Robust Control of Uncertain Linear Plants in Conditions of Signal Quantization and Time-delay

Alexey Margun[1] and Igor Furtat[1,2]
[1]Control Systems and Informatics Department, ITMO University, 49 Kronveksky av., 197101, Saint Petersburg, Russia
[2]Control of Complex Systems Lab., Institute for Problems of Mechanical Engineering, V.O., 61 Bolshoy str., 199178, Saint Petersburg, Russia

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Abstract: The paper is aimed to the robust control of parametric uncertain linear plants in conditions of signal quantization with time delay and bounded external perturbations. State vector of control plant is unmeasured. Output of control plant is measured only via quantizer with transport delay. Control algorithm is based on consecutive compensator method. Proposed algorithm provides convergence of tracking error to the reference signal with prespecified accuracy for bounded time delay in quantizer. Limitations on value of delay providing stability of closed-loop system are obtained.

1 INTRODUCTION

The great amount of modern electronic and computing devices uses digital technologies for data transfer and processing due to its high reliability and stability with respect to noises. However, digital technologies have several disadvantages: loss of information due to the time discretization, signal level quantization and other problems caused by channel restrictions.

Most of physical and technical processes have a continuous nature. Usually discrete models are used for description of such systems and their implementation on digital devices. High-performance computing systems allow to realize the discrete in time transformation of continuous signal with a prescribed accuracy for a large number of cases (Shannon, 1949). Quantization of signal level is a more difficult problem in practice because of the limited sensors accuracy and the high cost of precision measuring devices.

Most researches consider signal discretization as a random discrete noise in system (Widrow, 1961), (Gray and Neuhoff, 1988). However, such assumption is inapplicable if the quantization step is comparable with the range of signal variation (Delchamps, 1989).

The paper (Liberzon, 2003) is devoted to the synthesis of control law for providing of global asymptotic stabilization of continuous linear systems with known parameters in conditions of quantized estimation of state-space vector.

In (Sharon and Liberzon, 2013) the method of stability achieving for linear plants with known parameters and perturbations in the quantized state vector measurement is considered.

The results (Liberzon, 2003), (Brockett and Liberzon, 2000), (Sharon and Liberzon, 2013) and the algorithm of robust discrete control (Tsykunov, 2014) have been summarized in (Furtat et al., 2015). Proposed discrete robust controller is designed for continuous linear parametrically uncertain plants exposed to external disturbances when only quantized output measurements are available.

In (Margun and Furtat, 2015b) the consecutive compensator control law, which is described in (Bobtsov, 2008), is applied to the control of the class of linear continuous parametric uncertain plants in condition of external disturbances and quantized output measurement. This result is extended for the case of MIMO systems in (Margun and Furtat, 2015a).

The paper (Wang and Xue, 2010) proposes output feedback control method for time-delay systems with measurements quantized by logarithmic quantizer. In order to analyze the influence of the quantizer on the system the sector bound method is introduced. Quantization problem is considered as robustness problem. Condition of exponential stability is obtained using linear matrix inequalities techniques.

The paper (Mahmoud et al., 2011) proposes a quantized feedback stabilization algorithm for a class of interconnected continuous time-delay systems. $\mathcal{H}_\infty$

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approach and LMI-based method is used for synthesis of decentralized controller.

However, the problem of controller synthesis for uncertain systems with signal quantizing and time-delay remains urgent. The present research is devoted to the control of parametric uncertain plants in conditions of bounded external disturbances, unmeasured state vector, output signal time-delay and quantizing.

The paper is organized as follows: Section II describes mathematical problem statement, in Section III controller synthesis is considered, Section IV contains stability analysis with using of Lyapunov-Krasovskii function. An academic example confirming performance of proposed controller is given in Section V.

2 PROBLEM STATEMENT

Consider the control plant described by linear differential equation

$$Q(p)y(t) = R(p)u(t) + \tilde{f}(t), \tag{1}$$

where $Q(p), R(p)$ are linear differential operators with known degrees n and m respectively and unknown coefficients, $y(t) \in \mathbb{R}$ is an output signal, $u(t) \in \mathbb{R}$ is a control signal, $p = d/dt$ is a differential operator, $\rho = n - m \geq 1$ is a relative degree of a plant, $\tilde{f}(t)$ is a bounded external disturbance.

A reference model is described by linear differential equation

$$Q_m(p)y_m(t) = R_m(p)r(t), \tag{2}$$

where $Q_m(p), R_m(p)$ are linear differential operators with known coefficients, $y_m(t) \in \mathbb{R}$ is an output signal of a reference model, $r(t)$ is a piecewise smooth bounded reference signal, $Q_m(\lambda)$ and $R_m(\lambda)$ are Hurwitz polynomials, λ is a complex variable.

Assume, that the state vector of control plant is unmeasured and the plant output is measured only via quantizer (Liberzon, 2003) that converts the signal $y(t)$ according to (3):

$$q(y(t)) = \begin{cases} \tilde{q}(y(t-\tau)), |y(t-\tau)| \leq \bar{y}, \\ \bar{y} sign(y(t-\tau)), |y(t-\tau)| > \bar{y}, \end{cases} \tag{3}$$

where $\bar{y} > 0$ is a quantizer saturation value, τ is a bounded unknown time delay, $\tilde{q}(y) = \frac{1}{\chi p+1}\bar{q}(y)$, $\bar{q}(y)$ is a quantizer function illustrated on Fig. 1, $\chi > 0$ is a some positive number. It should be noted that signals $\bar{q}(y)$ and $\tilde{q}(y)$ are almost the same in the case of sufficiently small χ in comparison with quantization step, but $\tilde{q}(y)$ is differentiable in contradistinction to $\bar{q}(y)$. The presence of transport delay represents time which is necessary for signal processing in quantizer. In practice such plants are actual in technical systems with analog-digital converters.

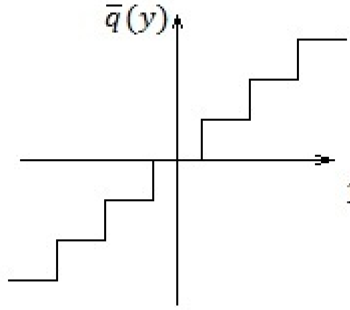

Figure 1: Quantizer output.

It is possible to represent structure of the system in the form of plant with time delay in output channel and quantizer without delay (Fig. 2).

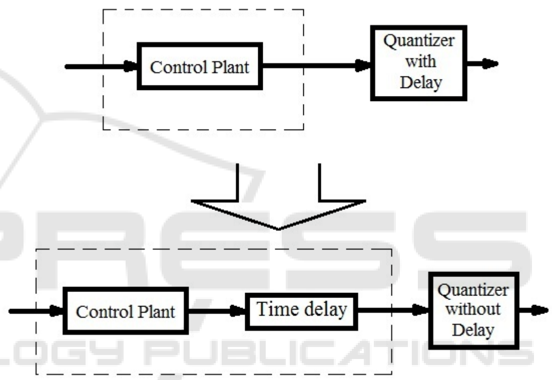

Figure 2: Plant structure transformation.

In this case, the control plant is described by the equation

$$Q(p)y(t-\tau) = R(p)u(t) + \tilde{f}(t). \tag{4}$$

Let the system (4) with quantizer (3) satisfies following assumptions.

Assumptions

1. Quantizer satisfies condition:

$$|\tilde{q}(y(t)) - y(t)| \leq \delta_1, \tag{5}$$

where δ_1 is a some positive number.

2. Unknown coefficients of the operators $Q(p)$ and $R(p)$ belong to the known bounded set Ξ.

3. Control plant (4) is minimum phase.

4. Initial conditions of output signal satisfies inequality

$$|y(0), \dot{y}(0), ..., y^{n-1}(0)| \leq \bar{y}.$$

It is necessary to synthesize control system that ensures the implementation of the goal condition:

$$|q(y(t)) - y_m(t)| < \delta, \forall t > T, \tag{6}$$

where $\delta > 0$ is a prespecified required accuracy, $T > 0$ is a transient time.

3 CONTROL LAW

Introduce consecutive compensator control law (Bobtsov, 2008) for plant (4) with quantization and time delay similarly to (Margun and Furtat, 2015b):

$$u(t) = -(\alpha+\beta)D(p)\hat{e}(t), \tag{7}$$

where α and β are some positive numbers, $D(\lambda)$ is Hurwitz polynomial of degree $\rho-1$ such that $(2Q(\lambda)+\alpha R(\lambda)D(\lambda))$ is Hurwitz polynomial, λ is a complex variable, $\hat{e}(t)$ is estimation of error $e(t) = q(y) - y_m(t)$.

Rewrite control plant (4) in the form

$$Q(p)y(t) = R(p)u(t) + \tilde{f}(t) + Q(p)(y(t) - y(t-\tau)) \tag{8}$$

Taking into account (2), (7) and (8) error dynamics can be represented as

$$\begin{aligned}&(2Q(p)+\alpha R(p)D(p))e(t) = R(p)D(p)\times\\ &((\alpha+\beta)(e(t)-\hat{e}(t)-\beta e(t)) + \tilde{f}(t)+\\ &+2Q_1(q(y(t))-y(t)) + Q_2(p)(\dot{q}(y(t))-\dot{y}(t))\\ &-2Qy_m(t)+Qy_m(t-\tau)-Q(q(y(t-\tau))-\\ &-y(t-\tau))+Qe(t-\tau),\end{aligned} \tag{9}$$

where $Q(p) = Q_1(p) + pQ_2(p)$, $degQ_2(p) = n-1$, $degQ_1(p) \leq n-1$.

Rewrite equation (9) in state-space form

$$\begin{cases}\dot{\varepsilon}(t) = A\varepsilon(t) + B(-\beta e(t) + (\alpha+\beta)(e(t)-\\ -\hat{e}(t))) + B_1(q(y(t))-y(t)) + B_2\times\\ \times(\dot{q}(y(t))-\dot{y}(t)) + B_3\varphi(t) + B_4\varepsilon(t-\tau),\\ e(t) = \bar{L}\varepsilon(t),\end{cases} \tag{10}$$

where $\varepsilon(t) \in \mathbb{R}^n$ is an error state vector, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^n$, $B_1 \in \mathbb{R}^n$, $B_2 \in \mathbb{R}^n$, $B_3 \in \mathbb{R}^n$, $B_4 \in \mathbb{R}^{n\times n}$ are matrices obtained from (9) to (10) transition, $\bar{L} = [1 \quad 0 \quad \dots \quad 0]$, $\varphi(t) = \tilde{f}(t) - 2Qy_m(t) + Qy_m(t-\tau) - Q(q(y(t-\tau)) - y(t-\tau))$ is a bounded function.

For implementation of control law (7) we use the observer algorithm

$$\begin{cases}\dot{\xi}(t) = \sigma\Gamma\xi(t) + \sigma G e(t),\\ \hat{e}(t) = L\xi(t),\end{cases} \tag{11}$$

where $\xi(t) \in \mathbb{R}^{\rho-1}$ is an observer state vector, $\Gamma = \begin{pmatrix} 0 & & I_{\rho-2} \\ -k_1 & \dots & -k_{\rho-1}\end{pmatrix}$ is Hurwitz matrix, $G = [0 \quad \dots \quad 0 \quad k]^T$, $I_{\rho-2}$ is identity matrix of $\rho-2$ order, $k_i, i = \overline{1,\rho-1}$ are positive numbers, $L = [1 \quad 0 \quad \dots \quad 0]$, $\sigma > \alpha+\beta$.

Introduce estimation error

$$\eta(t) = L^T e(t) - \xi(t), \tag{12}$$

where $L = [1 \quad 0 \quad \dots \quad 0]$.

Yield derivative of estimation error (12)

$$\dot{\eta}(t) = \sigma\Gamma\eta(t) + L^T\dot{e}(t). \tag{13}$$

Closed loop system is described by equations

$$\begin{cases}\dot{\varepsilon}(t) = A\varepsilon(t) + B(-\beta e(t) + (\alpha+\beta)(e(t)-\\ -\hat{e}(t))) + B_1(q(y(t))-y(t))+\\ +B_2(\dot{q}(y(t))-\dot{y}(t)) + B_3\varphi(t) + B_4\varepsilon(t-\tau),\\ \dot{\eta}(t) = \sigma\Gamma\eta(t) + L^T\dot{e}(t),\end{cases} \tag{14}$$

Taking into account

$$\varepsilon(t-\tau) = \varepsilon(t) - \int_{t-\tau}^{t}\dot{\varepsilon}(s)ds, \tag{15}$$

closed loop system takes the form

$$\begin{cases}\dot{\varepsilon}(t) = (A+B_4)\varepsilon(t) + B(-\beta\bar{L}\varepsilon(t)+\\ +(\alpha+\beta)L\eta(t) + B_1(q(y(t))-y(t))+\\ +B_2(\dot{q}(y(t))-\dot{y}(t)) + B_3\varphi(t)-\\ -B_4\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds,\\ \dot{\eta}(t) = \sigma\Gamma\eta(t) + L^T\dot{e}(t).\end{cases} \tag{16}$$

Rewrite (16) for brevity

$$\begin{cases}\dot{\varepsilon}(t) = \tilde{F}\varepsilon(t) + \bar{F}\eta + F_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds + F_2\phi(t),\\ \dot{\eta}(t) = \sigma\Gamma\eta(t) + L^T\dot{e}(t),\end{cases} \tag{17}$$

where $\tilde{F} = A + B_4 - \beta B\bar{L}, \bar{F} = (\alpha+\beta)BL, F_1 = -B_4, F_2\phi(t) = B_1(q(y(t)) - y(t)) + B_2(\dot{q}(y(t)) - \dot{y}(t)) + B_3\varphi(t)$ is a bounded perturbation function which depends on quantizer parameters, and external disturbances.

4 STABILITY ANALYSIS

Theorem.

Let Assumptions 1-4 hold. Then there exist polynomial $D(\lambda)$ and positive numbers $\alpha, \beta, \sigma > 0$ and $\bar{\tau} > \tau > 0$ such that the control system consisting of control law (7) and estimation algorithm (11) provides goal (6).

Proof of the Theorem.

Consider Lyapunov-Krasovskii functional $V = V_1 + V_2$, where V_1 is a functional for part of system without delay and V_2 is a functional for delay dependent part:

$$\begin{aligned}V_1 &= \varepsilon^T(t)H_1\varepsilon(t) + \eta(t)H_2\eta(t),\\ V_2 &= \int_{-\tau}^{0}\int_{t-\mu}^{t}\dot{\varepsilon}^T(s)N\dot{\varepsilon}(s)dsd\mu,\end{aligned} \tag{18}$$

where H_1 and H_2 are solutions of Lyapunov equations $\tilde{F}^T H_1 + H_1\tilde{F} = -Q_1$ and $\Gamma^T H_2 + H_2\Gamma = -Q_2$ respectively, Q_1, Q_2 and N are symmetric positive defined matrices.

In the case of the absence of the time-delay closed loop system takes the form

$$\begin{cases} \dot{\varepsilon}(t) = \tilde{F}\varepsilon(t) + \bar{F}\eta(t) + F_2\phi, \\ \dot{\eta}(t) = \sigma\Gamma\eta(t) + L^T\dot{e}(t). \end{cases} \tag{19}$$

Let us use the results of Theorem proof in (Margun and Furtat, 2015b). According to (Margun and Furtat, 2015b), the derivative of Lyapunov function V_1 along the trajectories of (19) is bounded by inequality

$$\dot{V}_1 \le -\varepsilon^T(t)R_1\varepsilon(t) - \eta^T(t)R_2\eta(t) + \theta, \tag{20}$$

where $R_1 = Q_1 - 2\upsilon H_1 B\bar{L} - \upsilon H_1\bar{F}\bar{F}^T H_1 - \upsilon\tilde{F}\bar{L}^T LL^T\bar{L}\tilde{F} - \beta H_1 B_1 B_1^T H_1 - \beta H_1 B_2 B_2^T H_1 - \beta H_1 B_3 B_3^T H_1 - \beta\upsilon$, $R_2 = \sigma Q_2 - 2L^T\tilde{F}^T\bar{L}^T LH_2 - \frac{1}{\upsilon}LL^T - \frac{1}{\upsilon}H_2H_2^T - \beta H_2L^T\bar{L}B_1B_1^T\bar{L}^T LH_2 - \beta H_2L^T\bar{L}B_2B_2^T\bar{L}^T LH_2 - \beta H_2L^T\bar{L}B_3B_3^T\bar{L}^T LH_2 - \frac{\beta}{\upsilon}H_2L^T\bar{L}B\bar{L}\bar{L}^T B^T\bar{L}^T LH_2$, $\theta = \frac{2}{\upsilon}\left(sup(\varphi^2(t)) + \delta_1^2\right)$, $\upsilon > 0$ is a sufficiently small number. R_1 and R_2 are positive defined matrices due to the choose of υ, Q_1 and Q_2.

Differentiating Lyapunov-Krasovskii function along the trajectories (17) we obtain

$$\dot{V} = -\varepsilon^T R_1\varepsilon - \eta^T R_2\eta + \theta + 2\varepsilon^T H_1F_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds + \\ + 2\varepsilon^T H_1F_2\phi + \tau\dot{\varepsilon}^T N\dot{\varepsilon} - \int_{t-\tau}^{t}\dot{\varepsilon}^T N\dot{\varepsilon}ds. \tag{21}$$

Using Jensen inequality

$$-\int_{t-\tau}^{t}\dot{\varepsilon}^T(s)N\dot{\varepsilon}(s)ds \le -\frac{1}{\tau}\int_{t-\tau}^{t}\dot{\varepsilon}^T(s)dsN\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds \tag{22}$$

bound Lyapunov-Krasovskii function:

$$\dot{V} \le -\varepsilon^T R_1\varepsilon - \eta^T R_2\eta + \theta + 2\varepsilon^T H_1F_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds + \\ + 2\varepsilon^T H_1F_2\phi + \tau(\varepsilon^T\tilde{F}^T N\tilde{F}\varepsilon + \eta^T\bar{F}^T N\bar{F}\eta + \\ + \int_{t-\tau}^{t}\dot{\varepsilon}^T(s)dsF_1^T NF_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds + \phi^T F_2^T NF_2\phi + \\ + 2\varepsilon^T\tilde{F}^T N\bar{F}\eta + 2\eta^T\tilde{F}^T NF_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds + \\ + 2\varepsilon^T\tilde{F}^T NF_2\phi + 2\eta^T\bar{F}^T NF_2\phi + 2\eta^T\bar{F}^T NF_1\times \\ \times\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds + 2\phi^T F_2^T NF_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds) - \\ - \frac{1}{\tau}\int_{t-\tau}^{t}\dot{\varepsilon}^T(s)dsN\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds. \tag{23}$$

The terms of the right side of (23) are bounded by the inequalities

$$2\varepsilon^T H_1F_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds \le \upsilon\varepsilon H_1F_1F_1^T H_1\varepsilon + \\ + \frac{1}{\upsilon}\int_{t-\tau}^{t}\dot{\varepsilon}^T(s)ds\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds, \\ 2\varepsilon^T H_1F_2\phi \le \upsilon\varepsilon^T H_1F_2F_2^T H_1\varepsilon + \frac{1}{\upsilon}\phi^T\phi, \\ 2\varepsilon^T\tilde{F}^T N\bar{F}\eta \le \upsilon\varepsilon^T\tilde{F}^T N\bar{F}\bar{F}^T N\tilde{F}\varepsilon + \frac{1}{\upsilon}\eta^T\eta, \\ 2\eta^T\tilde{F}^T NF_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds \le \upsilon\eta^T\tilde{F}^T NF_1F_1^T N\tilde{F}\eta + \\ + \frac{1}{\upsilon}\int_{t-\tau}^{t}\dot{\varepsilon}^T(s)ds\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds, \\ 2\varepsilon^T\tilde{F}^T NF_2\phi \le \upsilon\varepsilon^T\tilde{F}^T NF_2F_2^T N\tilde{F}\varepsilon + \frac{1}{\upsilon}\phi^T\phi, \\ 2\eta^T\bar{F}^T NF_2\phi \le \upsilon\eta^T\bar{F}^T NF_2F_2^T N\bar{F}\eta + \frac{1}{\upsilon}\phi^T\phi, \tag{24}$$

$$2\eta^T\bar{F}^T NF_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds \le \upsilon\eta^T\bar{F}^T NF_1F_1^T N\bar{F}\eta + \\ + \frac{1}{\upsilon}\int_{t-\tau}^{t}\dot{\varepsilon}^T(s)ds\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds, \\ 2\phi^T F_2^T NF_1\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds \le \upsilon\phi^T F_2^T NF_1F_1^T NF_2\phi + \\ + \frac{1}{\upsilon}\int_{t-\tau}^{t}\dot{\varepsilon}^T(s)ds\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds.$$

where υ is a some positive number.

In accordance with (24) rewrite (23)

$$\dot{V} \le -\varepsilon^T(R_1 - \tau\tilde{F}^T N\tilde{F} - \upsilon H_1(F_1F_1^T + F_2F_2^T)H_1 - \\ - \tau\upsilon\tilde{F}^T N(\bar{F}\bar{F}^T + F_2F_2^T)N\tilde{F})\varepsilon - \eta^T(R_2 - \tau(\bar{F}^T N\bar{F} + \\ + \frac{1}{\upsilon}I + \upsilon\tilde{F}^T NF_1F_1^T N\tilde{F} + \upsilon\bar{F}^T NF_2F_2^T N\bar{F} + \\ + \upsilon\bar{F}^T NF_1F_1^T N\bar{F}))\eta + \int_{t-\tau}^{t}\dot{\varepsilon}^T(s)ds(\tau F_1^T NF_1 + \\ + \frac{1}{\upsilon}I + \frac{3\tau}{\upsilon} - \frac{1}{\tau}N)\int_{t-\tau}^{t}\dot{\varepsilon}(s)ds + \theta + \phi^T(\tau F_2^T NF_2 + \\ + \tau\upsilon F_2^T NF_1F_1^T F_2 + \frac{1+2\tau}{\upsilon}I)\phi. \tag{25}$$

Assume that τ satisfies conditions

$$R_1 > \tau\tilde{F}^T N\tilde{F}, \\ R_2 > \tau\bar{F}^T N\bar{F} + \frac{\tau}{\upsilon}I, \\ \frac{1}{\tau}N > \tau F_1^T NF_1 + \frac{1+3\tau}{\upsilon}I. \tag{26}$$

Rewrite (25) in the form

$$\dot{V} \le -\varepsilon^T R_{1\tau}\varepsilon - \eta^T R_{2\tau}\eta - \int_{t-\tau}^{t} \dot{\varepsilon}^T(s)ds R_{3\tau}\times \times \int_{t-\tau}^{t} \dot{\varepsilon}(s)ds + \theta_\tau, \tag{27}$$

where $R_{1\tau} = R_1 - \tau\tilde{F}^T N\tilde{F} - \upsilon H_1(F_1F_1^T + F_2F_2^T)H_1 - \tau\upsilon\tilde{F}^T N(\bar{F}\bar{F}^T + F_2F_2^T)N\tilde{F}$, $R_{2\tau} = R_2 - \tau(\bar{F}^T N\bar{F} + \frac{1}{\upsilon}I + \upsilon\bar{F}^T NF_1F_1^T N\tilde{F} + \upsilon\bar{F}^T NF_2F_2^T N\bar{F} + \upsilon\bar{F}^T NF_1F_1^T N\bar{F})$ and $R_{3\tau} = \tau F_1^T NF_1 + \frac{1}{\upsilon}I + \frac{3\tau}{\upsilon} - \frac{1}{\tau}N$ are positive defined matrices due to the (26) and choose of Q_1, Q_2 and υ.

Transform (27) to the form

$$\dot{V} \le -\varsigma V + \theta_\tau \tag{28}$$

where $\varsigma = \frac{\lambda_{min}(R_{1\tau})}{\lambda_{max}(M)}$

Solving inequality (28) with respect to the V we obtain

$$V \le \left(V(0) - \frac{\theta_\tau}{\varsigma}\right)e^{\varsigma t} - \frac{\theta_\tau}{\varsigma}. \tag{29}$$

Taking into account $\lambda_{min}(P)e^2 \le \lambda_{min}(P)\varepsilon^T\varepsilon \le V$ we obtain tracking error bound:

$$|e| \le \sqrt{\frac{1}{\lambda_{min}(M)}\left[\left(V(0) - \frac{\theta_\tau}{\varsigma}\right)e^{-\varsigma t} + \frac{\theta_\tau}{\varsigma}\right]} \tag{30}$$

From (30) follows that the control system (7), (11) provides the execution of goal condition (6) for a time T with accuracy

$$\delta = \sqrt{\frac{1}{\lambda_{min}(M)}\left[\left(V(0) - \frac{\theta_\tau}{\varsigma}\right)e^{-\varsigma T} + \frac{\theta_\tau}{\varsigma}\right]}. \tag{31}$$

In infinite time control algorithm provides tracking of output for the reference signal with accuracy

$$\delta_\infty = \sqrt{\frac{1}{\lambda_{min}(M)}\frac{\theta_\tau}{\varsigma}}. \tag{32}$$

It follows from (31) that goal condition (6) holds. Theorem is proved.

Note.

Despite the fact that there is a quite rough estimates in proof of theorem, it is follows that the tracking error depends on quantizer parameter δ_1, external disturbances bounds $sup\tilde{f}(t)$ and value of time delay τ.

5 EXAMPLE

Consider numerical example. Control plant is described by equation

$$(p^3 + q_1p^2 + q_2p + q_3)y(t) = bu(t) + \tilde{f}(t).$$

Set of plant coefficients possible values is defined by inequalities:

$$1 \le q_1 \le 5,$$
$$-10 \le q_2 \le 10,$$
$$-10 \le q_3 \le 10,$$
$$1 \le b \le 10.$$

Plant state vector is unmeasured. Plant output is measured only via quantizer $q(y)$ with quantization step $\delta_1 = 0.05$ and time delay $\tau = 1$ ms.

Reference model is described by equation

$$(p^3 + 2p^2 + 2p + 1)y_m(t) = r(t),$$

where $r(t) = 1$.

Choose controller parameters $\alpha = 340$, $\beta = 10$ and $D(p) = p^2 + 12p + 35$. In this case control law (7) takes the form

$$u(t) = -350(p^2 + 12p + 35)\hat{e}(t).$$

Choose parameters $\sigma = 700$, $k_1 = 1$, $k_2 = 15$. Observer algorithm (11) takes the form

$$\begin{cases} \dot{\xi}_1(t) = 700\xi_2(t) \\ \dot{\xi}_2(t) = 700(-\xi_1(t) - 15\xi_2(t) + e(t)). \end{cases}$$

Let plant parameters are chosen as follows:

$$b = 3,$$
$$q_1 = 2,$$
$$q_2 = -2,$$
$$q_3 = 1.$$

Bounded external disturbance has a form of biased multiharmonic signal

$$\tilde{f}(t) = 0.5 + \sin(t - 0.5) + 0.2\sin(5t + 0.15).$$

Output transients of quantizer and reference model are illustrated on Fig. 3. Plant and reference model output transients are illustrated on Fig. 4. Tracking error transient is illustrated on Fig. 5. Modeling results show that tracking error is comparable with quantization step after 8 seconds.

6 CONCLUSIONS

Robust control algorithm for parametric uncertain plants in conditions of external perturbations and signal quantizing is considered. In the paper it is assumed that state vector of control plant is unmeasured and output of control plant is measured only via quantizer with bounded transport delay (transport delay is a time necessary for signal processing in quantizer).

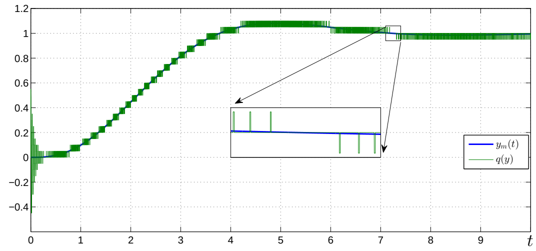

Figure 3: Quantizer and reference model output.

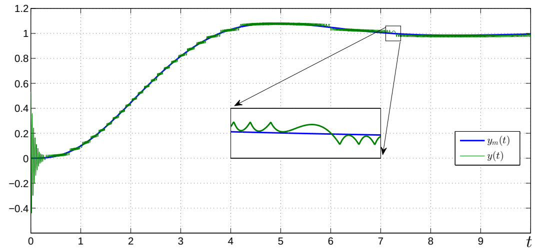

Figure 4: Plant and reference model output.

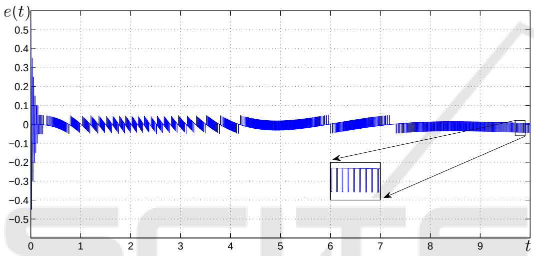

Figure 5: Tracking error.

Such system can be represented as a plant with transport delay in output channel and quantizer without delay. Proposed control algorithm provides convergence of tracking error to the reference signal with prespecified accuracy. Limitations on value of transport delay are obtained.

There are several advantages of the proposed method in comparison with classic controllers:

- for controller synthesis we need to know only relative degree of plant;
- only quantized output measurement is necessary for controller;
- algorithm provides prespecified accuracy for bounded time delay.

Numerical example confirms performance of proposed control method.

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