Soft Variable Structure Control in Sampled-Data Systems with Saturating Input

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Abstract: In the effort to achieve high convergence rate, at the same time avoiding implementation difficulties and poor robustness of time-optimal controllers, the concept of soft Variable Structure Control (VSC) may be applied. The classical formulation of soft VSC in continuous time domain assumes smooth switching among an infinite number of controllers. Since nowadays control laws are implemented digitally, changing the control structure is limited to sampling instances, which leads to *quasi*-soft VSC. The paper investigates how the favourable characteristics of dynamic soft VSC can be extended to input-constrained systems with finite sampling. The design procedure and stability analysis are conducted directly in discrete time domain. The resulting nonlinear control law is synthesised into a form substantially different from its continuous-time counterpart. However, smooth control action and fast convergence of continuous soft VSC is retained. The properties of the obtained control system are formally proved and confirmed experimentally.

1 INTRODUCTION

A combination of two or more control structures with switching logic results in new properties in thus formed variable structure control (VSC) system. As an example, one may consider two unstable systems which, when joint by an appropriate switching strategy, ensure asymptotic convergence to equilibrium (Utkin, 1977). Depending on the design requirements, the emphasis may be placed on different aspects and properties of the VSC system.

When robustness is of primary importance (with the quality of generated control signal a secondary objective), a popular approach is to introduce a high-gain switching element and create a sliding-mode control system. Once the system enters the sliding phase, any deviation from the prescribed manifold in the state space is compensated, yielding insensitivity to matched perturbations under ideal operating conditions. In practice, physical limitations do not permit achieving ideal sliding motion, yet high level of robustness can be achieved. Special considerations, however, need to be taken to mitigate the impact of chattering – unfavourable high-rate input oscillations that are destructive for mechanical components and inefficient from the point of energy budget (Lee and Utkin, 2007).

When a smooth control action becomes a priority, a different class of VSC systems may be considered. In particular, if high regulation rates are desired, one can apply the concept of *soft* VSC (Adamy and Flemming, 2004). Unlike sliding-mode control that relies on infinitely fast switching between a finite number of control configurations, in soft VSC, an infinite number of cooperating controllers is used in the effort to attain fast convergence to equilibrium. The input signal evolves smoothly within the range permitted by constraints.

The soft VSC was originally developed for continuous-time systems (Adamy and Flemming, 2004), and later explored also in continuous time domain (Lens et al., 2011; Kefferpütz et al., 2013; Liu et al., 2015). In now commonly applied digital control realizations (Ignaciuk and Bartoszewicz, 2011; Ignaciuk and Morawski, 2014), however, it is not possible to obtain switching at infinite rate. The smoothness of control structure transitions in discrete-time implementation of soft VSC is restricted by the sequence of sampling instants. In this paper, the design issues of soft variable structure controllers for sampled-data systems are considered. Although infinite switching rate among the control structures is not possible, the obtained *quasi*-soft VSC scheme ensures fast convergence to

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equilibrium with smoothly varying input signal. The closed-loop stability and control signal constraints are addressed explicitly and properties of the quasi-soft VSC system are formally demonstrated within discrete-time framework. The theoretical content is supported by experimental study – stabilization of an inverted pendulum-on-a-cart system.

2 PROBLEM SETTING

Let $t = 0, 1, 2, \ldots$ denote subsequent time instants in a system sampled with period T_s. The system dynamics are given by

$$\mathbf{x}[(t+1)T_s] = \mathbf{A}\mathbf{x}(tT_s) + \mathbf{b}u(tT_s), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input vector, $\mathbf{A} \in \mathbb{R}^{n\times n}$ and $\mathbf{b} \in \mathbb{R}^n$ for $n \in \mathbb{N}_+$. The initial state $\mathbf{x}_0 = \mathbf{x}(0)$ belongs to a bounded set $\mathbf{X}_0$. For notational brevity, the independent variable tT_s will be written shortly as t in a latter part of the text.

The control input needs to obey the constraint

$$|u| \leq u_0, \tag{2}$$

$u_0 > 0$. It is assumed that the control system is feasible, i.e. there exists control satisfying (2) that can bring any $\mathbf{x}_0 \in \mathbf{X}_0$ to 0. Equivalently, one may consider only a (nonempty) set of points $\mathbf{X}_0$ for which control system (1)–(2) is stabilizable (Hu and Lin, 2001).

3 SOFT VSC FOR SAMPLED-DATA SYSTEMS

3.1 Soft VSC Concept

When the linear control $u(t) = \mathbf{k}\mathbf{x}(t)$ with a fixed gain $\mathbf{k} \in \mathbb{R}^{1\times n}$ is applied to system (1), the convergence rate decreases as $\|\mathbf{x}\|$, $\|\cdot\|$ denoting the Euclidean norm, approaches zero. In order to speed up the performance, a nonlinear strategy, e.g. time-optimal control, can be used. However, time-optimal controllers, besides difficulties in obtaining convenient form in sampled-data systems (Gao, 2004), imply sudden changes of the control input at the extremity of allowed interval (2).

The idea behind soft VSC is to adjust the control system dynamics by *smoothly* changing the control structure so that high regulatory rate is maintained throughout the whole movement from $\mathbf{x}_0$ to equilibrium. However, unlike continuous-time systems, discrete-time implementation does not permit adapting the control structure infinitely fast. The inherent characteristics of discrete-time control call for special treatment to retain the desirable properties of soft VSC systems.

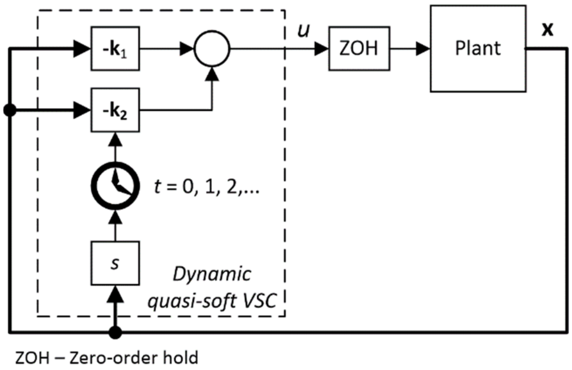

Figure 1: Dynamic soft VSC in sampled-data systems.

3.2 Quasi-soft VSC

The analysed dynamic VSC system is illustrated in Fig. 1. The control structure comprises two sub-controllers and selection logic that governs the overall gain adjustment. The input is determined as

$$u(t) = -[\mathbf{k}_1 + s(t)\mathbf{k}_2]\mathbf{x}(t), \tag{3}$$

where $\mathbf{k}_1$, $\mathbf{k}_2 \in \mathbb{R}^{1\times n}$ are the control gains and $s(t) \in \mathbb{R}$ is the selection variable used for gain adaptation. The controller design amounts to choosing suitable vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, and function $s(t)$.

The closed-loop system under control (3) becomes

$$\begin{aligned}\mathbf{x}(t+1) &= [\mathbf{A} - \mathbf{b}\mathbf{k}_1 - s(t)\mathbf{b}\mathbf{k}_2]\mathbf{x}(t) \\ &= [\mathbf{A}_1 - s(t)\mathbf{b}\mathbf{k}_2]\mathbf{x}(t)\end{aligned} \tag{4}$$

with gain $\mathbf{k}_1$ to be selected so that $\mathbf{A}_1 = \mathbf{A} - \mathbf{b}\mathbf{k}_1$ is stable and good closed-loop performance is achieved.

The system is required to have a single (asymptotically) stable equilibrium point

$$\begin{bmatrix}\mathbf{x} \\ s\end{bmatrix} = \begin{bmatrix}\mathbf{0} \\ 0\end{bmatrix}. \tag{5}$$

3.3 Selection Strategy

A possible choice of selection variable $s(t)$ so that (5) is the unique stable equilibrium for system (4) is given in the following theorem.

Theorem 1. If there exist positive definite matrices **P**, **Q**, and **R** $\in \mathbb{R}^{n\times n}$ satisfying

$$\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 - \mathbf{P} = -(\mathbf{Q}+\mathbf{R}), \tag{6}$$

for $\mathbf{A}_1 = \mathbf{A} - \mathbf{b}\mathbf{k}_1$, and the selection strategy is chosen as

$$\begin{aligned} &s(t+1) = \sqrt{r(t)}, \\ &r(t) = ws^2(t) + \frac{1}{v}\mathbf{x}^T(t)[\mathbf{R} + 2s(t)\mathbf{A}_1^T\mathbf{P}\mathbf{b}\mathbf{k}_2 \\ &\quad -s^2(t)\mathbf{k}_2^T\mathbf{b}^T\mathbf{P}\mathbf{b}\mathbf{k}_2]\mathbf{x}(t) \end{aligned} \tag{7}$$

with $v > 0$, $0 < w < 1$, and **R** adjusted so that $r(t) \geq 0$, then (5) is the stable equilibrium of system (4).

Proof. Consider the Lyapunov function candidate

$$V(t) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t) + vs^2(t). \tag{8}$$

Since **P** is positive definite and v positive, $V(t) > 0$ for $t > 0$ and $V = 0$ at equilibrium (5). Therefore, in order for (8) to be a Lyapunov function for system (4), the forward difference

$$\Delta V(t) = V(t+1) - V(t) \tag{9}$$

needs to be negative along the state trajectory.

Using (4) and (8), ΔV becomes

$$\begin{aligned} \Delta V(t) &= \mathbf{x}^T(t+1)\mathbf{P}\mathbf{x}(t+1) + vs^2(t+1) \\ &\quad -\mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t) - vs^2(t) \\ &= \mathbf{x}^T(t)[\mathbf{A}_1 - s(t)\mathbf{b}\mathbf{k}_2]^T\mathbf{P}[\mathbf{A}_1 - s(t)\mathbf{b}\mathbf{k}_2]\mathbf{x}(t) \\ &\quad +vs^2(t+1) - \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t) - vs^2(t) \\ &= \mathbf{x}^T(t)[\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 - \mathbf{P}]\mathbf{x}(t) + vs^2(t+1) - vs^2(t) \\ &\quad +\mathbf{x}^T(t)[-s(t)\mathbf{A}_1^T\mathbf{P}\mathbf{b}\mathbf{k}_2 - s(t)(\mathbf{b}\mathbf{k}_2)^T\mathbf{P}\mathbf{A}_1 \\ &\quad +s^2(t)(\mathbf{b}\mathbf{k}_2)^T\mathbf{P}\mathbf{b}\mathbf{k}_2]\mathbf{x}(t) \\ &= \mathbf{x}^T(t)[\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 - \mathbf{P}]\mathbf{x}(t) + vs^2(t+1) - vs^2(t) \\ &\quad +\mathbf{x}^T(t)[-2s(t)\mathbf{A}_1^T\mathbf{P}\mathbf{b}\mathbf{k}_2 + s^2(t)\mathbf{k}_2^T\mathbf{b}^T\mathbf{P}\mathbf{b}\mathbf{k}_2]\mathbf{x}(t). \end{aligned} \tag{10}$$

Substituting (7) for $s^2(t+1)$ in (10), yields

$$\begin{aligned} \Delta V(t) &= \mathbf{x}^T(t)[\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 - \mathbf{P}]\mathbf{x}(t) + \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t) \\ &\quad -(1-w)vs^2(t). \end{aligned} \tag{11}$$

Since $v > 0$ and $0 < w < 1$, using assumption (6) leads to

$$\Delta V(t) = -\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) - (1-w)vs^2(t) < 0. \tag{12}$$

Consequently, since $\Delta V(t) < 0$, $V(t)$ given by (8) is a Lyapunov function for system (4), and the system is stable. □

Note that for a stable matrix $\mathbf{A}_1$ (whose eigenvalues can be moved into the open unit disc by proper selection of vector $\mathbf{k}_1$), (6) represents a Lyapunov equation with positive definite solution **P** obtained for arbitrary positive definite matrix **Q** + **R**. Thus, since the sum of positive definite matrices is positive definite, one can always find positive definite matrices **P**, **Q**, and **R** satisfying relation (6). On the other hand, for sufficiently large **R** and v one can guarantee that the expression under the square root in (7) will be nonnegative, which results in a feasible function $s(t)$. **Q** can be arbitrary, e.g. an identity matrix.

3.4 Actuator Saturation

The selection variable needs to be chosen in such a way that the closed-loop system is stable, and input constraint (2) is satisfied at all times. Directly from (3), it follows that condition (2) is met whenever

$$-u_0 \leq -[\mathbf{k}_1 + s(t)\mathbf{k}_2]\mathbf{x} \leq u_0, \tag{13}$$

which is equivalent to the pair of inequalities

$$\begin{aligned} &\frac{-u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}} \leq s(t) \leq \frac{u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}} \quad \text{for} \quad \mathbf{k}_2\mathbf{x} > 0, \\ &\frac{u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}} \leq s(t) \leq \frac{-u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}} \quad \text{for} \quad \mathbf{k}_2\mathbf{x} < 0. \end{aligned} \tag{14}$$

When **x** approaches the equilibrium thus formed bounds extend to infinity. Therefore s should be further limited as

$$|s(t)| \leq s_0 \tag{15}$$

with s_0 being a positive constant. Combining (14) and (15) one arrives at

$$s_L(\mathbf{x}) \leq s(t) \leq s_U(\mathbf{x}) \tag{16}$$

where

$$s_L(\mathbf{x}) = \begin{cases} \dfrac{u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}}, & \mathbf{k}_2\mathbf{x} \leq \dfrac{-u_0 + \mathbf{k}_1\mathbf{x}}{s_0}, \\ -s_0, & \dfrac{-u_0 + \mathbf{k}_1\mathbf{x}}{s_0} < \mathbf{k}_2\mathbf{x} < \dfrac{u_0 + \mathbf{k}_1\mathbf{x}}{s_0}, \\ \dfrac{-u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}}, & \mathbf{k}_2\mathbf{x} \geq \dfrac{u_0 + \mathbf{k}_1\mathbf{x}}{s_0}, \end{cases} \tag{17}$$

and

$$s_U(\mathbf{x}) = \begin{cases} \dfrac{-u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}}, & \mathbf{k}_2\mathbf{x} \le \dfrac{-u_0 - \mathbf{k}_1\mathbf{x}}{s_0}, \\ s_0, & \dfrac{-u_0 - \mathbf{k}_1\mathbf{x}}{s_0} < \mathbf{k}_2\mathbf{x} < \dfrac{u_0 - \mathbf{k}_1\mathbf{x}}{s_0}, \\ \dfrac{u_0 - \mathbf{k}_1\mathbf{x}}{\mathbf{k}_2\mathbf{x}}, & \mathbf{k}_2\mathbf{x} \ge \dfrac{u_0 - \mathbf{k}_1\mathbf{x}}{s_0}. \end{cases} \tag{18}$$

Theorem 2. If there exist positive definite matrices **P**, **Q**, and **R** satisfying (6) with **R** and v adjusted so that $r(t)$ given by (7) is nonnegative, then the selection strategy

$$s(t+1) = \mu(r, \mathbf{x})\sqrt{r(t)}, \tag{19}$$

with

$$\mu(r, \mathbf{x}) = \begin{cases} \dfrac{s_L(\mathbf{x})}{\sqrt{r}}, & \operatorname{sgn}[s(t)]\sqrt{r} \le s_L(\mathbf{x}), \\ \operatorname{sgn}[s(t)], & s_L(\mathbf{x}) < \operatorname{sgn}[s(t)]\sqrt{r} < s_U(\mathbf{x}), \\ \dfrac{s_U(\mathbf{x})}{\sqrt{r}}, & \operatorname{sgn}[s(t)]\sqrt{r} \ge s_U(\mathbf{x}), \end{cases} \tag{20}$$

$s_L(\mathbf{x})$ and $s_U(\mathbf{x})$ given by (17) and (18), and

$$\operatorname{sgn}(s) = \begin{cases} -1, & s \le 0, \\ 1, & s > 0. \end{cases} \tag{21}$$

stabilises system (4) at equilibrium (5) while upholding input constraint (2).

Proof. First, note that (20) makes s given by (19) confined to interval (16), which is equivalent to the constraint $|u| \le u_0$.

Consider the Lyapunov function candidate

$$V(t) = \mathbf{x}^T(t)\mathbf{P}\mathbf{x}(t) + \frac{v}{\mu^2}s^2(t), \tag{22}$$

Since **P** is positive definite and v positive, $V(t) > 0$ for $t > 0$ and $V = 0$ at equilibrium (5). Using (4) and (19), the forward difference

$$\begin{aligned} &\Delta V(t) \\ &= \mathbf{x}^T(t)[\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 - \mathbf{P}]\mathbf{x}(t) + vr(t) - \frac{v}{\mu^2}s^2(t) \\ &+ \mathbf{x}^T(t)[-2s(t)\mathbf{A}_1^T\mathbf{P}\mathbf{b}\mathbf{k}_2 + s^2(t)\mathbf{k}_2^T\mathbf{b}^T\mathbf{P}\mathbf{b}\mathbf{k}_2]\mathbf{x}(t) \\ &= \mathbf{x}^T(t)[\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 - \mathbf{P} + \mathbf{R}]\mathbf{x}(t) + vws^2(t) - \frac{v}{\mu^2}s^2(t), \end{aligned} \tag{23}$$

which after applying (6) becomes

$$\Delta V(t) = -\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) - (1/\mu^2 - w)vs^2(t). \tag{24}$$

Since $v > 0$, for sufficiently small $w > 0$, $\Delta V(t) < 0$, and closed-loop system (4) with input constraint (2) is Lyapunov stable. □

3.5 Convergence

It remains to be determined whether the control system governed by the soft VSC strategy indeed results in faster convergence than a linear scheme with one controller. Note that

$$|s(t+1)| = |\mu s(t)| \times \sqrt{w + \frac{1}{v}\mathbf{x}^T(t)[\frac{\mathbf{R}}{s^2(t)} + \frac{2}{s(t)}\mathbf{A}_1^T\mathbf{P}\mathbf{b}\mathbf{k}_2 - \mathbf{k}_2^T\mathbf{b}^T\mathbf{P}\mathbf{b}\mathbf{k}_2]\mathbf{x}(t)}. \tag{25}$$

Assume $|s|$ to be initially small (and disregard the saturation effect). Then, the first term dominates the quadratic form under the square root and $|s|$ grows as

$$|s(t+1)| \cong |s(t)|\sqrt{w + \frac{1}{vs^2(t)}\mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)} \tag{26}$$

providing increasingly faster decrease of V according to (12). In consequence, the trajectory approaches the origin at a faster rate than in the case of static-gain linear control.

On the other hand, at the conclusion of the regulation process, as **x** approaches zero,

$$|s(t+1)| \cong |s(t)|\sqrt{w}. \tag{27}$$

Since $w < 1$, $|s|$ reduces to zero as well, effectively leaving the system regulated by $\mathbf{k}_1$ (which ensures stable performance by definition).

4 EXPERIMENTAL STUDY

The controlled plant, illustrated in Fig. 1, reflects a structurally unstable 4th-order inverted pendulum-on-a-cart system. The plant parameters are as follows: mass of the cart 0.768 [kg], mass of the pendulum 0.064 [kg], moment of inertia (around the centre of gravity) 0.00231 [kg·m^2], and distance between the pendulum gravity centre and the shaft 0.205 [m]. For the purpose of controller design a linearized plant model is considered – the neglected friction, nonlinearities, and actuator dynamics constitute the plant uncertainty. Thus obtained nominal plant dynamics are given by

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0.291 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 27.984 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1.166 \\ 0 \\ -3.429 \end{bmatrix} u, \qquad (28)$$

where $\mathbf{x} = [x_1 \ldots x_4]^T$ with x_1 – cart position, x_2 – cart velocity, x_3 – pendulum angular position, and x_4 – pendulum angular velocity. Input u is the motor driving force adjusted through a PWM wave generated from a microcontroller unit. The position of the cart and pendulum is obtained from incremental encoders with 1024 impulses per rotation. The remaining state variables – the cart and pendulum velocities – are determined from (noisy) position measurements using a differentiating filter with coefficients [1, –1]. Sampling time is set to $T_s = 10$ ms. The input constraint $|u| \leq 6$.

Performance of three control strategies is compared:

a) linear controller $u(t) = -\mathbf{k}\mathbf{x}(t)$ with the gain adjusted as $\mathbf{k}$ = [1.83, 2.58, 22.25, 4.12]. This setting corresponds to the closed-loop eigenvalues $\lambda_* = 0.98$ that yield the shortest transient time without violating the input constraint so that the system stabilizes in spite of uncertainties;

b) fast controller with saturation limiting the input to interval [–6, 6] [N] with the gain, set as $\mathbf{k}$ = [38.52, 25.10, 81.76, 15.27], that corresponds to the closed-loop eigenvalues $\lambda_* = 0.94$ in the linear region. The gain is selected so that the fastest convergence permitted by modelling inaccuracy and saturation nonlinearity is achieved;

c) dynamic quasi-soft VSC (3) with selection strategy (19): the control gains $\mathbf{k}_1$ = [0.15, 0.40, 12.11, 1.91] (closed-loop eigenvalues $\lambda_* = 0.985$) and $\mathbf{k}_2$ = [12.19, 10.65, 46.69, 8.74] (closed-loop eigenvalues $\lambda_* = 0.955$), $s_0 = 100$, $v = 100$, $w = 0.9$, $\mathbf{R} = \mathrm{diag}\{0.2, 1, 0.2, 1\}$.

The cart is initially at rest with the pendulum diverted from the upper unstable equilibrium by 30° (which further strains the test owing to larger inaccuracy in plant model linearization). The objective is to drive the state to zero.

The system output (pendulum position) is plotted in Fig. 3 and the corresponding input signal in Fig. 4. All three controllers bring the output to the vicinity of zero. As expected, the slowest convergence is attained by the linear controller, which also results in the largest limit cycle induced by the nonlinearities of the physical plant. The saturating and quasi-soft VSC strategies achieve

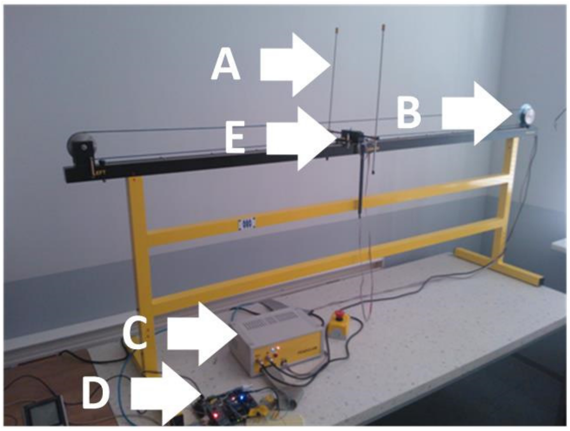

Figure 2: Experimental setup: A – inverted pendulum mounted on cart E; B – motor; C – signal manipulation device; D – microcontroller unit with the control logic.

similar convergence time with smaller overshoot exhibited by the latter. The quasi-soft VSC shows much improvement over the linear scheme in terms of convergence, at the same time avoiding oscillatory input generated by the saturating controller. The smoothness of input signal quantified through $J_s(t) = \sum_{i=0}^{t-1} |u(i+1) - u(i)|$, is illustrated in Fig. 5.

5 CONCLUSIONS

The paper investigates application of soft VSC concept in sampled-data control systems with saturating input. Unlike the classical continuous-time formulation, the control action is adjusted at finite intervals permitted by the sampling period, which results in quasi-soft behaviour. The presented design procedure, specific to discrete-time systems, allows one to preserve the favourable properties of continuous-time VSC. In particular, the quasi-soft VSC combines the benefits of fast convergence and smooth control signals, leading to an attractive solution to be implemented in digital control systems with magnitude-constrained inputs.

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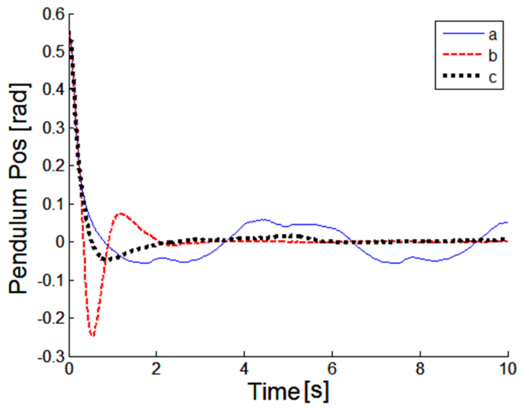

Figure 3: Pendulum angular position: a) linear, b) saturating, c) soft VSC strategy.

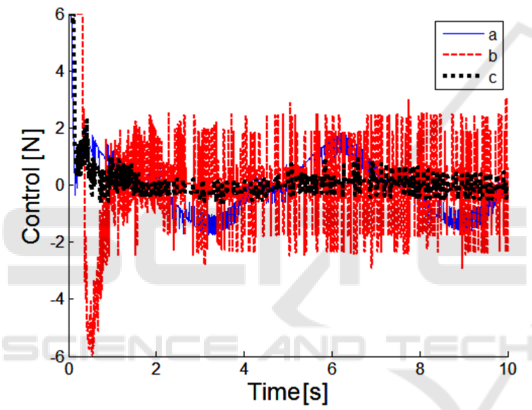

Figure 4: Control input: a) linear, b) saturating, c) soft VSC strategy.

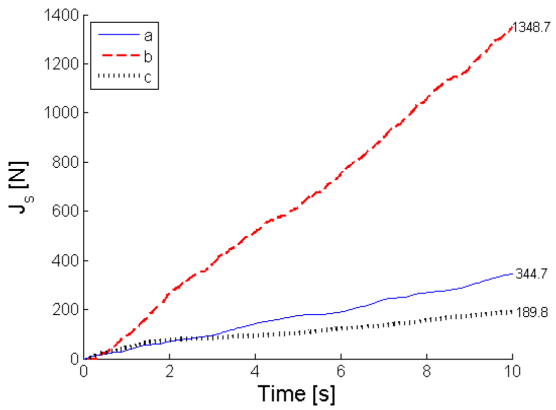

Figure 5: Input smoothness: a) linear, b) saturating, c) soft VSC strategy.

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