Passivity-based Control of Surge and Rotating Stall in Axial Flow Compressors

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Abstract: In this work, we address the stability of compression systems and the active control of performance limiting phenomena: surge and rotating stall. Despite considerable efforts to stabilize axial compressors at efficient operating points, preventing and suppressing rotating stall and surge are still challenging problems. Due to certain passivity properties of the widely used Moore and Greitzer model for axial compressors, a robust passivity-based control approach is applied here to tackle the problem. The main advantage of this approach is that robust stabilization and high performance control can be achieved by simple control laws and limited control efforts. Analytical developments and time-domain simulations demonstrate that the developed control laws can effectively damp out rotating stall and surge limit cycles by throttle and close-coupled valve actuations. The robust performance of the controller is validated in the presence of bounded mass flow and pressure disturbances, as well as model uncertainties.

1 INTRODUCTION

Passivity theory, which provides an energy based perspective in control theory, has been the subject of much research over the last decades (Byrnes et al., 1991; Sepulchre et al., 1997; Willems, 2007). The essential role of energy in the stability and performance of physical systems has resulted in the increasing attention to passivity. Basically, passive systems are a class of processes that dissipate a certain type of physical or virtual energy described by Lyapunov-like functions (Bao and Lee, 2007). The concept of passivity especially plays an important role in robust control. Since passive systems are easy to control, the first step in passive system theory is to render a process passive via either feedback or feedforward. Sufficient robustness to model uncertainties, parameter variations, and external disturbances can be ensured by passivity-based control (PBC) which guarantees the passivity of the system for the whole range of parameters. Achieving passivity with feedback is an appealing issue due to its input-output concept. However, one of the major challenges in feedback passification designs is to make it constructive. The key part of the design procedure is to select a proper output satisfying the required conditions (Sepulchre, et al., 1997). In 1991, Byrnes et al. (Byrnes et al., 1991) derived the conditions under which a nonlinear system can be rendered passive via smooth state feedback and in 2009, Tsai and Wu (Tsai and Wu, 2009) presented a constructive method for robust PBC (RPBC) of a certain class of weakly minimum phase nonlinear uncertain systems. They proposed a control law that renders the system passive and asymptotically stabilizes the closed loop system.

In this work, we utilize RPBC to effectively stabilize nonlinear phenomena in compression systems. Compression systems suffer from two types of nonlinearities with different natures: surge and rotating stall. Rotating stall is a non-axisymmetric perturbation that travels around the annulus of the compressor, while surge is a violent limit-cycle in compressor characteristic that can lead to a flow reversal and large axial oscillations (see (Gu et al., 1999) for more information). Despite the considerable efforts that have been made to

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investigate these phenomena, different aspects of the problem such as sensing, actuating and model-based control are still challenging issues.

From a control point of view, the nonlinear 2D model developed by Moore and Greitzer (Moore and Greitzer, 1986) for constant speed axial compressors (CSACs) dominates recent studies on rotating stall and surge control (Gu et al., 1999). The lumped parameter Moore and Greitzer model (so-called MG3) is based on the first harmonic approximation of rotating stall. This model was developed using Galerkin procedure applied to the original PDE form. In spite of the simple form of the model, it can capture surge and rotating stall nonlinearities and qualitative behavior of the system including bifurcations (see (Hős et al., 2002) for more information).

Remarkable efforts channeled into augmenting MG3 in different ways; among them obtaining higher order accurate model and including the force of actuators (Krstic and Wang, 1997; Leonessa et al., 1997; Mansoux et al., 1994). One of the most promising actuators is the close-coupled valve (CCV). The early work of Dussourd in 1977 (Dussourd et al., 1977) and the work of Simon and Valavani in 1991 (Simon and Valavani, 1991) addressed CCV in compression system control. In 1998, Gravdahl introduced an augmented MG3 model including CCV in error coordinates (Gravdahl, 1998). Recently, once again, this actuator attracted close attention of researchers in surge control ((Bartolini et al., 2008; Liaw et al., 2008; Shehata et al., 2009)).

Gravdahl demonstrated that the two-state simplified form of MG3 including CCV shows certain passivity properties and then applied PBC to develop a surge controller (Gravdahl and Egeland, 1998). This simple proportional PBC law effectively stabilized surge limit cycles. Although the controller was not able to damp out rotating stall, it showed promise for suppressing this hard-to-control nonlinearity. This interesting open problem was suggested as future work by Gravdahl.

Here, we address this problem and design a RPBC to suppress rotating stall in CSACs. The simple proportional and low order form of the developed controller is the first advantage of the applied method. It is not based on full-state feedback (the square amplitude of rotating stall as the third state of MG3 is practically hard to measure) and does not require the detailed knowledge of model parameters, which cannot be accurately estimated. The controller actuates the system with feedback from mass flow and pressure rise by using both the throttle valve and CCV. Simulation results corroborating the analytical developments demonstrate that the applied RPBC effectively damps out the developed rotating stall and stabilizes efficient operating points (OPs) in the presence of bounded external disturbances and model uncertainties. The utilized approach eliminates surge limit cycles as well.

The rest of the paper is organized as follows. In Section 2, we start by reviewing the Gravdahl model representing CSACs comprising CCV. Section 3 presents the control design and Section 4 reports time-domain simulations. Finally, some conclusions about this work are drawn in Section 5.

2 AXIAL COMPRESSORS MODELS

Here, we briefly review Gravdahl model for CSACs including CCV and throttle actuators. The compressor comprising CCV is shown in Figure 1 where the pressure rise over the equivalent compressor is the sum of the pressure rise of the compressor and the pressure drop over CCV: $\Psi_{ec}(\Phi) = \Psi_c(\Phi) - \Psi_v(\Phi)$ where Φ is the circumferentially averaged flow coefficient and Ψ is the total-to-static pressure rise coefficient. $\Psi_c(\Phi)$ is known as the compressor characteristic (map) which describes a nonlinear relationship (assumed cubic in (Gravdahl, 1998)) between Φ and Ψ:

$$\Psi_c(\Phi) = \psi_{c0} + H\left(1 + 1.5\left(\frac{\Phi}{W} - 1\right) - 0.5\left(\frac{\Phi}{W} - 1\right)^3\right) \quad (1)$$

Here, H is the compressor characteristic height factor, W is the compressor characteristic width factor, and ψ_{c0} is shut-off head. The CCV characteristic that describes the pressure drop over CCV as a function of flow is given by $\Psi_v(\Phi) = \frac{1}{\gamma_v^2}\Phi^2$ where γ_v is the gain of CCV. The throttle characteristic $\Psi_T(\Phi) = \frac{1}{\gamma_T^2}\Phi^2$ gives the pressure over the throttle as a function of flow, where γ_T is the throttle gain. The throttle can be thought as a simplified model of a power turbine.

For a given operating point (OP) (ϕ_0, ψ_0), the dynamic model is developed in the form of state-space equations $\dot{z} = f(z, u)$ $\dot{z} = \mathrm{f}(\mathrm{z}, \Gamma_\mathrm{t})$(Eqs. (2-4)), where $z \in \mathbb{R}^3, u \in \mathbb{R}^2$. $z = (\phi,\ \psi,\ J)^T$represents the state vector of the system and $u = (u_1, u_2)$ is the control vector. It is defined in error coordinates with respect to the coordinates of the operating point (ϕ_0, ψ_0). In this model, $\phi = \Phi - \phi_0$, and $\psi = \Psi - \psi_0$. J is the squared amplitude of the first harmonic of rotating stall. Control variables $u_1 = \gamma_T$ and $u_2 = \psi_v(\phi)$ include the effect of throttle and the pressure drop over CCV (in error coordinates) respectively. A partially closed CCV during normal operation of the compressor leads to a bidirectional control law u_2 .

$$\dot{\psi} = k_1\left(\phi + \Phi_0 - u_1\sqrt{\psi + \Psi_0} - \Delta_\phi\right) \quad (2)$$

$$\dot{\phi} = k_2\left(\psi_c - \psi - u_2 + \Delta_\psi - \frac{3H}{4}J\left(\frac{\phi+\phi_0}{W} - 1\right) - \frac{W^2 J}{2\gamma_v^2}\right) \quad (3)$$

$$\dot{J} = \varrho J\left(1 - \left(\frac{(\phi+\phi_0)}{W} - 1\right)^2 - \frac{J}{4} - \frac{4W(\phi+\phi_0)}{3H\gamma_v^2}\right) \quad (4)$$

The compressor characteristic given in Eq. (1) in global coordinates can be expressed in error coordinates as:

$$\psi_c(\phi) = -M_3\phi^3 - M_2\phi^2 - M_1\phi \quad (5)$$

where $M_1 = \frac{3H\phi_0}{2W^2}\left(\frac{\phi_0}{W} - 2\right)$, $M_2 = \frac{3H}{2W^2}\left(\frac{\phi_0}{W} - 1\right)$, and $M_3 = \frac{H}{2W^3} > 0$.

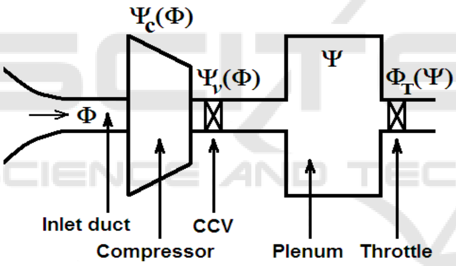

Figure 1: Compression system comprising CCV.

All derivatives are calculated with respect to a normalized time $\xi := {Ut}/{R}$ where t is the actual time, R is the mean compressor radius, and U is the constant compressor tangential speed. Here, $k_1 = \frac{1}{4B^2 l_c}$, $k_2 = \frac{1}{l_c}$, and l_c is the effective flow-passage nondimensional length of the compressor and ducts. B is a positive parameter (so-called Greitzer's B-parameter). The type of the developed nonlinear behavior to a great extent depends on the value of this parameter (small B can lead to rotating stall, and large B can cause surge).

In the model, $\Delta_\phi = \Phi_d + d_\phi$ and $\Delta_\psi = \Psi_d + d_\psi$ include model uncertainties and external disturbances. Mass flow disturbance $\Phi_d(\xi)$ and pressure disturbance $\Psi_d(\xi)$ are both considered as defined by Simon and Valavani (Simon and Valavani, 1991). The disturbances are time varying and bounded ($\|\Phi_d\|_\infty$ and $\|\Psi_d\|_\infty$ exist). In addition to time varying disturbances, constant or slow varying offsets d_ψ and d_ϕ are also introduced. These can be respectively thought of as an uncertainty in the compressor and throttle characteristics.

Setting $\dot{\phi} = \dot{\psi} = \dot{J} = 0$ leads to two equilibria: $J_{e1} = 0$ where the compressor is in its active operating point (ϕ_0, ψ_0) or $J_{e2} = 4\left(1 - \left(\frac{\Phi}{W} - 1\right)^2 - \frac{4W\Phi}{3H\gamma_v^2}\right)$ when the system is in fully developed rotating stall. By using J_{e2} in Eq. (3), one can obtain the equivalent stall characteristic $\Psi_{es}(\Phi)$, which is affected by pressure drop over CCV as can be seen in Eq. (6) (see (Gravdahl, 1998) for more information).

$$\Psi_{es}(\Phi) = \psi_{c0} + H\left(1 - \frac{3}{2}\left(\frac{\Phi}{W} - 1\right) + \frac{5}{2}\left(\frac{\Phi}{W} - 1\right)^3\right) + \frac{5}{H}\Psi_v(\Phi) - \frac{8W}{H\gamma_v^2}\left(1 - \frac{W^2}{3H^2\gamma_v^2}\right)\Phi \quad (6)$$

Figure 2a plots these characteristics: $\Psi_c(\Phi)$ (compressor map without CCV), $\Psi_{ec}(\Phi)$ (equivalent compressor map with CCV), $\Psi_v(\Phi)$ (pressure drop over CCV), $\Psi_T(\Phi)$ (pressure drop over throttle), $\Psi_s(\Phi)$ (stall characteristic without CCV), and $\Psi_{es}(\Phi)$ (equivalent stall characteristic with CCV) in (Φ, Ψ) plane. The OP of the compression system (ϕ_0, ψ_0) is the intersection of the throttle characteristic and the equivalent compressor map. An efficient and stable OP is normally located near the peak of the equivalent compressor map (corresponding to a high pressure rise). Moreover, it can be shown that this OP corresponds to fully damped stall (Hős et al., 2002).

Figure 2a shows how the pressure drop over CCV can modify the equivalent compressor map and equivalent stall characteristic as well (see (Gravdahl, 1998) for more details). This actuator can therefore be used to stabilize an unstable OP. Roughly speaking, when an OP is located in the negative slope area of the equivalent compressor map, it is stable (Willems, 1997). Figure 2b shows that due to the pressure drop over CCV an unstable initial OP in the positive slope area of the compressor map is changed to a stable OP in the negative slope area of the equivalent compressor map. Furthermore, throttle control can also be applied to move the OP. In this work, these two actuators are used to stabilize the system and eliminate rotating stall and surge.

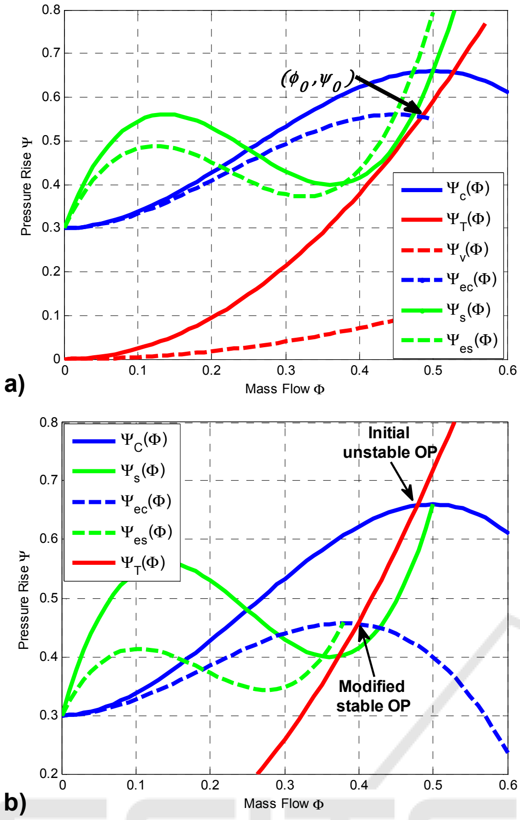

Figure 2: a) Plot of pressure drop over throttle and CCV and compressor and stall characteristics with and without CCV b) Effect of pressure drop over CCV on the equivalent compressor characteristic and the stability of the OP.

3 PASSIVITY-BASED CONTROL

The main objectives of this section are, firstly, to passificate the axial compression system model and, secondly, to achieve both input-to-state stability (ISS, see (Krstic et al., 1995) for definition) and disturbance rejection. In (Byrnes et al., 1991), conditions under which a nonlinear system can be rendered passive via smooth state feedback are driven. Based on this work, several authors have proposed to include uncertain terms (model uncertainties and external disturbances) in order to develop a RPBC (Lin and Shen, 1999; Tsai and Wu, 2009; Jiang and Hill, 1998). These works based on assumptions on uncertainties (vanishing perturbation) or measurable states (full-state feedback with stall as a state-variable), which are not applicable here. Consequently, we remove certain restrictions that are imposed on the uncertainties (e.g. output dependency) and propose a new Lyapunov function stability analysis. We demonstrate that the control law developed in *Theorem 3.1* below ensures robust asymptotic stabilization of the compression system model. Furthermore, this easy-to-implement RPBC does not require a full-state feedback.

Theorem 3.1:

Consider the following disturbed system:

$$\Sigma 1: \begin{cases} \dot{x} = f_0(x,0) + f_1(x,y)y \\ \dot{y} = b_0(x,y) + a_0(x,y)u + D(x,y) + \Delta(x,y) \end{cases} \quad (7)$$

where $f_0(x,y)$, $f_1(x,y)$, $b_0(x,y)$, and $a_0(x,y)$ are smooth functions and $a_0(x,y)$ is invertible for all x, y. $\Delta(x,y)$ is the system uncertainty and $D(x,y)$ is the external disturbance.

If $\Delta(x,y)$ and $D(x,y)$ are bounded and if the zero dynamics of the system are stable (i.e. there exists a positive storage function $S(x)$ such that: $S(0) = 0$ and $\frac{\partial S}{\partial x} f_0(x,0) \leq 0$) then the following feedback control law

$$u = -a_0(x,y)^{-1}\{b_0(x,y) + p(y)\} \quad (8)$$

where $p(y)$ satisfies $y^T p(y) > 0$, renders the closed-loop system input-to-state stable with respect to disturbances and model uncertainties, guarantees global uniform boundedness of $y(t)$, and ensures the convergence to a residual set. The size of the residual set can be arbitrarily made small by the choice of design parameters.

Proof:

Given a positive storage function for the system $\Sigma 1$ as:

$$V(x,y) = S(x) + \frac{1}{2} y^T y \quad (9)$$

Differentiating $V(x,y)$ gives:

$$\dot{V}(x,y) = \frac{\partial S}{\partial x}(f_0(x,0) + f_1(x,y)y) + y^T b_0(x,y) + y^T a_0(x,y)u + y^T(D + \Delta) \quad (10)$$

Since $\frac{\partial S}{\partial x} f_0(x,0) \leq 0$, the substitution of the control law Eq. (8) into Eq. (10) gives that:

$$\dot{V}(x,y) \leq -y^T p(y) + y^T \eta \quad (11)$$

where

$$\eta = [\eta_1 \dots \eta_n]^T = [\frac{\partial S}{\partial x} f_1(x,y)]^T + (D + \Delta) \quad (12)$$

Now, we use the simplified form of Young's inequality which states that for all $C > 0$ and all $(q_1, q_2) \in \mathbb{R}^2$:

$$q_1 q_2 \leq C q_1^2 + \frac{1}{4C} q_2^2 \quad (13)$$

By applying Eq. (13) to each term of $y^T\eta$, we have:

$$y_i\eta_i \le Cy_i^2 + \frac{1}{4C}\eta_i^2 \quad \forall C > 0, i = 1, \dots, n \quad (14)$$

Following the boundedness of uncertainties and assuming that $\left\|\frac{\partial S}{\partial x} f_1(x,y)\right\|_\infty$ exists, we have:

$$y^T\eta \le Cy^Ty + \frac{n}{4C}\|\eta\|_\infty^2 \quad (15)$$

therefore:

$$\dot{V}(x,y) \le -y^Tp_1(y) + \frac{n}{4C}\|\eta\|_\infty^2 \quad (16)$$

where

$$p_1(y) = p(y) - Cy \quad (17)$$

Appropriate choice of $p(y)$ can satisfy the condition $y^Tp_1(y) > 0$ (e.g. $p(y) = Ky$ with $K - CI$ positive definite). Since $y^Tp_1(y)$ and $V(x,y)$ are radially unbounded and positive definite, according to the work of Krstic et al. (Lemma 2.26) (Krstic et al., 1995), we can demonstrate that the control law of Eq. (8) renders the closed loop system ISS with respect to the uncertain terms and hence guarantees the global uniform boundedness of $y(t)$ and convergence to residual set U_Δ, outside which $\dot{V}(x,y) < 0$.

$$U_\Delta = \left\{y: |y| \le \alpha_1^{-1}.\,\alpha_2.\,\alpha_3^{-1}\frac{n}{4C}\|\eta\|_\infty^2\right\} \quad (18)$$

where α_1, α_2, and α_3 are $class-\mathcal{K}_\infty$ functions such that:

$$\alpha_1(|y|) \le V(x,y) \le \alpha_2(|y|) \quad (19)$$

$$\alpha_3(|y|) \le y^Tp_1(y) \quad (20)$$

The size of this set depends on $\|\eta\|_\infty^2$ and design parameter C. A smaller size of U_Δ requires a large C parameter, which implies higher controller gain.

4 PBC DESIGN FOR MG3

Here, it is supposed that mass flow ϕ and pressure rise ψ in the error coordinates can both be measured. Then $y = [\phi \;\; \psi]^T$ and the model (Eqs. 2-4) can be rewritten in the form of system Σ1 including matched uncertainties. Since J cannot be practically measured, the idea in this paper is to consider all the term containing J as part of the disturbances. This simplifies the control design and allows us to have an output feedback strategy. Here, a_0 and b_0 do not depend on J and the assumptions of perturbation boundedness of *Theorem 3.1* are satisfied.

$$\begin{cases} \dot{J} = f_0(J,0) + f_1(J,y)y \\ \dot{y} = b_0(y) + a_0(y)u + D(J,y) + \Delta(y) \end{cases} \quad (21)$$

where

$$a_0 = \begin{bmatrix} 0 & -k_2 \\ -k_1\sqrt{\psi+\psi_0} & 0 \end{bmatrix} \quad (22)$$

$$b_0 = \begin{bmatrix} k_2(-\psi+\psi_c) \\ k_1(\phi+\phi_0) \end{bmatrix} \quad (23)$$

$$D = \begin{bmatrix} -k_1\Phi_d \\ -k_2(\frac{3HJ}{4}\left(\frac{\phi+\phi_0}{W}-1\right) + \frac{W^2J}{2\gamma_v^2} - \Psi_d) \end{bmatrix} \quad (24)$$

$$\Delta = \begin{bmatrix} -k_1d_\phi \\ k_2d_\psi \end{bmatrix} \quad (25)$$

a_0 is nonsingular in the operating range of the compressor where $\psi + \psi_0 > 0$. Furthermore, Gravdahl showed that the squared amplitude of rotating stall and mass flow have upper bounds (Gravdahl, 1998):

$\exists\, J_{max} < \infty$ such that $J(\xi) \le J_{max}$ $\forall\, \xi > 0$, and $\phi_{min} \le \Phi \le \phi_{choke}$, where ϕ_{choke} is the choking value of the mass flow and ϕ_{min} is the negative flow during deep surge. The CCV gain is practically limited as well, in other words $\gamma_v \in [\gamma_{min}, \gamma_{max}]$. Consequently, D and Δ are both bounded. Similarly, $\left\|\frac{\partial S}{\partial x} f_1(x,y)\right\|_\infty$ exists since:

$$f_1(J,y) = \varrho J\phi(-\frac{\phi+2\phi_0}{W^2} + \frac{2}{W} - \frac{4}{3H\gamma_v^2}) \quad (26)$$

where W, H, and γ_v are nonzero.

To investigate the stability of zero dynamics, suppose that a nominal OP is initially located at the peak of the compressor map (which is ideally the case). It can be seen that the peak of the compressor map in Eq. (1) is located at $(\Phi,\Psi) = (2W, 2H + \psi_{c0})$. Therefore, $\phi_0 = 2W$ at this OP:

$$f_0(J,0) = \varrho J\left(-\frac{J}{4} - \frac{4W\phi_0}{3H\gamma_v^2}\right) \quad (27)$$

Considering $S = \frac{1}{2\varrho J_{max}}J^2$, one can show that:

$$\frac{\partial S}{\partial J}f_0(J,0) = \frac{J^2}{J_{max}}\left(-\frac{J}{4} - \frac{4W\phi_0}{3H\gamma_v^2}\right) \quad (28)$$

In Eq. (28) the parameters (W, H), ϕ_0, and J are all positive. Hence

$$\frac{\partial S}{\partial J}f_0(J,0) \le 0$$

This satisfies the first condition of *Theorem 3.1*. By choosing $p(y) = Ky$, *Theorem 3.1* states that the following control law Eq. (29) can stabilize the OP in the presence of the external disturbances and the model uncertainties.

$$u = [u_1, u_2]^T = \begin{bmatrix} \frac{\phi+\phi_0+k_1^{-1}P_2\psi}{\sqrt{\psi+\psi_0}} \\ -\psi+\psi_c+k_2^{-1}P_1\phi \end{bmatrix} \quad (29)$$

In the developed control law, $K = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ consists of two high enough positive design parameters (P_1 and P_2) that guarantee the convergence to U_Δ and limit the size of this residual convergence set.

Note that the control law Eq. (29) cancels all the nonlinearities in the model. Since the term M_3 is always positive in Eq. (5), we propose the following modification to avoid canceling the stabilizing nonlinearities ($-M_3\phi^3$):

$$u = [u_1, u_2]^T = \begin{bmatrix} \frac{\phi+\phi_0+k_1^{-1}P_2\psi}{\sqrt{\psi+\psi_0}} \\ -\psi-M_2\phi^2-M_1\phi+k_2^{-1}P_1\phi \end{bmatrix} \quad (30)$$

It is worth noting that all uncertainties in the model parameters are considered in the terms of Δ, therefore the parameter set used in Eq. (30) is only a reasonable estimation.

Remark:

In the case of deep surge, the system does not include the zero dynamics and the simplified form of control system can be derived by putting $J = 0$ in Eq. (21). It can be seen that $J = 0$ considerably relaxes the boundedness conditions; however, the developed control laws Eq. (30) remains effective.

5 RESULTS AND DISCUSSION

All of the numerical constants and model parameters, which are used in this section, are mentioned in Table 1. At first, we demonstrate that external disturbances and model uncertainties can lead to rotating stall when the controller is deactivated (B-parameter in this case is 0.1.). The system initially starts from OP1 (the intersection of throttle characteristic $\gamma_T = 0.62$ and compressor map at $(\phi_i, \psi_i) = (0.51, 0.66)$ (see Figure 3)). This OP is located in the negative slope area and the system is initially stable. As seen in Figure 4f, disturbances including time varying sinusoidal and constant offsets are applied at $\xi = 50$.

Consequently, the system develops rotating stall (Figure 4c) and output pressure drops (Figure 4a). This spells trouble for normal operation of the axial compressor. In Figure 3, the disturbed trajectory (blue line) settles down at OP2 consisting of the effect of rotating stall and disturbances. The disturbances last until $\xi = 200$, but due to the hysteresis in the qualitative behavior of the system, rotating stall cannot be automatically removed (see Figure 4c). When disturbances disappear, uncontrolled trajectory (magenta line) ends up in OP3 which is located on the stall characteristic where pressure is considerably reduced.

At $\xi = 300$, the controller starts and rapidly damps out rotating stall and imposes the controlled trajectory (green line) toward the initial efficient OP1 where output pressure is high. In this simulation, $P_1 = 10$ and $P_2 = 0.2$. Figure 4d and 4e respectively report the control laws u_1 and u_2. trajectory (green line) toward the initial efficient OP1 where output pressure is high. In this simulation, $P_1 = 10$ and $P_2 = 0.2$. Figure 4d and 4e respectively report the control laws u_1 and u_2.

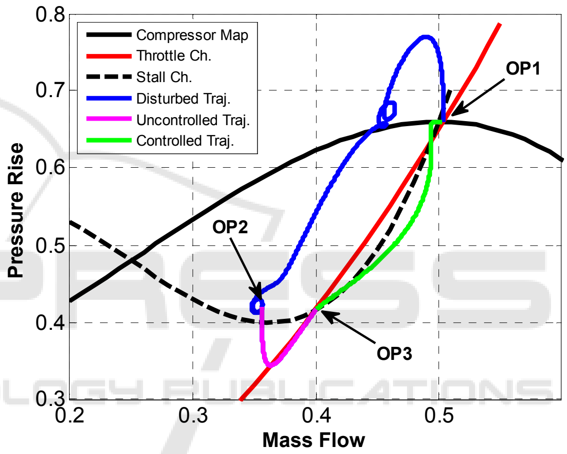

Figure 3: Perturbations lead to rotating stall, but RPBC effectively damps it out. OP1: efficient OP, OP2: developed rotating stall and disturbances, OP3: rotating stall OP.

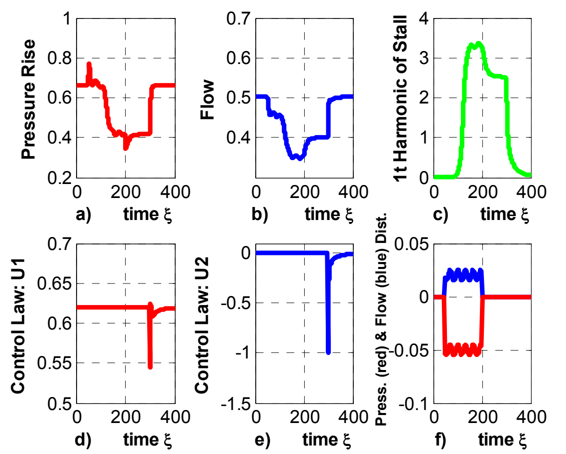

Figure 4: RPBC returns the system to its initial efficient OP and removes rotating stall.

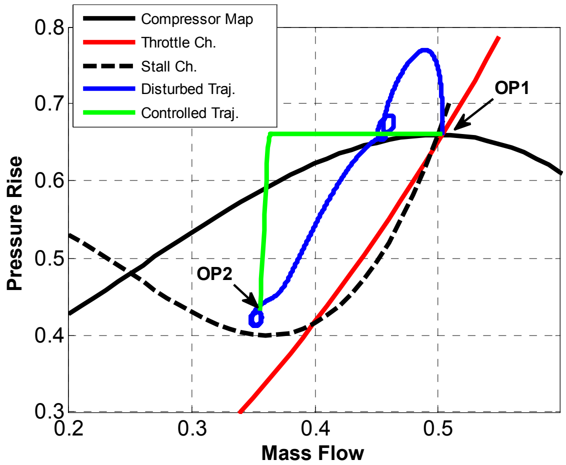

Figure 5: RPBC removes rotating stall and returns the system to its desired initial OP1. OP1: initial efficient OP, OP2: OP including rotating stall and disturbances.

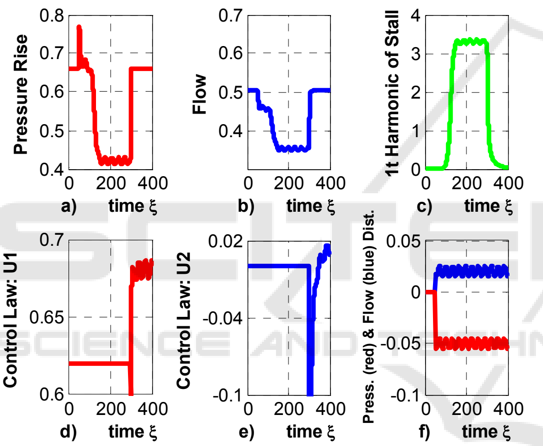

Figure 6: RPBC increases the output pressure and eliminates rotating stall.

To investigate the effectiveness of the controller in the presence of perturbations, long lasting disturbances are applied to the system for $\xi > 50$ (Figure 6f). Again, Figure 5 and 6 show that the controller, which is activated at $\xi = 300$, stabilizes the system at the desired OP1. In Figure 5, the controlled system trajectory finally reaches to the initial desired OP1. Figure 6c shows that rotating stall is rapidly damped out and Figure 6a reports the corresponding pressure increase after the activation of the controller at $\xi = 300$. In this case, $P_1 = P_2 = 20$. These two design parameters also modify the transient response of the system (e.g. the fall time of rotating stall). The scale of Figure 6d and 6e are adjusted to show the variation of control laws due to the time varying sinusoidal disturbances.

Compressors suffer from deep surge as well. For surge simulations, the system initially starts at an efficient OP at the peak of compressor map. In this case, at $\xi = 50$, we apply only the offset disturbances (thought of as model uncertainties) that move the system toward surge condition. Deep surge can be simulated by choosing a high enough value of B-parameter (e.g. $B = 2$ leads to surge).

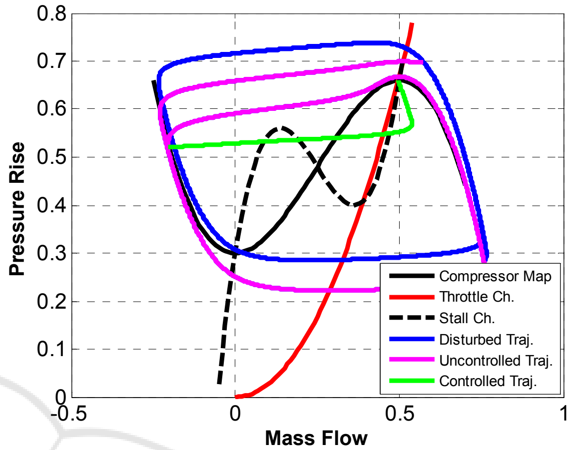

Figure 7: RPBC stabilizes the compression system and eliminates deep surge.

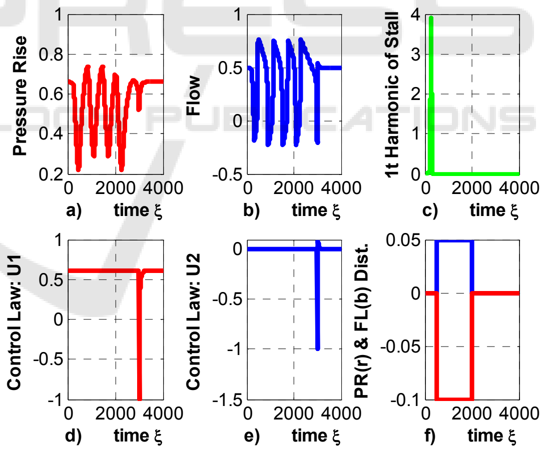

Figure 8: Deep surge including flow reversal and pressure oscillation is damped out due to RPBC activation.

During deep surge, flow reversal occurs (see Figure 8b with negative flow values). Although perturbations are removed at $\xi = 2000$, the system remains in surge condition (see Figure 8f and 8b). Then at $\xi = 3000$, the controller starts and quickly stabilizes deep surge as shown in Figure 8a and 8b. Control efforts are shown in Figure 5d and 5e.

Figure 7 reports disturbed and uncontrolled trajectories showing a limit cycle. Finally, when the control is applied at $\xi = 3000$, the controlled system trajectory settles down at the initial efficient OP1 (green trajectory). This time-domain simulation shows that the developed control law Eq. (30) can robustly stabilize deep surge as well.

6 CONCLUSIONS

In this paper, the effectiveness of RPBC in stabilizing compression systems is demonstrated. Here, surge and rotating stall being potentially able to cause mechanical damages and performance reduction are robustly controlled in the presence of external disturbances and model uncertainties. The controller derives the control signal from pressure and flow measurements and applies it to the system by CCV and throttle actuations. The main contribution of this paper is to propose a simple and easy-to-implement RPBC algorithm that only relies on a small number of design parameters and does not require accurate knowledge of the model parameters.

Analytical developments demonstrate that RPBC accomplishes the ISS property of the closed-loop disturbed system. The size of the residual convergence set and the transient response can be adjusted by control parameters. Time-domain simulation evaluates the performance of the control system and widely supports analytical outcomes.

This brings us to the conclusion that by taking advantage of control methods based on the passivity of compression systems, a wide range of machines using compressors can obtain higher performance and greater operational reliability. Among these machines, gas turbines play an essential role both in aerospace and energy industries.

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APPENDIX

Table 1: Numerical values used in simulations.

l_c	3
W	0.25
H	0.18
B for rotating stall	0.1
B for deep surge	2
ϱ	0.425
d_ϕ	-0.05
d_ψ	0.02
$\Psi_d(\xi)$	$0.01 sin(0.2\xi)$
$\Phi_d(\xi)$	$0.01 sin(0.2\xi)$