Magnetohydrodynamics Simulation in a Sphere by Yin–Yang–Zhong Grid

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Abstract: For numerical simulations in a sphere, we have recently proposed a new spherical grid system called Yin–Yang–Zhong grid. The Yin–Yang–Zhong grid is composed of three components—Yin, Yang, and Zhong—that are combined to cover a spherical region with partial overlaps on their borders. Mutual interpolations are applied to sew the components together, following the overset grid methodology. We review the idea of the Yin–Yang–Zhong grid and its applications to magnetohydrodynamics (MHD) simulations in a sphere. We also present visualization methods employed to analyze the Yin–Yang–Zhong simulations.

1 INTRODUCTION

Magnetohydrodynamics (MHD) is a theory for electrically conducting fluid flows (Davidson, 2001). Computer simulations of MHD in spheres are important in astro- and planetary physics because many stars and planets have electrically conducting fluids in their bodies.

One of the most popularly used methods to discretize the basic equations of MHD, i.e., MHD equations, in the spherical geometry is the spectral methods in which physical variables are expanded by orthonormal functions defined by the spherical harmonics. The time development of a set of mode amplitudes is numerically integrated. In this kind of spectral approach, it is common that nonlinear terms appearing in the equations are calculated in the real space as products, to avoid the costly computations of convolutions in the spectral space. This approach, called the pseudo-spectral approach, requires transformations of variables between the real space and the spectral space every time step. There is, however, no de facto standard of "fast" algorithm for the spherical harmonics transformations for massively parallel computers. It means that the computational speed of the spherical harmonics expansion method does not linearly scale as functions of the maximum mode number, i.e., spatial resolution, and the processor number used in the parallel computation.

On the other hand, the grid-based approaches, that are exemplified by the finite difference method and the finite volume method, are relatively easy to attain the linear scaling in massively parallel computations. However, it is impossible to discretize a sphere with a structured grid system without a coordinate singularity. Take the spherical polar coordinate system (r,ϑ,φ), for example, where r, ϑ, and φ are the radius, colatitude, and longitude. It has two types of coordinates singularities: One is at the poles ($\vartheta = 0$ and π) and the other is at the origin ($r = 0$).

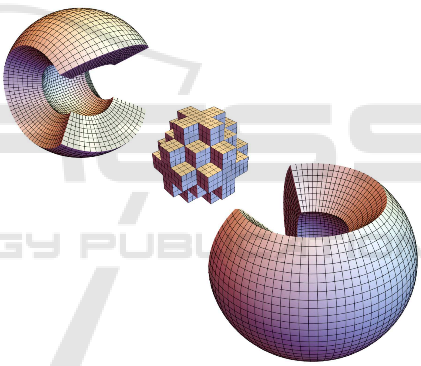

Figure 1: Yin–Yang–Zhong grid. Three component grids, Yin, Yang, and Zhong, are combined to cover a full sphere including the origin. The overset grid method is used to stitch up the three component grids together.

The coordinate singularity itself is not a serious problem because one can always apply L'Hôpital's rule to convert an equation on a coordinate singularity into a non-singular form. The challenge resides *around* a coordinate singularity, rather than *on* it. In a structured grid system, a coordinate singularity inevitably leads to a nearby concentration of grid points

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which degrades computational efficiency, especially when one uses an explicit scheme for temporal integration. The Courant–Friedrichs–Lewy (CFL) condition (de Moura, 2012) imposes an impractically severe limit on the time step due to the small grid spacings. Even if an implicit time integration scheme is used, the grid concentration implies unphysical high resolution of the numerical accuracy around there.

We proposed a grid system, Yin–Yang grid, to avoid the coordinate singularities on the poles of the spherical polar coordinates (Kageyama and Sato, 2004; Kageyama, 2005). The Yin–Yang grid is a kind of the overset grid (Chesshire and Henshaw, 1990) that is applied to the spherical geometry. It has two congruent grid elements—Yin and Yang—that are combined to cover a two-dimensional spherical surface or a three-dimensional spherical shell volume between two concentric spheres.

We have applied the Yin–Yang grid to geodynamo simulations (Kageyama et al., 2008; Miyagoshi et al., 2010), solar dynamo simulations (Masada et al., 2013; Mabuchi et al., 2015), and mantle convection simulations (Kameyama et al., 2008). The Yin–Yang grid is also used in other fields and by other groups, from geophysics to astrophysics, from climate models to image proccessings. The spherical tessellation problem (Yan et al., 2016) would be one of the most promising applications in future in which the Yin–Yang grid is potentially useful.

While the Yin–Yang grid system avoids the coordinate singularities at the poles ($\vartheta = 0$ and π), another singularity at the origin ($r = 0$) is laid aside. Yin–Yang simulations have, therefore, a "cavity" at the center of the sphere, unless some symmetries are assumed on the solutions at $r = 0$.

We have recently proposed an overset grid system, Yin–Yang–Zhong grid, for the spatial discretization of a full sphere, or a ball, including the origin (Hayashi and Kageyama, 2016). The Yin–Yang–Zhong grid has three components; Yin, Yang, and Zhong (see Figure 1). The new component grid (Zhong) is a set of cuboid blocks based on the Cartesian grid. ("Zhong" stands for "center" in Chinese language.) The Zhong grid component is placed to cover the "cavity" of the Yin–Yang grid. The three component grids cover the full sphere with partial overlaps on their borders. The boundaries are sewed together by mutual interpolations, following the general overset grid methodology (Chesshire and Henshaw, 1990). We performed a couple of validation tests of the Yin–Yang–Zhong grid (Hayashi and Kageyama, 2016). For example, we compared damping rates of various eigenfunctions of the diffusion equation in a sphere with analytical solutions.

The Yin–Yang–Zhong grid is a straightforward extension of the Yin–Yang grid, by just adding a new component grid (Zhong) at the center. Therefore, it is relatively easy to modify an existing Yin–Yang code into a Yin–Yang–Zhong code.

In the following, we summarize our recent applications of the Yin–Yang–Zhong grid for MHD simulations in a sphere. We then briefly review visualization methods that we have developed for those simulations.

2 SIMULATIONS OF MHD IN A SPHERE

2.1 MHD Relaxation in a Sphere

MHD relaxation is a fundamental process in MHD physics. When an MHD fluid with a magnetic field is placed in a vessel (with no initial flow), the MHD system shifts spontaneously toward another state if the initial state is unstable. After a short period of transition, the system calms itself down to a quasi-equilibrium state. This process is called MHD relaxation (Ortolani and Schnack, 1993). Various plasma experiments show surprisingly good agreements with a relaxation theory proposed by Woltjer (Woltjer, 1958) and Taylor (Taylor, 1986). Although plasma instabilities, and therefore flows, play essential roles in the Woltjer-Taylor theory, the flow velocity is assumed to be absent in the relaxed state in the theory.

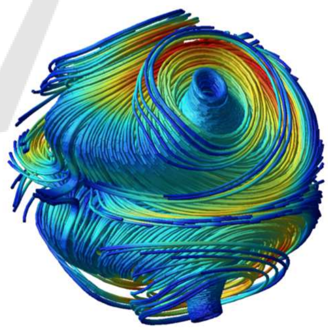

Figure 2: Streamline visualization of the flow of a quasi-stationary state of an MHD relaxation simulation in a sphere. The color denotes the velocity amplitude (blue to red for slow to fast). The simulation is performed using the Yin–Yang–Zhong grid.

We have performed an MHD simulation inside a sphere using the Yin–Yang–Zhong grid to investigate the MHD relaxation processes that has a flow in the relaxed state. Figure 2 shows streamlines in a relaxed state obtained by the simulation. The quasi-stationary, relaxed state has both the magnetic field and flow field with the same levels of energy; this is a solution beyond the Woltjer-Taylor theory.

2.2 MHD Convection in a Thin Shell

We have also performed an MHD simulation of thermal convection in a thin spherical shell layer with the Yin–Yang–Zhong grid. The layer is between two concentric spheres of radii $r = 0.9$ and $r = 1.0$, whose temperatures are kept hot and cold, respectively. A central gravity toward the center is assumed. The purpose of this simulation is to investigate the pattern formation of the MHD convection and the MHD dynamo effect by the flow. (The MHD dynamo is an energy conversion process from the flow's kinetic energy into the magnetic energy through the electromagnetic induction effect.)

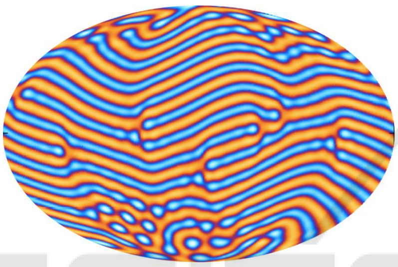

Figure 3: A flow pattern of thermal convection of an MHD fluid in a thin spherical shell. It shows the radial component of the flow at the average radius of the shell. Magnetic field is generated by this convection flow and the magnetic field diffuses into the inner conductive sphere under the convection layer.

The MHD convection exhibits a roll-like pattern in the spherical shell as shown in Figure 3. The Zhong grid component is critically important in this simulation because the dynamo-generated magnetic field diffuses into the inner sphere of $r \leq 0.9$, in which we solve the diffusion equation for the magnetic field on the Zhong grid.

Magnetic field is generated by the MHD dynamo action by the flows in the convection rolls. Drawing magnetic field lines, we have found that they wind around the rolls and the magnetic energy is converted from the flow's kinetic energy through the work done by the flow against the field line tension force in the windings. Generated magnetic energy is concentrated in dislocations of the columns, i.e., the Y–shaped forks of the rolls in Fig. 3. The field lines are anchored to the inner core.

3 VISUALIZATIONS OF MHD IN A SPHERE

As in other simulations, data visualization is a crucial step in analyzing the Yin–Yang–Zhong simulations. Visualization methods are, in general, divided into two categories, i.e., post-process visualization and co-process visualization. A post-process visualization is applied to numerical data that are saved to a disk drive system after a simulation job. A co-process visualization is, on the other hand, applied while a simulation is running. The output data of the co-process visualization is a set of images.

3.1 Post-process Visualization on Supercomputer

We use *Armada* as a post-process visualization tool. *Armada* was originally developed by N. Ohno for Yin–Yang simulation data. We have recently improved this program so that it can visualize Yin–Yang–Zhong data, too. *Armada* is a software rendering program that is parallelized with MPI and OpenMP. Visualization methods implemented in *Armada* are volume rendering, contour colors on cross sections, vector glyphs, and stream tubes. Since it does not need GPU (Graphics Processing Unit), we can execute *Armada* on general supercomputers. Figure 4 shows sample snapshots of the visualization by this software.

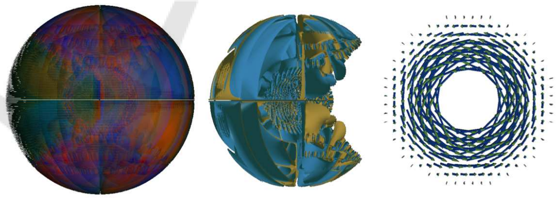

Figure 4: Post-process visualizations by *Armada*. *Armada* is a parallel visualization program with volume rendering (left), isosurface (middle), vector arrow glyphs (right), and other visualization methods.

3.2 Co-process Visualization using ParaView

In the post-process visualizations, we need to save three-dimensional numerical data for the visualization. The required storage size and the network bandwidth degrade the usefulness of the post-process visualization. As a result, another approach to the visualization, i.e., co-process visualization, is getting attentions of simulation researchers these days.

ParaView[1] is one of the most popularly used general purpose visualization programs. Although it is basically for post-process visualizations, ParaView can also be used for co-process visualizations by making use of a special library called Catalyst[2]. We use ParaView/Catalyst as a co-process visualization tool to analyze Yin–Yang–Zhong simulations. Shown in Figure 5 are sample snapshots of movies obtained by this approach. This is a powerful approach for the visualization of large scale parallel simulations since ParaView has a rich set of advanced visualization methods.

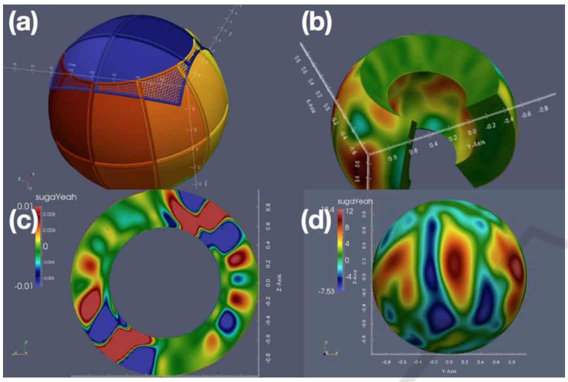

Figure 5: Snapshots of visualization movies taken by co-process visualizations of an MHD simulation by ParaView/Catalyst. (a) Allocations of MPI process in the simulation & visualization. (b) Vorticity amplitude in the Yin grid component. (c) Vorticity amplitude in the equatorial plane. (d) Vorticity amplitude in a sphere.

3.3 Co-process Visualization by Vector Graphics Format

We have also developed our original co-process visualization tool that is much simpler than ParaView/Catalyst. The tool, *insitu2d*, is implemented as a Fortran90 module. It visualizes only two-dimensional cross sections (the equatorial plane and meridian planes) of Yin–Yang–Zhong grid simulations. Consequently, it enables us to perform a quick rendering without damaging the simulation speed.

The output images of *insitu2d* are stored in EPS (Encapsulated PostScript) format. We can magnify the images keeping sharp outlines thanks to the vector graphics format of EPS. Sample figures of *insitu2d* visualizations are shown in Figure 6.

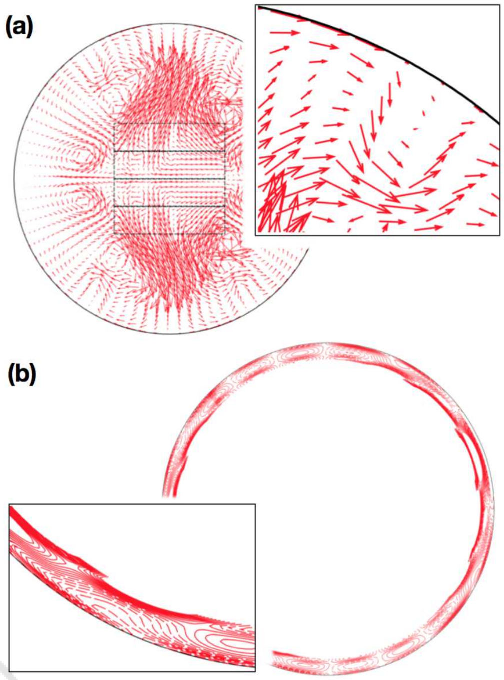

Figure 6: Co-process visualization of MHD simulations by our original tool *insitu2d*. (a) A meridional cross section of the MHD relaxation simulation presented in Section 2.1. (b) An equatorial cross section of the MHD simulation presented in Section 2.2.

4 CONCLUSIONS

We have recently proposed a new overset grid system, Yin–Yang–Zhong grid, for numerical simulations in a sphere (Hayashi and Kageyama, 2016). The Yin–Yang–Zhong is an extension of the Yin–Yang grid that is for the spherical shell geometry between two concentric spheres. In many cases, Yin–Yang simulations have a cavity at the center of the sphere because of the coordinate singularity at the origin $r = 0$. The Zhong component grid is placed to cover the cavity region. Three component grids (Yin, Yang, and Zhong) are combined to cover a full sphere with partial overlap between them on the borders that are stitched by mutual interpolations based on the standard overset grid method. The Yin–Yang–Zhong grid enables us to perform a simulation in a full sphere without any care about the severe CFL conditions caused by concentrated points.

We presented in this paper two MHD simulations as application examples of the Yin–Yang–Zhong grid: One is MHD relaxation simulation, and the other is MHD convection simulation in a sphere. For three-dimensional visualizations of those simulations, we use *Armada* for post-process visualizations and Par-

[1]http://www.paraview.org
[2]http://www.paraview.org/in-situ/

aView/Catalyst for co-process visualizations. For two-dimensional, co-process visualizations, we have developed a simple and concise library, *insitu2d*.

The combination of the Yin–Yang–Zhong grid and the specially designed visualization tools for the grid system will be useful for various simulations in the sphere.

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