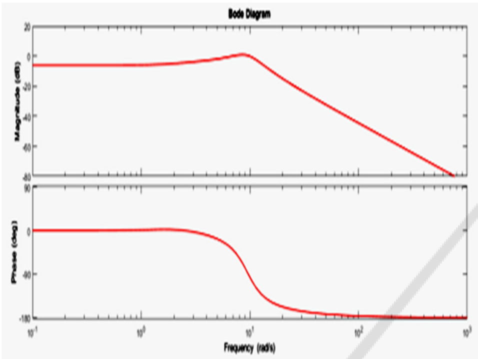
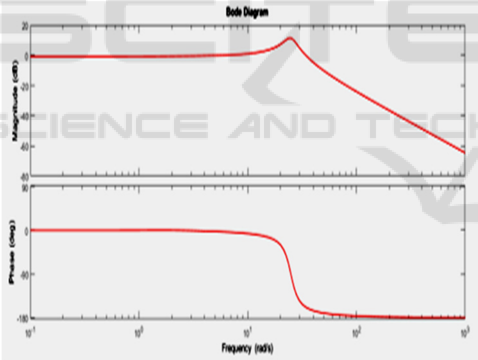


Figure 4: Inverse problem response θ1 for M=4, N=6 k=1 and ω = (a) 0.1 Hz, (b) 1 Hz, (c) 5 Hz, (d) 10 Hz, (e) 20 rad/sec.

For ω = 20, the amplitude is significantly lower than in for lower frequencies in Figure 2 (e) and somewhat lower in Figure. 3(e). Bode diagrams of open loop control transfer function in Figure. 5 explain this by indicating significantly lower magnitudes for ω > 11 rad/sec in Figure 5 (a) for k=1 and for ω >20 rad/sec in Figure 5 (b) for k=10. In the case of open loop control, since there is no feedback from the output, parameter uncertainty and disturbance effects cannot be reduced [9]. Figure 5 shows the Bode diagram of closed loop control transfer function for N=6 and M=4. and for k=1 in (a) and k=10 in (b).



(a)



(b)

Figure 5: Bode diagram of closed loop control transfer function for N=6 and M=4.and (a) k=1and (b) k=10.

4 CONCLUSIONS

The temperature on the one face of a plate can be controlled in real-time to a desired value from the other face using the proposed closed loop approach, based on inverse problem solution. Simulation results indicate the advantages and the limitations of this approach. Closed loop control has to be further investigated to improve the performance and range of applications to multi-layer plates.

REFERENCES

Beck J. et al., 1985, Inverse Heat Conduction, *John Wiley& Sons*.

Fan C. et al., 2009, "A simple method for inverse estimation of surface temperature distribution on a flat plate," *Inverse Problems in Science and Engineering, Vol. 17, No. 7*, pp. 885-899.

Feng Z. et al., 2010, "Real-time solution of heat conduction in a finite slab for inverse analysis," *International Journal of Thermal Science, vol. 49*, pp. 762-768.

Feng Z. et al., 2010, "Temperature and heat flux estimation from sampled transient sensor measurements," *International Journal of Thermal Sciences, vol. 49*, pp. 2385-2390.

Feng Z. et al., 2011, "Estimation of front surface temperature and heat flux of a locally heated plate from distributed sensor data on the back surface," *International Journal of Heat and Mass Transfer, vol. 54*, pp. 3431-3439.

Maillet D. et al., 2000, Thermal Quadrupoles: Solving the Heat Equation through Integral Transforms, *John Willey & Sons*.

Monde M.,2000, "Analytical method in inverse heat transfer problem using Laplace transform technique," *International Journal of Heat and Mass Transfer, vol. 43*, pp. 3965-3975.

Necsulescu D., 2009, Advanced Mechatronics: Monitoring and Control of Spatially Distributed Systems, *Word Scientific*.

Necsulescu D., Jarrah B., 2016, Temperature Control of Thin Plates, *CDSR Conf.*, Ottawa.

Piazzi A., Visioli A., 2001, Robust Set-Point Constrained Regulation via Dynamic Inversion, *Int. Journal of Robust and Nonlinear Control, 11*, pp. 1-22.