Experimental Results of Trajectory Tracking Control of Robot Manipulator using Time Varying Sliding Mode Controller

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Abstract: The author et al. proposed the design method of the sliding mode controller for the trajectory tracking control problem of nonlinear systems. This controller consists of the conventional sliding mode control and the pole placement controller for the the linear time varying approximate model of the nonlinear system around the desired trajectory. In this paper, this controller is applied to the trajectory tracking control of the actual 2-link robot manipulator, and, experimental results are shown.

1 INTRODUCTION

The pole placement control for linear time varying systems is argued in (Nguyen,1987)(Valášek,1995).

The author et al. proposed the simple design method of the pole placement controller for linear time varying systems using the concept of the relative degree of the system (Mutoh,2011) (Mutoh and Kimura,2011). Using this controller, a time varying closed loop system becomes equivalent to some linear time invariant system that has desired constant eigenvalues, by the state feedback. This implies that if we apply the pole placement controller to a linear time varying system, any control technique for linear time invariant systems can be applied to the equivalent time invariant closed loop system.

From this point of view, the authors (Mutoh and Kogure,2014) proposed to make use of this technique for designing the sliding mode controller for linear time varying systems and this controller was applied to the trajectory tracking control of nonlinear systems. The nonlinear system has a linear time varying approximate model around some desired trajectory. The design procedure is as follow. The first step is to find the pole placement state feedback for the linear time varying approximate system, by which the closed loop system is equivalent to some linear time-invariant system. Then, by using the conventional sliding mode control technique, the sliding mode control input for this linear time invariant system can be obtained (Utkin,1992). Finally, using an equivalent time varying transformation matrix, this control input can be transformed into the sliding mode control for the original linear time varying system. By this controller, the linear time varying approximate model around the desired trajectory ia stabilized, which implies the trajectory tracking controller for this nonlinear system is obtained.

In this paper, this type of controller is applied to the actual 2-link robot manipulator, and, the experimental results are shown. Here, both of continuous and discrete sliding mode controllers are used. In the following, the pole placement controller for linear time varying systems and a continuous and discrete types of the sliding mode controllers are summarized in Section 2 and 3, respectively. Then, Section 4 presents how these controllers are used for the trajectory tracking control of nonlinear systems. And, finally, some experimental results are shown in Section 5.

2 POLE PLACEMENT FOR LINEAR TIME VARYING SYSTEMS

2.1 Controllability of Linear Time Varying Systems

Consider the following linear time-varying multi-input system.

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \qquad (1)$$

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Here, $x(t) \in R^n$ and $u(t) \in R^m$ are the state variable and the input signal, respectively. $A(t) \in R^{n\times n}$ and $B(t) \in R^{n\times m}$ are time varying coefficient matrices, which are bounded and smooth functions of t.

The matrix $B(t)$ is written as follows, using its column vectors $b_i(t) \in R^n$ $(i = 1, \cdots, m)$.

$$B(t) = \begin{bmatrix} b_1(t) & b_2(t) & \cdots & b_m(t) \end{bmatrix} \tag{2}$$

Let $b_k^i(t) \in R^n$ be defined by the following recursive equations.

$$\begin{cases} b_k^0(t) = b_k(t) \\ b_k^i(t) = A(t)b_k^{i-1}(t) - \dot{b}_k^{i-1}(t) \end{cases} \tag{3}$$
$$k = 1, 2 \cdots m,\ i = 1, 2 \cdots$$

Then, the controllability matrix of the system (1) is defined as follows.

$$U_c = [b_1^0(t) \cdots b_m^0(t)| \cdots |b_1^{n-1}(t) \cdots b_m^{n-1}(t)] \tag{4}$$

Theorem 1. *The system* (1) is completely controllable if and only if

$$rankU_c(t) = n \quad {}^{\forall}t \tag{5}$$

If the system (1) is completely controllable, the controllability indices, $\mu_1, \mu_2, \cdots, \mu_m$, can be defined, and

$$\sum_{i=1}^{m} \mu_i = n. \tag{6}$$

Using these controllability indices, the following nonsingular matrix, $R(t)$, can be defined.

$$R(t) = \begin{bmatrix} b_1^0(t) \cdots b_1^{\mu_1-1}(t)| \cdots |b_m^0(t) \cdots b_m^{\mu_m-1}(t) \end{bmatrix} \tag{7}$$

In this paper, it is assumed that the system is completely controllable, and, its controllability indices satisfy the inequality, $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_m$, without loss of generality.

2.2 Time Varying Pole Placement Control

The problem of pole placement control for the time-varying system (1) is to find the state feedback so that the time-varying closed loop system is equivalent to some time-invariant system which has desired constant eigenvalues.

Suppose that the system (1) is completely controllable and has the controllability indices, $\mu_1, \mu_2, \cdots, \mu_m$. Let $\tilde{C}(t) \in R^{m\times n}$ be defined by

$$\tilde{C}(t) = W(t)R^{-1}(t) \tag{8}$$

where

$$\begin{cases} W(t) = diag(w_1(t), w_2(t), \cdots, w_m(t)) \\ w_i(t) = [0, \cdots, 0, \lambda_i(t)] \in R^{1\times\mu_i}(i = 1, \cdots, m) \end{cases} \tag{9}$$
$$\lambda_i(t) \neq 0.$$

Using $\tilde{C}(t)$, a new output signal $\tilde{y}(t)$ is defined as follows.

$$\tilde{y}(t) = \tilde{C}(t)x(t) \tag{10}$$

Then, the vector relative degree from $u(t)$ to $\tilde{y}(t)$ becomes $(\mu_1, \mu_2, \cdots, \mu_m)$ (Mutoh and Kimura,2011).

Let $\tilde{y}(t)$ and $\tilde{C}(t)$ be defined by

$$\tilde{y}(t) = \begin{bmatrix} \tilde{y}_1(t) \\ \vdots \\ \tilde{y}_m(t) \end{bmatrix}, \tilde{C}(t) = \begin{bmatrix} \tilde{c}_1(t) \\ \vdots \\ \tilde{c}_m(t) \end{bmatrix} \tag{11}$$

where, $\tilde{y}_i(t) \in R^1$ and $\tilde{c}_i(t) \in R^{1\times n}$.

From this, the pole placement state feedback is obtained in the following procedure. Let α_j^i be the coefficients of a desired stable polynomial of the differential operator p.

$$\alpha^i(p) = p^{\mu_i} + \alpha_{\mu_i-1}^i p^{\mu_i-1} + \cdots + \alpha_0^i \tag{12}$$
$$(i = 1, \cdots, m)$$

Since, the vector relative degree from $u(t)$ to $\tilde{y}(t)$ is $\mu_1, \mu_2, \cdots, \mu_m$, there exist a matrix $D(t) \in R^{m\times n}$ and a nonsingular matrix $\Lambda(t) \in R^{m\times m}$ satisfying the following equation.

$$\begin{bmatrix} \alpha^1(p) & & \\ & \ddots & \\ & & \alpha^m(p) \end{bmatrix} \tilde{y}(t) = D(t)x(t) + \Lambda(t)u(t) \tag{13}$$

Actually, $D(t)$ and $\Lambda(t)$ are given by

$$D(t) = \begin{bmatrix} D_1(t) \\ D_2(t) \\ \vdots \\ D_m(t) \end{bmatrix}, \quad \Lambda(t) = \begin{bmatrix} \Lambda_1(t) \\ \Lambda_2(t) \\ \vdots \\ \Lambda_m(t) \end{bmatrix} \tag{14}$$

and

$$D_i(t) = [\alpha_0^i, \alpha_1^i, \cdots \alpha_{\mu_i-1}^i, 1] \begin{bmatrix} \tilde{c}_i^0(t) \\ \tilde{c}_i^1(t) \\ \vdots \\ \tilde{c}_i^{\mu_i}(t) \end{bmatrix} \tag{15}$$
$$\Lambda_i(t) = [0, \cdots, 0, \lambda_i(t), \gamma_{i(i+1)}(t), \cdots, \gamma_{ij}(t)]$$

where $\tilde{c}_i^k(t) \in R^{1\times n}$ and $\gamma_{ij}(t) \in R^1$ are defined as follows.

$$\begin{cases} \tilde{c}_i^0(t) = \tilde{c}_i(t) \\ \tilde{c}_i^{j+1}(t) = \tilde{c}_i^j(t)A(t) + \dot{\tilde{c}}_i^j(t) \end{cases} \tag{16}$$
$$i = 1, 2 \cdots m, \quad j = 1, 2 \cdots$$

$$\gamma_{ij}(t) = c_i^{\mu_i-1}(t)b_j(t) \quad j = i+1,\cdots,m \tag{17}$$

Thus, by the state feedback

$$u(t) = -\Lambda^{-1}(t)D(t)x(t) \tag{18}$$

the closed loop system becomes

$$\begin{bmatrix} \alpha^1(p) & & \\ & \ddots & \\ & & \alpha^m(p) \end{bmatrix} \tilde{y}(t) = 0. \tag{19}$$

This system has the following state realization,

$$\dot{\omega}(t) = A^*\omega(t) = \begin{bmatrix} A_1^* & & 0 \\ & \ddots & \\ 0 & & A_m^* \end{bmatrix} \omega(t) \tag{20}$$

where

$$A_i^* = \begin{bmatrix} 0 & 1 & & & 0 \\ \vdots & & \ddots & & \vdots \\ \vdots & & & & 1 \\ -\alpha_0^i & \dots & \dots & & -\alpha_{\mu_i-1}^i \end{bmatrix} \quad (i = 1,\dots,m) \tag{21}$$

and,

$$\det(sI - A^*) = \alpha^1(s)\cdot\alpha^2(s)\cdots\alpha^m(s). \tag{22}$$

Here, $\omega(t) \in R^n$ is a new state vector, and is defined by

$$\omega(t) = \begin{bmatrix} \tilde{y}_1(t) \\ \vdots \\ \tilde{y}_1^{(\mu_1-1)}(t) \\ \hline \vdots \\ \hline \tilde{y}_m(t) \\ \vdots \\ \tilde{y}_m^{(\mu_m-1)}(t) \end{bmatrix}. \tag{23}$$

This implies that $\omega(t)$ and the original state variable $x(t)$ satisfy the relation

$$\omega(t) = T(t)x(t) \tag{24}$$

where

$$T(t) = \begin{bmatrix} \tilde{c}_1^0(t) \\ \vdots \\ \tilde{c}_1^{\mu_1-1}(t) \\ \hline \vdots \\ \hline \tilde{c}_m^0(t) \\ \vdots \\ \tilde{c}_m^{\mu_m-1}(t) \end{bmatrix}. \tag{25}$$

Hence, the closed loop system is equivalent to the time invariant linear system which has the desired closed loop poles, i.e.,

$$T(t)(A(t) - B(t)\Lambda^{-1}(t)D(t))T^{-1}(t) - T(t)\dot{T}^{-1}(t) = A^* \tag{26}$$

The non-singularity of $T(t)$ is guaranteed by the controllability of the system (1).

Note that $T(t)$ is Lyapunov transformation if it is non-singular and both of $T(t)$ and $T^{-1}(t)$ are continuous and bounded for all t. It is well known that the exponential stability is preserved between two equivalent linear time-varying systems if the transformation matrix is Lyapunov transformation (Rugh,1993)(Chen,1999). Then, to guarantee the stability of the closed loop system, $T(t)$ should be the Lyapunov transformation.

From the above, the pole placement procedure is summarized as follows.

STEP 1 Using the controllability matrix, $U_c(t)$, check the controllability of the system (1) and find the controllability indices μ_i $(i = 1,\cdots,m)$.

STEP 2 Calculate $\tilde{C}(t)$ using (8).

STEP 3 From $\tilde{C}(t)$, calculate $\tilde{c}_i^k(t)$ and $\gamma_{ij}(t)$ using (16) and (17).

STEP 4 Determine the coefficients, α_j^i of desired closed loop characteristic polynomials in (12).

STEP 5 Using (14) and (15) with the parameters obtained in the above STEP 3 and 4, the pole placement state feedback is given in (18).

3 SLIDING MODE CONTROLLER DESIGN

3.1 Continuous Sliding Mode Control

In this section, the sliding mode controller design for the linear time varying system (1) is summarized. In the previous section, for the given time varying system (1), we design the pole placement state feedback (18). Here, the new input signal $v(t)$ is added to it, then the pole place ment state feedback becomes as follows.

$$u(t) = \Lambda^{-1}(t)(-D(t)x(t) + v(t)) \tag{27}$$

Hence, the closed loop system,

$$\dot{x}(t) = (A(t) - B(t)\Lambda^{-1}(t)D(t))x(t) + B(t)v(t) \tag{28}$$

becomes equivalent to

$$\dot{\omega}(t) = \begin{bmatrix} A_1^* & & 0 \\ & \ddots & \\ 0 & & A_m^* \end{bmatrix} \omega(t) + B^*v(t) \tag{29}$$

where

$$B^* = diag[b_1^*,\ b_2^*,\ \cdots,\ b_m^*]$$
$$b_i^* = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{\mu_i}, \quad i = 1,\ \cdots,\ m \quad (30)$$

and the transformation matrix $T(t)$ between $x(t)$ and $\omega(t)$ are defined by (24), (25).

To design the sliding mode controller for the time varying system (28), we first design the conventional sliding mode control input $v(t)$ for the equivalent linear time invariant system (29), and then, transform $v(t)$ into the sliding mode control input for the system (28), using the relation (24)(25).

$\omega(t)$ and $v(t)$ can be written as follows.

$$\omega(t) = \begin{bmatrix} \omega_1(t) \\ \vdots \\ \omega_m(t) \end{bmatrix}, \quad v(t) = \begin{bmatrix} v_1(t) \\ \vdots \\ v_m(t) \end{bmatrix} \quad (31)$$
$$\omega_i(t) \in R^{\mu_i},\ v_i(t) \in R^1 \quad (i = 1, \cdots, m)$$

Then, the system (29) is presented as the set of the following m subsystems.

$$\dot{\omega}_i(t) = A_i^* \omega_i(t) + b_i^* v_i(t), \quad (i = 1, \cdots, m) \quad (32)$$

Here, A_i^* is defined by (21). As well known, the conventional sliding mode controller for i-the subsystem is obtained as follows.

First, let the desired stable characteristic polynomial of the i-th sliding dynamics be chosen as follows.

$$s^i(p) = p^{\mu_i - 1} + s^i_{\mu_i - 2} p^{\mu_i - 2} + \cdots + s^i_0 \quad (33)$$

Then, the i-th stable sliding surface is given by the following hyper surface,

$$S_i^T \omega_i(t) = [s^i_0,\ \cdots, s^i_{\mu_i - 2},\ 1\ ]\omega_i(t) = 0. \quad (34)$$

And, it is also well known that the i-th sliding control input $v_i(t)$ which makes the state variable move toward the sliding surface can be obtained by

$$v_i(t) = -(S_i^T b_i^*)^{-1}\{S_i^T A_i^* \omega_i(t) + q_i\ sgn(\sigma_i) + k_i f_i(\sigma_i)\}$$
$$= -\{S_i^T A_i^* \omega_i(t) + q_i\ sgn(\sigma_i) + k_i f_i(\sigma_i)\} \quad (35)$$

where

$$\sigma_i = S_i^T \omega_i(t) \quad (36)$$

and $q_i > 0$ and $k_i > 0$ are constant parameters and $f_i(\sigma_i)$ is a function such that $\sigma_i f_i(\sigma_i) > 0$. In fact, it is readily shown that, using (35), we have the following Lyapunov function.

$$V = \frac{1}{2}\sum_{i=1}^{m} \sigma_i^2 > 0, \quad \dot{V} = \sum_{i=1}^{m} \sigma_i \dot{\sigma}_i < 0 \quad (37)$$

Hence, from the above, the pole placement and the sliding mode control input $u(t)$ for the original system (1) becomes as follows, using the original state variable $x(t)$.

$$u(t) = \Lambda^{-1}(t)(-D(t)x(t) + v(t)) \quad (38)$$

Here, the i-th element of $v(t)$ is

$$v_i(t) = -\{S_i^T A_i^* T_i(t)x(t) + q_i\ sgn(\sigma_i) + k_i f_i(\sigma_i)\} \quad (39)$$

and

$$\sigma_i = S_i^T T_i(t)x(t) \quad (i = 1, \cdots, m) \quad (40)$$

where from (24)(25),

$$T_i(t) = \begin{bmatrix} \tilde{c}_i^0(t) \\ \vdots \\ \tilde{c}_i^{\mu_i - 1}(t) \end{bmatrix}. \quad (41)$$

3.2 Discrete Sliding Mode Control

In general, the continuous sliding mode controller has a chattering problem. On the other hand, there is a discrete type of sliding mode controller, which can avoid the chattering problem. In this paper, we also apply this type of discrete sliding mode controller from the practical point of view.

Suppose that the following is a discretized system of i-th subsystem ($i = 1, \cdot, m$)), (32), with the sampling period, T_s.

$$\omega_i(T_s(k+1)) = F_i^* \omega_i(T_s k) + g_i^* v_i(T_s k), \quad (i = 1, \cdots, m) \quad (42)$$

Here, F_i^* and g_i^* have the following forms.

$$F_i^* = \begin{bmatrix} 0 & 1 & & & 0 \\ \vdots & & & \ddots & \vdots \\ \vdots & & & & 1 \\ -\beta_0^i & \dots & \dots & & -\beta^i_{\mu_i - 1} \end{bmatrix} \in R^{\mu_i \times \mu_i}$$
$$g_i^* = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{\mu_i}, \quad (i = 1,\ \cdots,\ m) \quad (43)$$

As the continuous case, let the desired discrete stable characteristic polynomial of the i-th sliding dynamics be chosen as follows.

$$\xi^i(z) = z^{\mu_i - 1} + \xi^i_{\mu_i - 2} z^{\mu_i - 2} + \cdots + \xi^i_0 \quad (44)$$

Here, z is the forward shift operator. Then, the i-th stable sliding surface is given by the following highper surface,

$$S_i^T \omega_i(T_s k) = [\ \xi^i_0,\ \cdots, \xi^i_{\mu_i - 2},\ 1\ ]\omega_i(T_s k) = 0. \quad (45)$$

And, it is also known that the i-th sliding control input $v_i(T_sk)$ which makes the state variable move toward the sliding surface can be obtained as

$$v_i(T_sk) = -S_i^T(F_i^* - I)\omega(T_sk) - \eta_i\sigma_i(T_sk) \quad (46)$$

where

$$\sigma_i(T_sk) = S_i^T\omega_i(T_sk). \quad (47)$$

and

$$0 \leq \eta_i \leq 2. \quad (48)$$

Further more, if $0 \leq \eta_i \leq 1$, the state variable approaches the sliding surface without chattering, and, if $1 \leq \eta_i \leq 2$, with chattering.

From the above, the pole placement and the discrete sliding mode control input $u(t)$ for the original system (1) becomes as follows, using the original state variable $x(t)$.

$$\begin{aligned} u(t) &= \Lambda^{-1}(t)(-D(t)x(t) + Zoh[v(T_sk)]) \\ & \qquad (\ T_sk \leq t \leq T_s(k+1)\ ) \end{aligned} \quad (49)$$

here the i-th element of $v(T_sk)$ is

$$v_i(T_sk) = -S_i^T(F_i^* - I)\omega(T_sk) - \eta_i\sigma_i(T_sk) \quad (50)$$

and

$$\sigma_i = S_i^T T_i(T_sk)x(T_sk) \quad (i = 1, \cdots, m) \quad (51)$$

where from (24)(25),

$$T_i(T_sk) = \begin{bmatrix} \tilde{c}_i^0(T_sk) \\ \vdots \\ \tilde{c}_i^{\mu_i - 1}(T_sk) \end{bmatrix}. \quad (52)$$

Here, $Zoh[\cdot]$ is the zero-order hold.

4 TRAJECTORY TRACKING CONTROL OF NONLINEAR SYSTEMS

Consider the following non-linear system.

$$\dot{x}(t) = f(x(t), u(t)) \quad (53)$$

Here, $x(t) \in R^n$ and $u(t) \in R^m$ are the state variable and the input signal. Let $x^*(t)$ and $u^*(t)$ be some desired trajectory and the desired input for $x^*(t)$.

The problem is to design a controller to track this desired trajectory $x^*(t)$ stably around it. This can be done by stabilizing this trajectory in the neighborhood of $x^*(t)$ and $u^*(t)$. Let $\Delta x(t)$ and $\Delta u(t)$ be defined by

$$\begin{cases} \Delta x(t) = x(t) - x^*(t) \\ \Delta u(t) = u(t) - u^*(t). \end{cases}$$

Figure 1: Two-Link Manipulator(SR-402DDII).

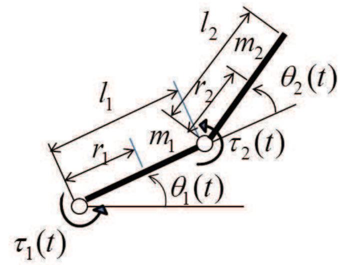

Figure 2: A Model of the 2-Link Manipulator.

Then, we have a linear time-varying approximate model around $x^*(t)$ and $u^*(t)$ as follows.

$$\Delta\dot{x}(t) = A(t)\Delta x(t) + B(t)\Delta u(t) \quad (54)$$

$$\begin{cases} A(t) = \dfrac{\partial}{\partial x} f(x^*(t), u^*(t)) \\ B(t) = \dfrac{\partial}{\partial u} f(x^*(t), u^*(t)) \end{cases} \quad (55)$$

Then, using time-varying sliding mode control technique, error equation can be stabilized around the desired trajectory $x^*(t)$ and $u^*(t)$. For this purpose, the time varying pole placement control is first applied to this linear time varying approximate system. By which, the closed system is equivalent to some linear time invariant system. Next, verious types of sliding mode controllers are applied to this time invariant system to obtain the robustness against disturbance.

5 EXPERIMENTAL RESULTS

In this Section, the time varying sliding control technique was applied to the trajectory tracking problem of the actual 2-link robot manipulator, and some experimental results are shown.

The manipulator used for the experiment is shown in Fig.1 and its model is depicted in Fig.2. The motion equation of the manipulator is described as follows.

$$M(\theta(t))\ddot{\theta}(t) + C(\theta(t), \dot{\theta}(t))\dot{\theta}(t) + D(\dot{\theta}(t)) = \tau(t) \quad (56)$$

where,

$$\theta(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix}$$

$$M(\theta(t)) = \begin{bmatrix} J_1 + J_2 + 2m_2r_2l_1cos\theta_2(t), & J_2 + m_2r_2l_1cos\theta_2(t) \\ J_2 + m_2r_2l_1cos\theta_2(t), & J_2 \end{bmatrix}$$

$$C(\theta(t),\dot{\theta}(t)) = \begin{bmatrix} -2m_2r_2l_1\dot{\theta}_2(t)sin\theta_2(t), & -m_2r_2l_1\dot{\theta}(t)_2sin\theta_2(t) \\ m_2r_2l_1\dot{\theta}_1(t)sin\theta_2(t), & 0 \end{bmatrix}$$

$$D(\dot{\theta}(t)) = \begin{bmatrix} 2sgn(\dot{\theta}_1(t)) \\ 0.25sgn(\dot{\theta}_2(t)) \end{bmatrix}$$

$$J_i = J_{l_i} + m_ir_i^2 \ \ (i = 1,2). \quad (57)$$

Here, $\theta_i(t)$ and $\tau_i(t)$ are a joint angle and an input torque of i-th joint, l_i and r_i are a length of the i-th link and the distance between the i-th joint and the center of gravity of i-th link, and J_{l_i} is the moment of inertia of i-th link about its center of gravity. $D(\dot{\theta}(t))$ is a friction term.

Define a state variable $x(t)$ and an input vector $u(t)$ by

$$x(t) = \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix} \in R^4, \quad u(t) = \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix} \in R^2$$

then, the system, (55), can be rewritten as the following state equation.

$$\begin{aligned} \dot{x}(t) &= f(x(t),u(t)) \\ &= \begin{bmatrix} 0 & I \\ 0 & \Gamma(x(t)) \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \Phi(x(t)) \end{bmatrix} u(t) \end{aligned} \quad (58)$$

where

$$\Gamma(x(t)) = -M(\theta(t))^{-1}C(\theta(t),\dot{\theta}(t)) \in R^{2\times 2}$$

$$\Phi(x(t)) = M(\theta(t))^{-1} \in R^{2\times 2}. \quad (59)$$

$I \in R^{2\times 2}$ is the identity matrix. The values of physical parameters of this system are shown in Table 1.

Table 1: Parameters of the Robot Manipulator.

	variable $(i = 1,2)$	link1 $i = 1$	link2 $i = 2$
Mass[kg]	m_i	3.43	1.55
Length[m]	l_i	0.2	0.2
Center of Gravity[m]	r_i	0.1	0.1
Inertia[kgm^2]	J_{l_i}	0.208	0.03

The desired trajectory of the end portion of the manipulator is given by the following equation, which

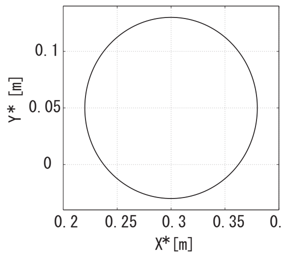

Figure 3: The Desired Trajectory of the End Portion.

is shown in Fig.3

$$X^* = 0.08\cos\frac{\pi}{5}t + 0.3 \quad (60)$$

$$Y^* = 0.08\sin\frac{\pi}{5}t + 0.05 \quad (61)$$

The experimental results are shown in Fig.4 - 14. The results of the combination of the continuous time pole placement and the continuous time sliding mode controller are shown in Fig.4 - 6. The response of the end portion of the manipulator and the control input signals are shown in Fig.4 and Fig.5, respectively. The responses of σ_1 and σ_2 are shown in Fig.6 which

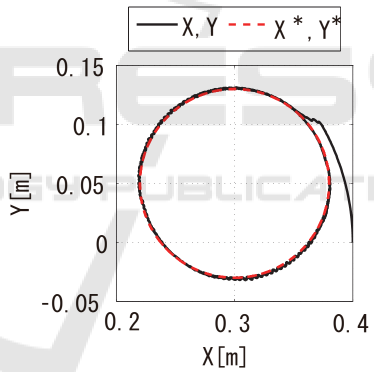

Figure 4: Control Response and Desired Trajectory of the End Portion.

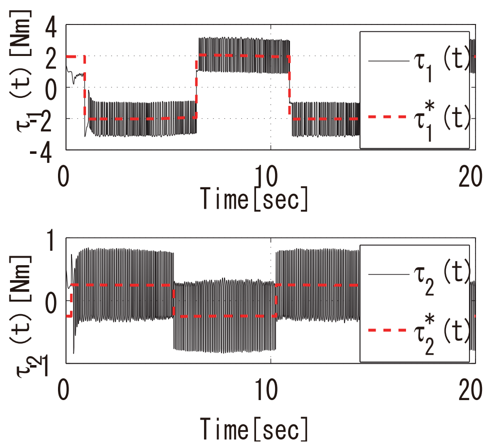

Figure 5: Control Input $u(t)$.

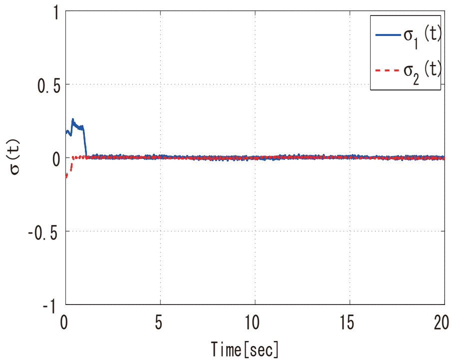

Figure 6: Response of $\sigma_1(t)$ and $\sigma_2(t)$.

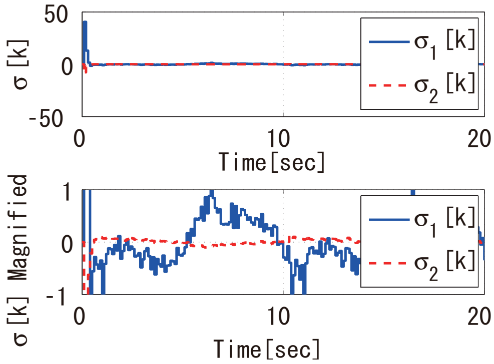

Figure 9: Response of $x(t)$.

implies the sliding mode control works well. However, as shown in Fig.5, the continuous sliding mode controller has chattering problem.

Fig.7 - 9 show the experimental results of the combination of the continuous time pole placement and the discrete sliding mode controller with $\eta_i = 1$. The response of the end portion of the manipulator and the control input signals are shown in Fig.7 and Fig.8, respectively. The responses of σ_1 and σ_2 are shown in Fig.9 which implies the sliding mode control also works well. Compare to the continuous sliding mode case, the chattering of the discrete version of the sliding mode control is very small.

The experimental results of the same controller (the discrete version) with a disturbance are shown in Fig.10-13. The response of the end portion of the manipulator is shown in Fig.10. In this experiment, the disturbance in Fig.11 is added to the input channel. The control input signals is shown in Fig.12. And, the responses of σ_1 and σ_2 are shown in Fig.13.

Fig.14 shows the response of the end portion of the manipulator with only the pole placement controller with the same disturbance as the previous case.

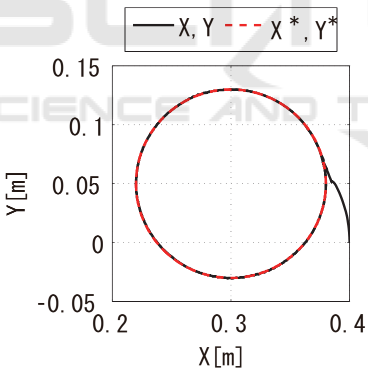

Figure 7: Control Response and Desired Trajectory of the End Portion.

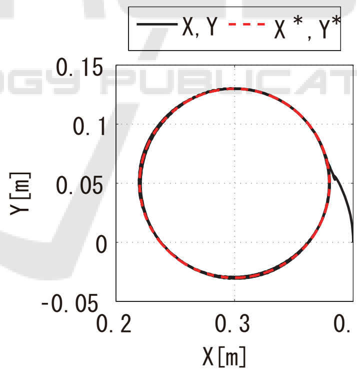

Figure 10: Response of $\Delta x(t)$.

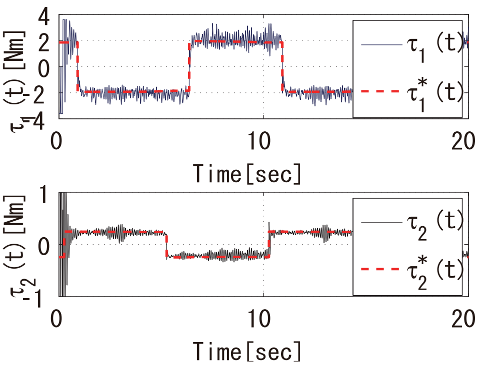

Figure 8: Response of $\Delta u(t)$.

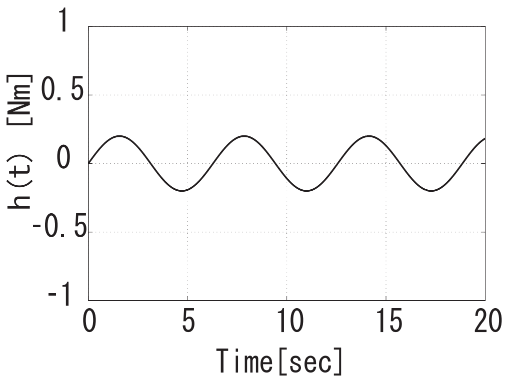

Figure 11: Response of $\Delta u(t)$.

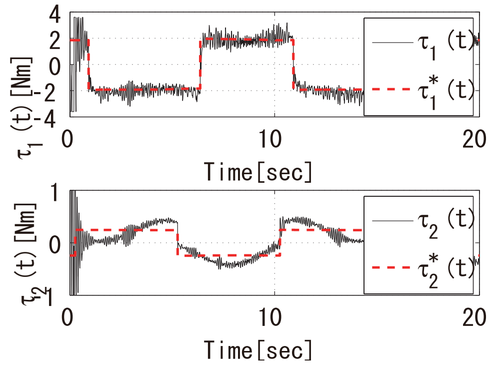

Figure 12: Response of $x(t)$.

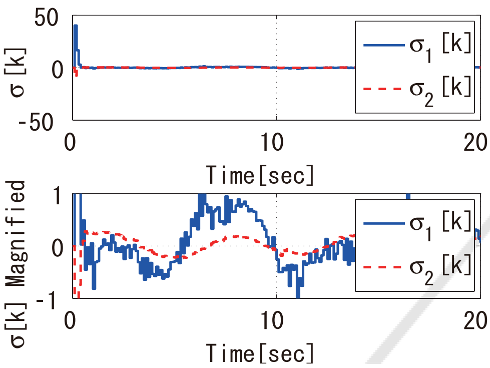

Figure 13: Response of $\Delta u(t)$.

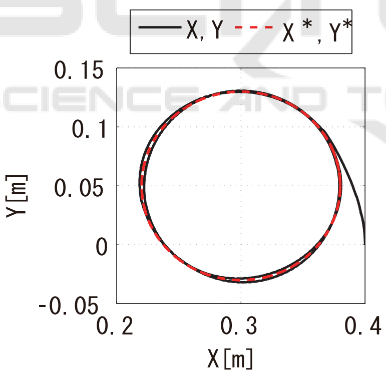

Figure 14: Response of $\Delta u(t)$.

The controller with only the pole placement also has a good performance under the disturbance, but, if we need high accuracy performance, the combination of the continuous pole placement and the discrete sliding mode control is one practical choice for the trajectory tracking control of a practical nonlinear systems.

6 CONCLUSIONS

In this paper, the design procedure of sliding mode controller for linear time-varying system is presented. For this purpose, the time-varying pole placement feedback is used so that the closed loop system is equivalent to some linear time invariant system. Then, the conventional design method of the sliding mode control can be applied to this time invariant system. In this paper, this controller is applied to the actual 2-link robot manipulator, and experimental results were shown. The results show the controller has a good performance with high accuracy under disturbance.

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