Pedestrian Trajectory Prediction in Large Infrastructures
A Long-term Approach based on Path Planning

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Abstract: This paper presents a pedestrian trajectory prediction technique. Its mail novelty is that it does not require any previous observation or knowledge of pedestrian trajectories, thus making it useful for autonomous surveillance applications. The prediction requires only a set of possible goals, a map of the scenario and the initial position of the pedestrian. Then, it uses two different path planing algorithms to find the possible routes and transforms the similarity between observed and planned routes into probabilities. Finally, it applies a motion model to obtain a time-stamped predicted trajectory. The system has been used in combination with a pedestrian detection and tracking system for real-world tests as well as a simulation software for a large number of executions.

1 INTRODUCTION

Predicting the future trajectory of a pedestrian in a given environment is a very useful task for many applications, both in the robotics world as in social sciences. This work presents a novel approach for this task, that can be used in any given scenario without previous observation or data-collection.

This work is focused on a very common application of pedestrian trajectory prediction: the autonomous surveillance of critical infrastructures, such as harbours, power plants or security facilities. Nowadays, surveillance systems are mostly static and they require continuous monitoring by a human. Moreover, they are usually limited to provide reactive information (*e.g.* motion sensors, alarms on doors and windows, etc). Therefore, they require a response from the users which in some cases may not have enough time to effectively address the incident. In this context, predicting the possible goal of a given pedestrian, as well as the route and time that will take him or her to the goal can help to increase the effectiveness of the surveillance system because the prediction can provide the ability of responding efficiently and on time to potential vulnerabilities, such as the presence or approximation of intruders to restricted or vulnerable locations.

When performing autonomous surveillance of a critical infrastructure, a series of important or vulnerable locations can be clearly defined. Moreover, it can be assumed that an intruder moving on the infrastructure has a knowledge of the complete map, and therefore it can also perform a *long-term* planning of his or her own route. Furthermore, the prediction does not need to be highly accurate, because the objective of the prediction algorithm will be to find the most probable destination and its route so a robot can be sent to obtain a detailed image of the intruder. Moreover, in order to obtain the prediction, only a few requisites, analogous those required by any path planning algorithm, are needed. Namely: The initial position of the pedestrian, a map of the scenario and a list of possible goals.

Once the initial position, and the set of possible goals are defined, an efficient route to each one of them can be obtained. Those possible routes depend on the cost-map and they are the base of the trajectory prediction. After this, the objective of the prediction algorithm will be to find the probability of the pedestrian following any of the possible trajectories. This probability can be computed based on the similarity between the observed trajectory and each one of the possible routes. The probability distribution, as well as the prediction output, can be updated every time a new observation arrives or at a fixed time-step.

For this work, two different planning techniques are used and compared: Fast Marching Method (FMM) (Sethian, 1999) and the widely used A Star

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Algorithm (A^*). Both of them take into account the map of the scenario and the possible destination of the pedestrians but differ on the optimal solution obtained due the heuristically component of A^* and the fact that FMM uses only a 4-connected neighbourhood to obtain the route.

The main novelty in this work is that is does not require any previous observation of pedestrian trajectories to obtain the prediction, and it can be adapted to any scenario by requiring only a cost-map, making it useful for security applications where it is not likely to have previous observations. Furthermore, the prediction presented is integrated with a previously-developed pedestrian detection and tracking algorithm, so it is capable of handling errors or noisy measurements present in any real-world detection system. Finally, by integrating a motion model of the pedestrian, it is possible to obtain a time-stamped prediction, which is required for its integration with autonomous surveillance with mobile robots.

2 RELATED WORK

Most of the pedestrian trajectory prediction techniques that have been proposed are focused on obtaining a short-time prediction, mainly for robot navigation in scenarios where people and robots may found themselves together.

An early work was based on multi-layer Bayesian dynamic structures, where each layer represents a path trough the environment (Bui et al., 2001). A different work was based on Hidden Markov Models, using clustering obtained form expectation maximization (Bennewitz et al., 2005). However, it requires a high load of previous observations, it is scenario dependent and it does not consider time or velocities. A prediction based on genetic algorithms and an agent-based was tested on a large shopping centre, however, it does not take into account any time constraint because it is oriented to recreate trajectories in social studies (Kitazawa and Batty, 2004).

Some works have proposed to predict the pedestrian trajectory using a goal-directed prediction. The goals may be obtained from clustering large amounts of observations (Yen et al., 2008) or by topological places in a map (Ikeda et al., 2012). Then data from observations was used to obtain probability of transition between sub-goals. The main drawback of these works is their dependence on scenario specific information and their limitation on time and length of the prediction.

A two level prediction process has been also proposed (Foka and Trahanias, 2010), it defines a *short-term* prediction based on Polynomial Neural Networks, and a *long-term* prediction based on the probability of transition between a series of manually defined ”Hot Points”. This technique however only directs the future position of the pedestrian and does not take into account time or velocity issues.

A more recent approach proposes a probabilistic method of determining pedestrian trajectory (Tamura et al., 2013). It classifies the behaviours of pedestrians into definite patterns, learned through observation. Then compares a new one by likelihood calculation. This technique however does not take into account the environment and it only predicts simple trajectories based on the direction of the movement.

A more complex algorithm models goal-directed trajectories of pedestrians using maximum entropy inverse optimal control (Ziebart et al., 2009). This approach describes the environment by using generic features, which then can be moved. Then, it creates a cost-map based on previously observed trajectories, and uses it to plan a future trajectory of the newly observed pedestrian. This work was later extended by adding vision based physical scene features and noisy tracker observations (Kitani et al., 2012). Those works are similar to the one proposed in this paper in the sense that they also use a planning step to predict the future position of the pedestrian, however, their long-term prediction is not very clear, because it only directs its future position and therefore it is only valid in relatively short distances, furthermore, time or velocity issues are not accounted for, and as with all other prediction techniques they require a large amount of observations.

The main novelty of the prediction technique proposed in this paper, is that it does not require any previous observations of pedestrians in order to obtain the prediction, as does every previous work. This means that can be used in any scenario, only having its map. Another difference with previous works is that the work presented here uses a Kalman filter to obtain the pedestrian velocity and combines it with a path planning based prediction. This combination results in a long-term prediction that can take into account not only the position but also time and velocity constraints. Finally, it is possible to obtain predictions that model the different behaviours of pedestrians by modifying the cost-map and using different path planning techniques, therefore increasing the adaptability of the prediction to any infrastructure or depending on the user’s necessities.

3 METHODOLOGY OVERVIEW

As aforementioned, the prediction process has three inputs: The list of possible goals, the map of the scenario and the pedestrian position. The process is initialized by defining the first two while the third one will change continuously as the pedestrian moves. Moreover, the other two are only processed once at the initialization of the algorithm and then only if a re-initialization is executed.

Two independent threads are used, one for processing the income data and other for computing the prediction. Therefore the predicted trajectory can be produced at a constant frequency, independently from the rate at which observations are received. Once the target position is received, it is sent simultaneously to both the Kalman filter and the prediction process.

When the first position is received an instance of a Kalman filter is created. Then, every time a new observation arrives, the filter is updated and a new cycle of its loop is executed, this allows to keep an accurate model of the movement of the pedestrian.

The prediction process also receives the initial position, then it uses a planner (A^* or FMM) to obtain an efficient path from the initial position to each one of the possible destinations, the initialization is completed by assigning an equal probability value to each one of the possible routes. Then, when new observations arrive, the trajectory followed by the pedestrian is compared to the possible routes. The result of this comparison is translated into probabilities so the most *similar* route will have the highest probability.

Finally, the long-term prediction is obtained by projecting the position and velocity observed into the planned route. The prediction can be as long as the complete route or according to a given parameter. In order to account for changes in the direction or unpredicted movements, the process is restarted if the pedestrian moves far away from the route or after a given lapse of time. The complete prediction process is summarized in Figure 1.

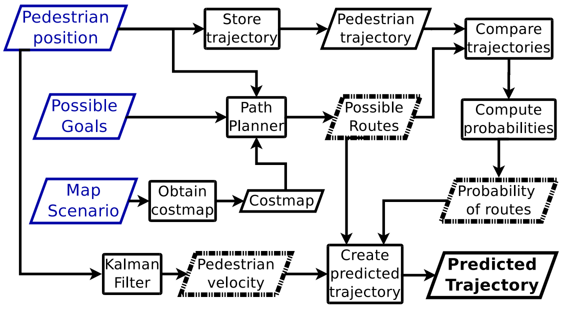


Figure 1: Overview of the proposed approach. The inputs of the algorithm are shown in blue, the main components, as well as the trajectory output, are highlighted.

4 TRAJECTORY PREDICTION BASED ON PATH PLANNING

The core of the prediction algorithm is the use of a path planning strategy. Its main objective is to obtain the most likely destination of the pedestrian as well as its possible trajectory towards it. The problem is simplified by defining a fixed number of possible destinations, and then compare the observed trajectory of the pedestrian with the possible routes obtained from a path planning algorithm. After this, the similarities are translated into probabilities and the most likely route is extracted out and considered as the predicted trajectory. The main components of the algorithm will be described next.

4.1 Obtaining the Cost-map

The first input to be processed is the map of the scenario, which is used to obtain the cost-map required for the path planner. This cost-map is of high importance, because it will determine the behaviour of the trajectory prediction. Moreover, the process used to obtain it can be taken as a replacement to the observations of past pedestrian trajectories required by most of the prediction algorithms in the literature.

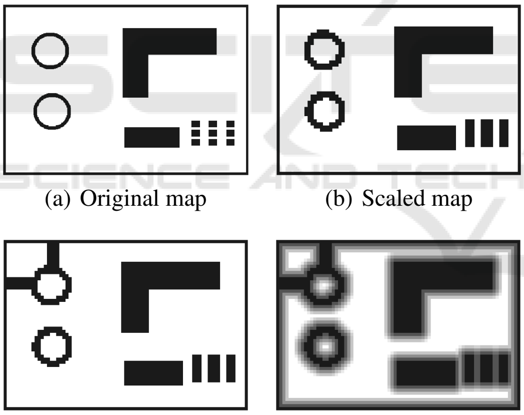
The algorithm uses the standard ROS map format[1] which defines the map in two files: a *yaml* file which contains the meta-data and defines the name of the map image. The second file is the image, which describes the occupancy state of each cell of the map. It marks free cells with whiter colours and occupied ones with black. The map can be obtained from a SLAM algorithm or it can be created using information from a Geographical Information System (GIS).

The first step in processing the map is to scale it (*i.e* change its resolution). Maps used in ROS have usually high resolution because they are used for autonomous navigation (*e.g.* 0.05m per pixel). However, a much lower resolution can be used for this task. Taking into account the frequency of the detection algorithm, the expected mean size of pedestrians and the required precision of the algorithm, the resolution can be much lower, for the presented experiments a value of 0.35m per pixel was defined. This decrease in resolution will speed up and facilitate the complete process, moreover other prediction techniques also use low resolution maps(Ikeda et al., 2012). The scaling process is done by using a bilinear interpolation technique, ensuring that positions in the resulting map have a correct translation to the original one. This allows using the result of the prediction in the original, unscaled, map.

[1] http://wiki.ros.org/map_server

The second step in processing the map is to modify it by incorporating additional obstacles. Those obstacles may represent static objects not included in the original map, or they can also represent virtual obstacles used to define zones where the pedestrian is not expected to cross, or to block passages that need to be avoided. This obstacles can be manually added by defining its position and radius, or they can be given as a new map of obstacles that can be added to the original one.

After the obstacles map is completed, the map is modified by applying transformations or filters, such as a distance transformation, median or low pass filters as well as any other geometrical transformation. This transformation allows to change the costs in the surroundings of the obstacles, and therefore modifying the planned routes to common pedestrian behaviours (*i.e.* generate trajectories by the center of halls, closer to walls, or avoid crossroads or areas where other pedestrians may be found). This transformations are applied homogeneously to the complete map, so it does not include any preference on the predicted trajectories. An example of the map processing with an artificial map is presented on Figure 2.



(a) Original map

(b) Scaled map

(c) Map with obstacles

(d) Resulting Costmap

Figure 2: Steps in the process of obtaining the prediction cost-map from a given (artificial) map.

4.2 Path Planning for Prediction

Since this work is not focused on developing a new planning technique, two well known planners (A^* and Fast Marching Method) have been used. However, as aforementioned, any planner based on a cost-map can be used. It should be clarified that the implementation of the path planning algorithms used here does not take into account the orientation of the pedestrian, nor it poses any kinematic restriction to its movements. There are two reasons for this: First, pedestrians can move in any direction in a plane and second, the detection and tracking algorithm used does not provide information about the pedestrian orientation.

The planner uses the same three inputs as the complete algorithm: The list of possible goals, which are defined in map coordinates and can be pre-fixed or given manually to the algorithm. The second input is the cost-map, which is received after the pre-process described in Section 4.1. The third input is the first position received from the detection algorithm.

Once these inputs are defined, the planner obtains an effective route from the initial position to each one of the possible goals in a sequential manner. Each time a route is found, it is stored and the algorithm remains on stand-by until all possible routes are found. An example of the result using the A^* planner, which uses the euclidean distance to the goal as heuristic cost, is depicted on Figure 3.

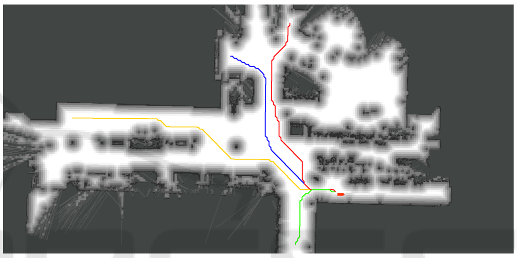


Figure 3: Possible pedestrian routes (shown in colours) from an initial position (red rectangle) towards different possible goals.

4.3 Comparing Two Trajectories

Once the possible routes are defined, the next step is to compare them with the trajectory that pedestrian is following so the results can be later translated into probabilities. There are several issues to take into account when comparing the trajectories, the first one is that the possible routes may have different length and they do not have any time constraints. Also, the distance should be computed with the information available at each moment (*i.e.* the trajectory observed so far) and it should be updated every time a new observation arrives.

It was necessary to define a method to measure the similarity between two trajectories, one that can be used without requiring time-stamped positions, or complex computations. The measurement technique used is based on the Fréchet distance (Fréchet, 1906), which can be defined as follows: “A man is walking a dog on a leash: the man can move on one curve, the dog on the other; both may vary their speed, but backtracking is not allowed. What is the length of the shortest leash that is sufficient for traversing both curves?”(Alt and Godau, 1995).

The distance computation proposed here also relies on the sum of the point-to-point distance between each of possible routes. However some differences are introduced. First of all, since the planned routes do not have time constraints, and their length can be different, the minimum distance between each point in the pedestrian trajectory and the routes is used. Another difference is that the distance values are updated every time a new observation arrives, meaning that the distance between the trajectories D_{traj} changes accordingly at every time-step, as defined in Equation (1).

$$\begin{aligned} D_{traj}(n-1) &= \sum_{i=0}^{n-1} d_i \\ D_{traj}(n) &= D_{traj}(n-1) + d_n \end{aligned} \tag{1}$$

Where d_i represents the distance from the observation i to the possible route. $D_{traj}(n-1)$ is the distance up to the $n-1$ observation. The complete distance $D_{traj}(n)$ is obtained by adding the distance from the last observation to the predicted route d_n whenever a new observation is received

4.4 Obtain Likelihood and Probabilities

The next step in the algorithm is to translate the similarities into probabilities. A first approach was to normalize them so the result can be directly translated to probability values. Taking into account that the distance value is punctual and defined at each time-step, the normalization is straightforward. First, the total distance value D_{total} is obtained, by adding the distance to each one of the possible routes D_j. Then the probability of each route P_G will be the ratio between each distance and the total distance value, as expressed by (2).

$$P_1 = \frac{D_1}{D_{total}}; \quad P_2 = \frac{D_2}{D_{total}}; \quad \dots \quad P_j = \frac{D_j}{D_{total}}; \tag{2}$$

However, after some initial tests, it was found that using the similarity of all the observed trajectory induce errors, because the pedestrians may not follow the predicted route, they can change direction or even go back. In order to solve this, it was necessary to propose a different computation, one that allows to give more relevance to the more recent observations without completely disregarding the previous ones.

The new approach consists on using a *memory* of the trajectory observed so far, which will slowly *fade* as new observations arrive. To obtain this, a combined probability with two components is computed. The first component will be obtained from the last observation only, and the second one will be the *memory* of the previous prediction. This approach has a twofold advantage, firstly it uses a very simple computation. Secondly, it allows to control the weight of both current and previous values, so as to adapt them to different types of behaviours or scenarios. The combined probability computation is expressed in Equation (3).

$$P_j(t) = \alpha P_j(t-1) + (1-\alpha)\hat{P}_j \tag{3}$$

Where $P_j(t)$ represents the probability of the pedestrian following any given trajectory P_j at time t. And α is the weight factor (0.6 for the experiments) that allows to take into account the previously observed trajectory, and finally $\hat{P}_j$ is the probability value obtained using only the last observation. This process is also illustrated in Figure 4.

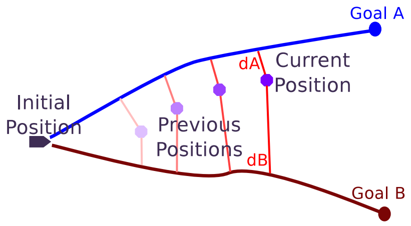


Figure 4: Current and previous distances from pedestrian position to possible routes, clearer colours represent the lower weight of those values in the probability computation.

4.5 Create Predicted Trajectory

After the probability computations, the trajectory with higher value is selected as the predicted route/goal. In order to generate the trajectory prediction, it is necessary to take into account not only the goal but also the velocity of the pedestrian. Furthermore, it should be possible to control the time step and the temporal scope of the prediction.

This is achieved by projecting the current position and velocity of the pedestrian, extracted from the state of the Kalman filter, into the route with the higher probability. The (x, y) velocity is added to the position on the path previously found, this results in a new position that is projected back to the planned route. This procedure is repeated until the length of the prediction is achieved, this process is depicted in figure 5.

Although this trajectory may introduce some deviation or inaccuracies, its prediction is valid for the

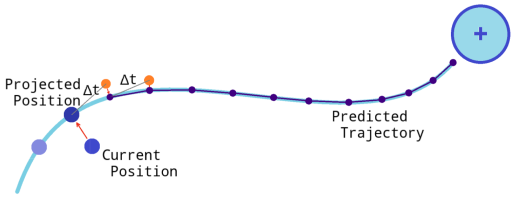


Figure 5: Generation of the predicted trajectory, clear blue shows the most probable route. The predicted trajectory is shown with connected purple dots.

security application, because the objective is not to collide with the pedestrian but rather be able to reach it with a mobile robot. Furthermore, changes on direction or variation of velocity can be correctly handled by continuously updating the prediction.

5 EXPERIMENTS AND RESULTS

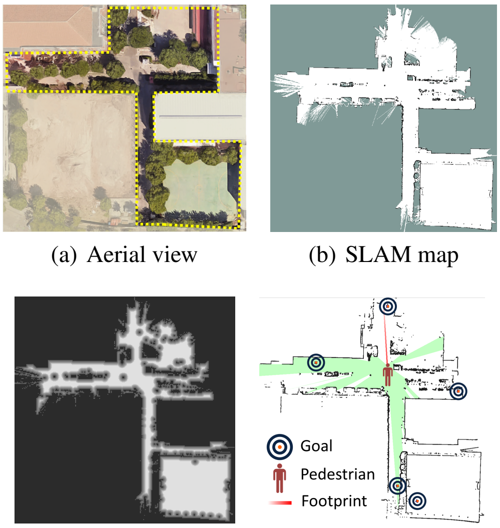
This section describes the experiments performed to evaluate the proposed prediction method. Both simulations and real-world pedestrian trajectories have been tested. First, the scenario and common features for both experiments are described. Then, the acquisition of simulated and real trajectories is explained, along with the performance of the prediction system in every case.

5.1 Scenario

The same scenario was used in both simulated and real experiments. The location is a real street intersection with many obstacles and possible routes for pedestrian motion. The 2D map was pre-built using SLAM algorithms. Then, the cost-map was generated from the map, as described in Section 4.1, and it was used for getting the predicted routes in simulated and real-world tests. The original scenario, the map reconstruction as well as the cost-map and the possible goals are shown in Figure 6.

5.2 Simulations

The pre-built map and a mobile robot, acting as simulated pedestrian, were loaded in Stage simulator. Five positions, according to their proximity to important locations or exits, were selected as possible goals (See Figure 6(d)). Then, the trajectories were obtained by teleoperating the pedestrian from one possible goal to each one of the other four. In all cases the operator knew the map of the infrastructure and choose the route for the simulated pedestrian, thus resulting in arbitrary suboptimal routes. This process was repeated five times for each possible combination, resulting on a total of 100 recorded trajectories. Finally, the A^* and Fast Marching planners to generate the predicted routes, and the prediction algorithm was applied, resulting in a total of 200 simulations.



(a) Aerial view (b) SLAM map

(c) Cost-map (d) Possible goals

Figure 6: Scenario for simulations and real world tests.

This amount of registers allows obtaining valid information about the efficiency of the prediction methodology as well as its behaviour when it is used in conjunction with the A^* or the FMM planner. Three parameters were studied to validate the results: the number of changes in the most probable route over the time, the percentage of trajectory covered before finding the correct route/goal and the percentage of trajectory covered before the probability of the correct route reaches 0.5. Moreover, the mean and standard deviation were calculated for all parameters named above. The results for the complete data are shown in Table 1.

Table 1: Results for trajectory prediction. *A* changes in the most probable route; *B* Trajectory covered (%) before finding the correct route/goal; *C* Trajectory covered (%) before probability of route reaches 0.5.

	A^* Planner		FMM Planner	
	μ	σ	μ	σ
A	4.93	2.79	3.01	1.64
B	51.81%	18.58%	50.46%	18.71%
C	61.61%	13.91%	58.39%	16.27%

It was found that the A^* planner have more changes selecting the most probable route over the time. This fact was associated to the heuristic component of this technique. The euclidean distance minimization derives in early differentiation of the possible routes to reach every goal, which causes that randomly movements of the pedestrian have high impact in the probability distribution since the first time of the displacement. On the other hand, due to the shared starting point for the FMM plans, they tend to follow the same routes for a considerable percentage of the displacement, resulting in a more stable probability distribution.

In both cases, the percentage of trajectory covered before finding the correct route and the percentage before its probability reaches 0.5 is nearly 50%. This value is related to the goals positions in the map (Figure 6(d)). Several plans go through the intersection at the centre of the scenario before they split into very distinct paths. It can be said that this result is strongly dependent on the map and for that reason this is not significant to determinate the speed of achieving a definitive prediction. However, it gives relevant qualitative information about high interest positions in the scenario.

5.3 Real World Experiments

As aforementioned, in order to use the prediction algorithm, it is necessary to have the pedestrian position in a given map. This was achieved by using a previous work, where pedestrians are detected by fusing information of a camera and a laser scanner, and their position on a map is given by a tracking algorithm running on-board a mobile robot (Garzón et al., 2015). An image of the detection and tracking algorithm and the corresponding position in the map is presented on Figure 7.

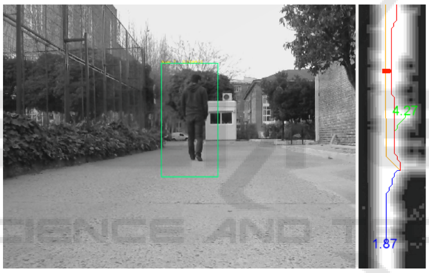


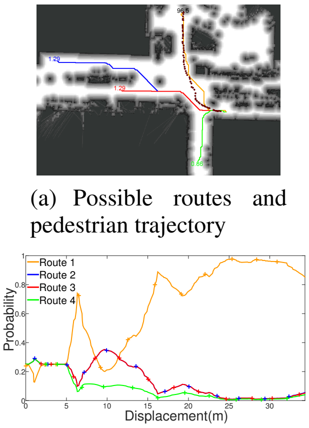
Figure 7: Screen capture of the pedestrian detection and tracking algorithm and the corresponding position on the map.

Several experiments, having similar start and end points as with the simulations were carried out. The prediction algorithm was executed on-board the mobile robot, thus testing the real-time capabilities of the implementation, and the observed results were consistent to those of the simulations.

The behaviour of prediction based on the A^* planner algorithm could be observed in Figure 8(b) where the most probable route changes several times due to unexpected variations in the direction of the pedestrian at the intersection, as can be seen in Figure 8(a).

The results for the prediction based on the *FMM* algorithm were also consistent with the simulations, the number of changes in the predicted route is lower because the possible routes share the same path for a longer period. This can be seen in Figures 9(b) and 9(a) respectively.

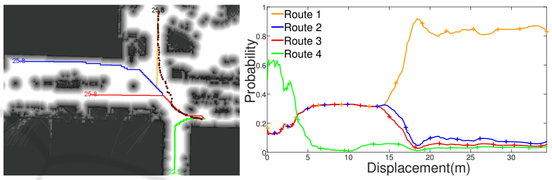
In order to compare the proposed prediction with previous works, the Modified Hausdorff Distance (MHD) can be computed. This measurement provides a standardized a-posteriori information about the similarity of the predicted routes and the trajectory followed by the pedestrian. Figure 10 shows the evolution of the MHD for the prediction using A^* and *FMM*, which has an analogous behaviour as that of the prediction, which was expected because of the probability computation process is based on comparing distances.



(a) Possible routes and pedestrian trajectory

(b) Probabilities evolution

Figure 8: Real experiments using A^* Planner.



(a) Possible routes and pedestrian trajectory

(b) Probabilities evolution

Figure 9: Real experiments using FMM Planner.

A comparison with previous works can not be directly performed, because they provide their results in pixels, not it meters as is done here. However, a relative estimation could be calculated based on the difference between maximum and minimum MHD. The work of (Kitani et al., 2012) has a relative reduction of 75% and in the results presented here, this relative reduction is 86%. Although this is not conclusive, it shows that the proposed technique produces a useful prediction, even when working in large scenarios.

6 CONCLUSIONS

An approach for pedestrian trajectory prediction was presented. The proposed methodology, as well as the experiments and results obtained show that it is pos-

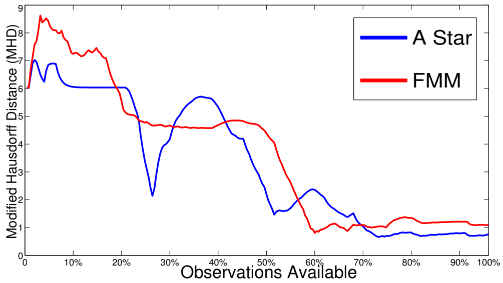


Figure 10: Modified Hausdorff Distance evolution with the observations available for A^* and FMM planners.

sible to model the approximate behaviour of pedestrians based on using path planning techniques.

The tests have shown that a valid pedestrian trajectory prediction can be obtained without requiring a large set of previously observed trajectories. This allows to use the proposed algorithm in any scenario, requiring only a map and a list of possible goals.

The prediction algorithm was successfully integrated with a pedestrian detection system and it was executed on-board a mobile robotic platform, thus validating its capabilities of working in real-time in both simulations and real-world applications.

Two different path planning algorithms (A^* and FMM) were implemented and tested for prediction. It was shown that the prediction based on the A^* planner is more prone to be affected by variations in the movement of the pedestrian, whereas the FMM based prediction is more stable in this sense. Moreover, it can be concluded that in terms of the length of the trajectory required to determine the correct route, both planning techniques produce a similar result, requiring about 50% of the trajectory, although this results are may be conditioned by the test scenario.

Furthermore, it was possible to use a simple motion model, based on a Kalman filter to estimate the time it will take to reach any given goal, thus making the time-stamped prediction very useful for any autonomous surveillance system.

This work has two main lines for future work, the first one is to try to autonomously find the possible destinations and the second is to provide a map or set of maps of predictions where the or different possibilities, uncertainties or variances of the prediction can be expressed in a better way.

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