Contingency Table Analysis Applying Fuzzy Number and Its Application

Needs Analysis for Media Lectures

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Abstract: Generally, we could efficiently analyse the inexact information by applying fuzzy theory. We would extend contingency table, and propose type-2 fuzzy contingency table. In this paper, we would discuss about type-2 fuzzy contingency table and a needs analysis method applying type-2 fuzzy contingency table.

1 INTRODUCTION

With the spread PCs, tablet PCs and high-capacity Internet communication, recognition of university students for the media class has been changed significantly. In order to the better media class, it is important to know what students are feeling.

Today, there are some of the needs of the students, for example, teaching aid, homework, feedback and so on. In this paper, we propose a questionnaire analysis that applies type-2 fuzzy contingency table.

2 FUZZY CONTINGENCY TABLE

Def. 1. Cardinality of Type-1 Fuzzy Set

Consider the type-1 fuzzy set A in universe $U = \{x_i \mid i = 1, \cdots, n\}$. Cardinality $|A|$ of type-1 fuzzy set A is defined as follows;

$$|A| = \sum_{i=1}^{n} \mu_A(x_i)$$

where, $\mu_A(x_i)$ is a membership function of a type-1 fuzzy set A.

Def. 2. Type-1 Fuzzy $m \times n$ Contingency Table

Consider the type-1 fuzzy set A in universe $U = \{x_i \mid i = 1, \cdots, n\}$. The type-1 fuzzy $m \times n$ contingency table of type-1 fuzzy set $A_1, \cdots, A_n, B_1, \cdots, B_m$ is defined as follows;

	A_1	$\cdots$	A_n	Sum
B_1	f_{11}	$\cdots$	f_{1n}	$\lvert B_1\rvert$
$\vdots$	$\vdots$		$\vdots$	$\vdots$
B_m	f_{m1}	$\cdots$	f_{mn}	$\lvert B_m\rvert$
Sum	$\lvert A_1\rvert$	$\cdots$	$\lvert A_n\rvert$	$n(U)$

where,

$$\sum_{i=1}^{n} \mu_{A_i}(x_k) = 1, \qquad \sum_{i=1}^{m} \mu_{B_i}(x_k) = 1$$

and,

$$f_{ij} = |A_j \cap B_i|$$
$$\mu_{A_j \cap B_i}(x) = \mu_{A_j}(x) \cdot \mu_{B_i}(x).$$

Here, we would expand the definition, we define a type-2 fuzzy contingency table. For the definition of type-2 fuzzy contingency table, we need the mean value of fuzzy numbers, the product value of fuzzy numbers and the intersection of type-2 fuzzy sets. Then, we could clarify these definitions.

Def. 3. Mean Value of Fuzzy Numbers

Let $x_1^*, x_2^*, x_3^*, \ldots, x_n^*$ be fuzzy numbers with α −cuts

$$C_\alpha(x_i^*) = [a_{\alpha,i}, b_{\alpha,i}] \; (\alpha \in \mathbb{R}, 0 \le \alpha \le 1)$$

then the mean value $\overline{x^*}$;

$$\overline{x^*} = \bigcup_{\alpha \in (0,1]} \alpha C_\alpha(\overline{x^*})$$

$$C_\alpha(\overline{x^*}) = \left[\frac{1}{n}\sum_{i=1}^{n} a_{\alpha,i}, \frac{1}{n}\sum_{i=1}^{n} b_{\alpha,i}\right]$$

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Def. 4. Product Value of Fuzzy Numbers

Let u_1^*, u_2^* be fuzzy numbers with α −cuts

$$C_\alpha(u_i^*) = [a_{\alpha,i}, b_{\alpha,i}] \quad (\alpha \in \mathbb{R}, 0 \leq \alpha \leq 1)$$

then the product value $u_1^* \cdot u_2^*$;

$$u_1^* \cdot u_2^* = \bigcup_{\alpha \in (0,1]} \alpha C_\alpha(u_1^* \cdot u_2^*)$$

$$C_\alpha(u_1^* \cdot u_2^*) = \left[\min_{(x_1,x_2) \in C_\alpha(u_1^*) \times C_\alpha(u_2^*)} x_1 \cdot x_2, \max_{(x_1,x_2) \in C_\alpha(u_1^*) \times C_\alpha(u_2^*)} x_1 \cdot x_2\right]$$

Def. 5. Intersection of Type-2 Fuzzy Sets

Consider the type-2 fuzzy sets $\tilde{A}, \tilde{B}$ in universe $U = \{x_i \mid i = 1, \cdots, n\}$;

$$\tilde{A} = \{(x_i, u_i^*) | i = 1, \dots, n\},$$
$$\tilde{B} = \{(x_i, v_i^*) | i = 1, \dots, n\}$$

where, let u_i^*, v_i^* be fuzzy numbers. Then the intersection $\tilde{A} \cap \tilde{B}$;

$$\tilde{A} \cap \tilde{B} = \{(x_i, u_i^* \cdot v_i^*) | i = 1, \dots, n\}$$

Here, we would define the type-2 fuzzy contingency table by these definitions.

Def. 6. Type-2 Fuzzy $m \times n$ Contingency Table

Consider the type-2 fuzzy sets

$$\widetilde{A_1}, \cdots, \widetilde{A_n}, \widetilde{B_1}, \cdots, \widetilde{B_m}$$

in universe

$$U = \{x_i \mid i = 1, \cdots, k\};$$
$$\widetilde{A_p} = \{(x_{i,p}, u_{i,p}^*) | i = 1, \dots, k\},$$
$$\widetilde{B_q} = \{(x_{i,q}, u_{i,q}^*) | i = 1, \dots, k\}$$
$$(1 \leq p \leq n, 1 \leq q \leq m)$$

	$\widetilde{A_1}$	$\cdots$	$\widetilde{A_n}$
$\widetilde{B_1}$	$\overline{f_{11}}$	$\cdots$	$\overline{f_{1n}}$
$\vdots$	$\vdots$		$\vdots$
$\widetilde{B_m}$	$\overline{f_{m1}}$	$\cdots$	$\overline{f_{mn}}$

where, let $\overline{f_{ij}}$ be mean value $\overline{u^* \cdot v^*}$ of grades of intersection $\widetilde{A_i}, \widetilde{B_j}$.

Def. 7. Entropy of Fuzzy Number

Let v^* be fuzzy numbers with membership function $\mu_{v^*}(x)$, then the entropy $S(v^*)$ of fuzzy number v^* ;

$$S(v^*) = \int_{-\infty}^{\infty} D(\mu_{v^*}(x))\, dx$$

$$D(u) = \begin{cases} -u \log u - (1-u) \log(1-u) & , 0 < u < 1 \\ 0 & , otherwise \end{cases}$$

3 ANALYSIS METHOD

The Kano model[1] is a theory of product development and customer satisfaction developed in the 1980s by Professor Noriaki Kano, which classifies customer preferences into five categories.

Figure 1: Kano Model Illustrated.

- Must Be (Expected Quality):
  Requirement that can dissatisfy (expected, but cannot increase satisfaction)
- One-Dimensional (Desired Quality):
  The more of these requirements that are met, the more a client is satisfied
- Delighters (Excited Quality):
  If the requirement is absent, it does not cause dissatisfaction, but it will delight clients if present
- Indifferent:
  Client is indifferent to whether the feature is present or not
- Reverse:
  Feature actually causes dissatisfaction

The authors propose a method to analyse Kano model style questionnaire to the media classroom, analysed by type-2 fuzzy $m \times n$ contingency table.

1. We execute questionnaire, we ask two questions for one requirement. Two questions are a positive question and a negative question.

- Positive question:
  "How does customer feel if the requirement can be met?"
- Negative question:
  " How does customer feel if the requirement can't be met?"

And, we prepare the answer choices of 13 steps to each question.

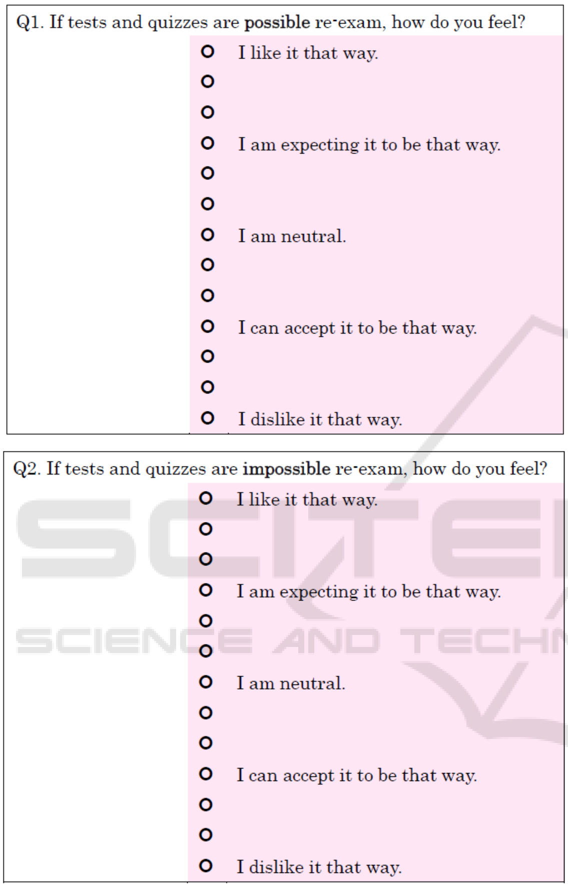


Figure 2: Positive Question and Negative Question.

2. We count the answer of questionnaire by fuzzy number, and create type-2 fuzzy sets.

   For example, when a student S_1 checks for second step (Fig.3.), we interpret as follows:

   The degree of truth of the statement "a student S_1 answers 'I like it that way' " is grade $\frac{\tilde{2}}{3}$, the degree of truth of the statement "a student S_1 answers 'I am expecting it to be that way'" is grade $\frac{\tilde{1}}{3}$.

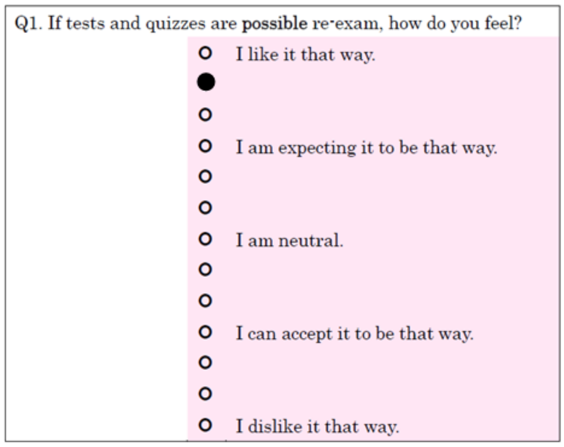


Figure 3: Example.

Here, we define a membership function $\mu_{\tilde{a}}(x)$ of the fuzzy number $\tilde{a}$ as follows:

$$\mu_{\tilde{a}}(x) = \max\{0, 1 - |3(x-a)|\}$$

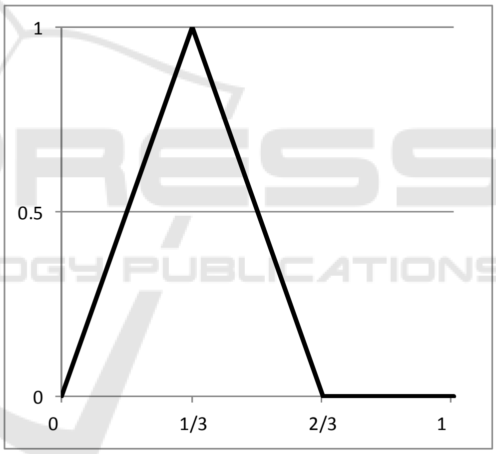


Figure 4: Membership Function $\mu_{\frac{\tilde{1}}{3}}(x)$.

3. We create a 5×5 fuzzy contingency table(Table I.)

   For example, let $\overline{f_{23}}$ be mean value of grades of intersection "Expect(Functional)" and "Neutral(Dysfunctional)". Consider the type-2 fuzzy sets $\widetilde{A_2}, \widetilde{B_3}$ in universe $U = \{x_1, x_2\}$. If

   "Expect(Functional)" : $\widetilde{A_2} = \left\{\left(x_1, \frac{\tilde{2}}{3}\right), \left(x_2, \frac{\tilde{1}}{3}\right)\right\}$

   "Neutral(Dysfunctional)": $\widetilde{B_3} = \left\{\left(x_1, \tilde{1}\right), \left(x_2, \frac{\tilde{2}}{3}\right)\right\}$,

   then $\widetilde{A_2} \cap \widetilde{B_3} = \left\{\left(x_1, \frac{\tilde{2}}{3} * \tilde{1}\right), \left(x_2, \frac{\tilde{1}}{3} * \frac{\tilde{2}}{3}\right)\right\}$, and the membership function of fuzzy number $\frac{\tilde{2}}{3} * \tilde{1}$ and $\frac{\tilde{1}}{3} * \frac{\tilde{2}}{3}$ as follows:

Table 1: 5×5 Fuzzy Contingency Table.

Functional\Dysfunctional	Like	Expect	Neutral	Tolerate	Dislike
Like	$\overline{f^*_{11}}$	$\overline{f^*_{12}}$	$\overline{f^*_{13}}$	$\overline{f^*_{14}}$	$\overline{f^*_{15}}$
Expect	$\overline{f^*_{21}}$	$\overline{f^*_{22}}$	$\overline{f^*_{23}}$	$\overline{f^*_{24}}$	$\overline{f^*_{25}}$
Neutral	$\overline{f^*_{31}}$	$\overline{f^*_{32}}$	$\overline{f^*_{33}}$	$\overline{f^*_{34}}$	$\overline{f^*_{35}}$
Tolerate	$\overline{f^*_{41}}$	$\overline{f^*_{42}}$	$\overline{f^*_{43}}$	$\overline{f^*_{44}}$	$\overline{f^*_{45}}$
Dislike	$\overline{f^*_{51}}$	$\overline{f^*_{52}}$	$\overline{f^*_{53}}$	$\overline{f^*_{54}}$	$\overline{f^*_{55}}$

- $\frac{\tilde{2}}{3} * \tilde{1}$:

$$\mu_{\frac{\tilde{2}}{3}*\tilde{1}}(x) = \max\left\{\min\left\{\frac{-3+\sqrt{1+36x}}{2}, \frac{7-\sqrt{1+36x}}{2}\right\}, 0\right\}$$

- $\frac{\tilde{1}}{3} * \frac{\tilde{2}}{3}$:

$$\mu_{\frac{\tilde{1}}{3}*\frac{\tilde{2}}{3}}(x) = \max\left\{\min\left\{\frac{-1+\sqrt{1+36x}}{2}, \frac{5-\sqrt{1+36x}}{2}\right\}, 0\right\}$$

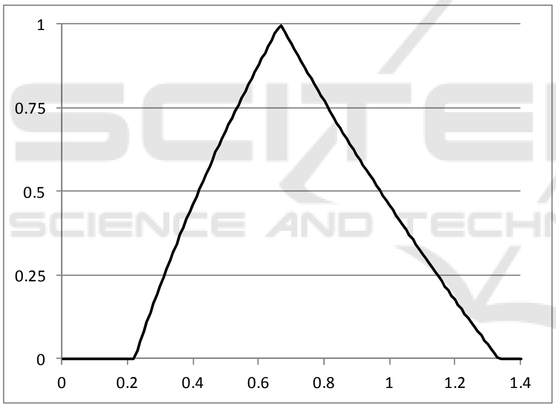


Figure 5: Membership Function $\mu_{\frac{\tilde{2}}{3}*\tilde{1}}(x)$.

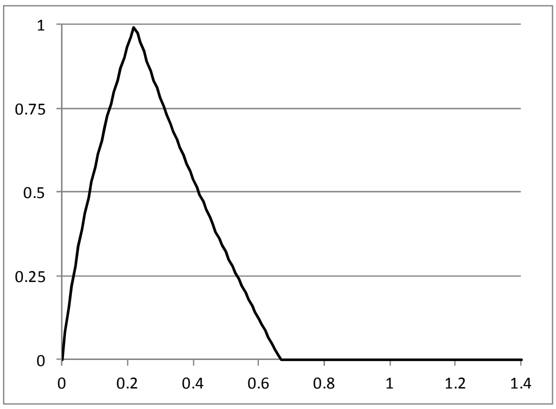


Figure 6: Membership Function $\mu_{\frac{\tilde{1}}{3}*\frac{\tilde{2}}{3}}(x)$.

Fuzzy number $\frac{\tilde{2}}{3} * \tilde{1}$ with α −cuts

$$C_\alpha\left(\frac{\tilde{2}}{3} * \tilde{1}\right) = \left[\frac{\alpha^2+3\alpha+2}{9}, \frac{\alpha^2-7\alpha+12}{9}\right]$$

and fuzzy number $\frac{\tilde{1}}{3} * \frac{\tilde{2}}{3}$ with α −cuts

$$C_\alpha\left(\frac{\tilde{1}}{3} * \frac{\tilde{2}}{3}\right) = \left[\frac{\alpha^2+\alpha}{9}, \frac{\alpha^2-5\alpha+6}{9}\right]$$

then, α −cuts of $\overline{f_{23}}$ is calculated as follows:

$$C_\alpha\left(\overline{f_{23}}\right) = \left[\frac{\alpha^2+2\alpha+1}{9}, \frac{\alpha^2-6\alpha+9}{9}\right]$$

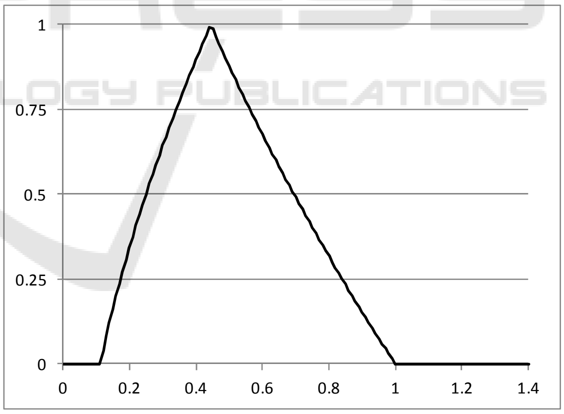


Figure 7: Membership Function $\mu_{\overline{f_{23}}}(x)$.

4. From 5 × 5 fuzzy contingency table, we create a cardinality table(Table II.).

Table 2: Cardinality Table.

Dysfunctional				Functional		
One-Dimensional	Must-Have	Attractive	Indifferent	Attractive	Must-Have	One-Dimensional
$\overline{a^*}$	$\overline{b^*}$	$\overline{c^*}$	$\overline{d^*}$	$\overline{e^*}$	$\overline{f^*}$	$\overline{g^*}$

Where, Let $\overline{f_{ij}^*}$ be fuzzy numbers with α −cuts;

$$C_\alpha(\overline{f_{ij}^*}) = [a_{\alpha,ij}, b_{\alpha,ij}] \quad (\alpha \in \mathbb{R}, 0 \leq \alpha \leq 1)$$

then,

$$\overline{a^*} = \overline{f_{51}^*}, \quad \begin{cases} \overline{b^*} = \bigcup_{\alpha\in(0,1]} \alpha C_\alpha(\overline{x_b^*}) \\ C_\alpha(\overline{x_b^*}) = \left[\sum_{j=2}^{4} a_{\alpha,5j}, \sum_{j=2}^{4} b_{\alpha,5j}\right] \end{cases}, \quad \begin{cases} \overline{c^*} = \bigcup_{\alpha\in(0,1]} \alpha C_\alpha(\overline{x_c^*}) \\ C_\alpha(\overline{x_c^*}) = \left[\sum_{i=2}^{4} a_{\alpha,i1}, \sum_{i=2}^{4} b_{\alpha,i1}\right] \end{cases}$$

$$\begin{cases} \overline{d^*} = \bigcup_{\alpha\in(0,1]} \alpha C_\alpha(\overline{x_d^*}) \\ C_\alpha(\overline{x_d^*}) = \left[\sum_{i=2}^{4}\sum_{j=2}^{4} a_{\alpha,ij}, \sum_{i=2}^{4}\sum_{j=2}^{4} b_{\alpha,ij}\right] \end{cases}, \quad \begin{cases} \overline{e^*} = \bigcup_{\alpha\in(0,1]} \alpha C_\alpha(\overline{x_e^*}) \\ C_\alpha(\overline{x_e^*}) = \left[\sum_{j=2}^{4} a_{\alpha,1j}, \sum_{j=2}^{4} b_{\alpha,1j}\right] \end{cases},$$

$$\begin{cases} \overline{f^*} = \bigcup_{\alpha\in(0,1]} \alpha C_\alpha(\overline{x_f^*}) \\ C_\alpha(\overline{x_f^*}) = \left[\sum_{i=2}^{4} a_{\alpha,i5}, \sum_{i=2}^{4} b_{\alpha,i5}\right] \end{cases}, \quad \overline{g^*} = \overline{f_{15}^*}$$

5. From a cardinality table, we calculate the fuzzy weighted average. These weights are as follows:

Table 3: Weight Table.

weight	input	conclusion
−1	O(N)	not actively implement this requirement
− 2/3	M(N)	not implement as much as possible this requirement
− 1/3	A(N)	can afford to not implement this requirement
0	I	can't decided either way
1/3	A(P)	can afford to implement this requirement
2/3	M(P)	implement as much as possible this requirement
1	O(P)	actively implement this requirement

The weight are fuzzy number $\tilde{a}$, the membership function is defined as follows:

$$\mu_{\tilde{a}} = \max\{0, 1 - |3(x - a)|\}$$

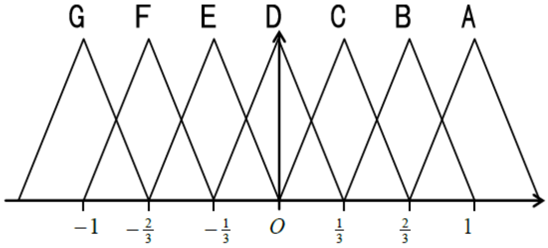


Figure 8: Membership Functions of weight.

We determine a comprehensive evaluation from this fuzzy weighted average.

4 APPLICATION

We executed questionnaires about the function for the media class for 244 students.

Questionnaires:

Q1,Q2 : ToDo list
Q3,Q4 : Reminder Mail
Q5,Q6 : Test's Deadline

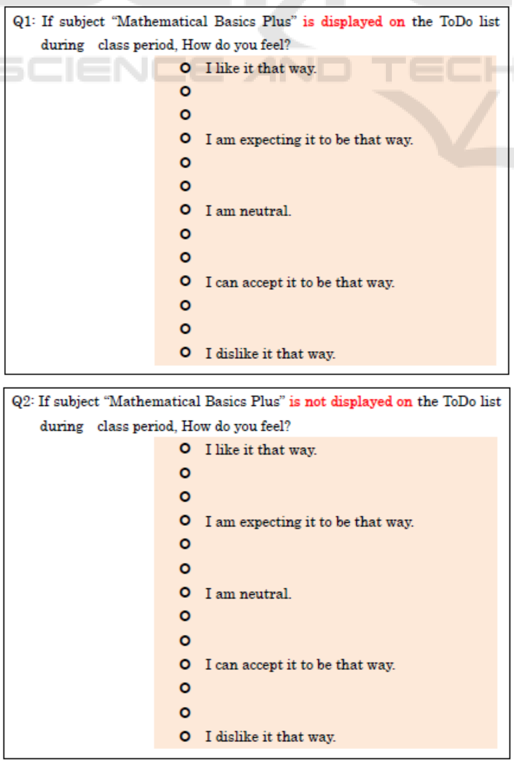


Figure 9: Questionnaires (Q1,Q2).

Then, we obtain the response table(Table IV.).

Table 4: Response Table.

No.	Q1	Q2	Q3	Q4	Q5	Q6
1	7	7	7	7	13	1
2	7	7	10	4	7	7
3	7	7	4	7	13	2
4	10	1	1	9	10	3
5	7	7	7	7	7	7
6	7	7	7	7	7	7
7	7	12	7	7	3	8
8	7	7	11	3	7	7
9	11	6	10	4	7	7
10	1	10	1	10	7	7
11	7	7	1	7	8	1
12	7	7	4	10	13	1
13	1	10	1	10	10	1
14	1	13	1	13	13	1
15	7	7	7	7	9	6
234	2	10	6	7	7	7
235	1	13	1	13	7	7
236	7	7	7	10	12	2
237	7	7	1	13	7	7
238	1	1	1	1	1	1
239	0	0	0	0	0	0
240	7	7	2	10	7	7
241	7	7	7	7	7	7
242	7	7	2	7	7	7
243	7	7	6	7	7	7
244	7	7	1	10	10	2

By using the previous method, we obtain a cardinality table (Table V.).

Next, we calculate the fuzzy weighted average.

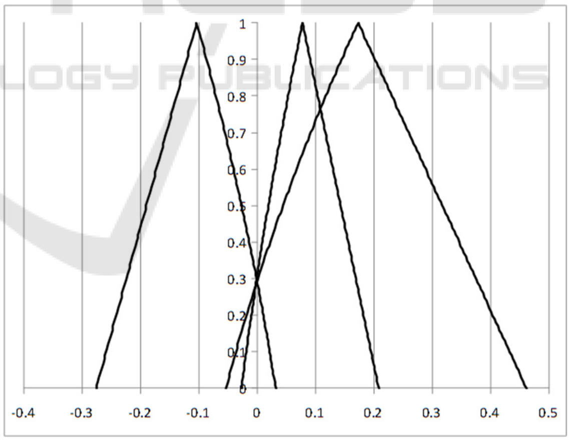
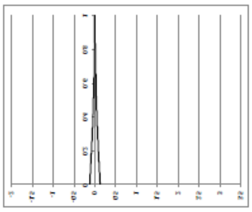
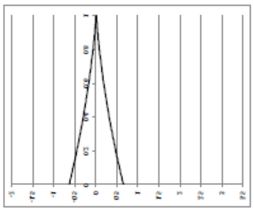
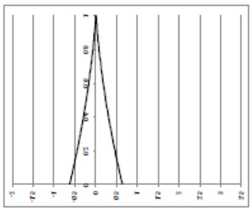
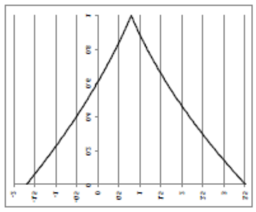


Figure 10: Fuzzy Weighted Average.

Then, we determine a comprehensive evaluation by calculating gravity of this fuzzy weighted average.

Table 5: Cardinality Table (Q1,Q2).

Dysfunctional			
One-Dimensional	Must-Have	Attractive	Indefferent
			
0	0.01	0.01	0.79

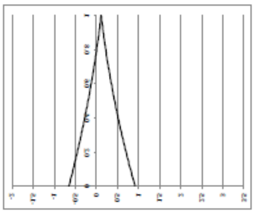
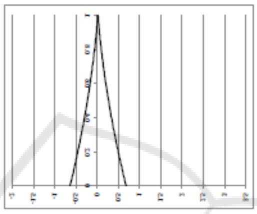
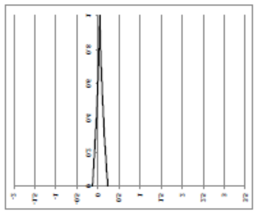
Functional		
Attractive	Must-Have	One-Dimensional
		
0.11	0.02	0.04

Table 6: Result.

	ToDo List	Reminder Mail	Test's Deadline
Weighted Average	0.078324	0.173297	-0.10338
Center of Gravity	0.084673	0.188755	-0.11257
Fuzzy Entropy	0.170076	0.372188	0.222228

5 CONCLUSION

We executed a needs analysis of the students applying type-2 fuzzy 5 × 5 contingency table. As a result, it was able to confirm its effectiveness as a method. Further, we would like to improve analytical methods in the future.

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