Hidden Markov Model Traffic Characterisation in Wireless Networks

Evangelos Spyrou and Dimitris Mitrakos
School of Electrical and Computer Engineering, Aristotle University of Thessaloniki,
Egnatia Odos, Panepistimioupoli, 54124 Thessaloniki, Greece
{evang spyrou, mitrakos}@eng.auth.gr

Keywords: Information Based Similarity, Euclidean distance, Wireless Traffic, Shannon Entropy, Signal-to-Interference-and-Noise-Ratio.

Abstract: Quality of service wireless traffic that often exhibits burstiness, occasionally occurring due to mobility, provides a critical networking issue. Traffic patterns in wireless networks are not of a traditional nature. Nodes transmit their information in batches over a short period of time, before they lose connection. Prediction of wireless incoming load plays an important role in the design of wireless local area networks. The issues of load balancing and Quality of Service constraints are a major problem, which is responsible for the increase of throughput of the network; thus, predicting traffic can be of a great assistance in the aforementioned research directions, leading to a significant optimisation of the wireless network operation. This paper addresses the problem of traffic prediction using Hidden Markov Models. The data is clustered using the Information Based Similarity index that classifies different types of traffic. We show the limitation of this approach and we finally select Euclidean distance for data clustering. Together, they provide an efficient solution towards the solution of wireless traffic characterisation and prediction. We show the efficiency of our scheme in a series of simulations

SCITEPRESS SCIENCE AND TECHNOLOGY PUBLICATIONS

1 INTRODUCTION

In the field of wireless networks, requirements such as quality of service over normal or bursty links, often originating from mobility, are of great significance (Jiang and Dovrolis, 2005; Alizai et al., 2009). Wireless devices do not always follow specific traffic patterns; on the contrary, they attempt to transmit their packets in a bursty fashion, meaning all of them in a short period of time, before they lose or make the connection not reliable (Papadopoulos et al., 2015). Such dynamic and bursty traffic cause certain deficiencies in the network and provide an interesting problem to investigate.

Wireless traffic prediction is a key factor in the analysis and design of wireless local area networks (WLAN)s (Papadopouli et al., 2005). For example, the well-known problem of load balancing can be addressed using Access Point (AP) traffic prediction. In this way, a new connection with this AP may be decided based on the prediction of the AP load. This is an improvement of the throughput of the network; furthermore, Quality of Service (QoS) constraints can be satisfied.

Recently, wireless network traffic has proven to exhibit self-similarity or long-range dependence (Park and Willinger, 2000; Jiang et al., 2001). Self-similarity in traffic is crucial and cannot be analysed by traditional models. Traffic prediction from past measurements constitutes an efficient way to acquire traffic control when self-similar loads occur. Optimality in forecasting still remains a major issue (Beran, 1994). This constitutes the location of efficient self-similar models for prediction of future traffic fluctuations a fundamental problem.

In this paper we address the problem of classifying wireless traffic by attempting to cluster the data utilising the information based similarity (IBS) index (Yang et al., 2003). We show the efficiency of this approach as well as the problems that may arise when integrating it to a Hidden Markov Model (HMM). Based on the limitations of this

Spyrou E. and Mitrakos D.
Hidden Markov Model Traffic Characterisation in Wireless Networks.
DOI: 10.5220/0006227400780085
In *Proceedings of the Fifth International Conference on Telecommunications and Remote Sensing (ICTRS 2016)*, pages 78-85
ISBN: 978-989-758-200-4
Copyright © 2016 by SCITEPRESS – Science and Technology Publications, Lda. All rights reserved

approach we select the Euclidean distance to cluster the data in our machine learning approach.

We attempt to characterise the pattern of the traffic by feeding the packets generated by a wireless network to a Hidden Markov Model. The distances we find using the Euclidean distance assist us to cluster the data to normal, bad and bursty traffic. Combining the two, we are able to provide substantial information and a good characterisation and prediction on the traffic we will be experiencing. More specifically, we show the following contributions:

- Wes show the difference in similarity of different types of wireless traffic using the IBS index;
- We show that the IBS index can be a good methodology to cluster the incoming data and characterise traffic;
- We show the limitations of this approach when integrating it to an HMM
- We employ Euclidean distance to cluster the traffic data.
- We utilise the HMM to characterise and predict the upcoming traffic and show indexes of the efficiency of our approach;
- We show in a series of simulations the efficiency of our approach;

This paper is structured as follows: Section 2 provides the related work, Section 3 gives the background of the IBS index and some results, Section 4 provides a brief background on the Euclidean distance, Section 5 gives a background on our HMM proposal, Section 6 provides the simulation results of our approach, and Section 7 provides the conclusions.

2 RELATED WORK

In (Ni et al., 2015), the authors address the resource management for dynamic control of channel resources and energy efficiency in modern cellular systems. They identify that early monitoring and forecasting if basestation traffic volumes play a key roles in the management. Spatiotemporal network traffic analysis is the means to a successful prediction of traffic. To this end, they examine the spatiotemporal features of cellular traffic generated in a practical scenario in China. The authors analyse basestations that exhibit similar features and they cluster them. They elaborate on the sliding windows sizes and encapsulate the Elman Neural Network with wavelet transform to accomplish traffic precision.

In (Maheshwari et al., 2013), the authors aim to tackle the issue of Quality of Service of end-user calls for realistic traffic models. They mainly address modelling wireless internet traffic using realistic traffic traces. This traffic are collected from networks and they perform forecasting on the end-to-end Quality of Service parameters for the networks. The traffic model is designed based on Hidden Markov Model by taking into account the joint distribution of end-to-end delay, packet variation and packet size. The states that are identified are mapped to four traffic classes. These are conversational, streaming, interactive, and background.

In (Yadav and Balakrishnan, 2014), the issue of traffic modeling is investigated. Typical networking issues, such as resource allocation, quality of service, bandwidth and congestion control are addressed. A comparison of modeling techniques is carried out of adaptive neuro fuzzy inference system (ANFIS) and autoregressive integrated moving average (ARIMA) for modeling of wireless network traffic in terms of typical statistical indicator and computational complexity. Furthermore, a comparative performance evaluation is undertaken in traffic modeling showing that ANFIS constitutes a good methodology for prediction with respect to statistical indicators. Moreover, it provides a reasonable description of the conditions of a wireless network in the time domain. The main drawback is the complexity ANFIS introduces, even though it performs better than the ARIMA model in the scenarios they investigate.

In (Li et al., 2014), the authors aim to examine traffic prediction on cellular radio networks. The need for a traffic-aware energy efficient architecture is highlighted. To this end, traffic prediction is modeled a traffic-aware networking is addresses. For the former, entropy theory is utilised towards the analysis of traffic predictability. In terms of the latter practical prediction performance is demonstrated using the best methodologies in the literature. Finally, the authors suggest a blueprint regarding a traffic-oriented software-defined cellular radio network architecture and they show the potential applications of traffic prediction in this architecture.

In (Rutka and Lauks, 2015), the utilisation of neural networks for internet traffic prediction is investigated. Specifically, the investigate traffic prediction in the presence of self-similarity, which is an important feature of traffic in high-speed networks that may not be obtained by traditional traffic models. The major aim of this work is the performance and prediction error investigation using feed forward neural networks.

In (Loumiotis et al., 2014), the efficient management of the backhaul resources in 4G networks is examined. The authors raise this issue in the case that the backhaul network has been leased by the mobile operator. Hence, the backhaul resource allocation issue at the basestation is investigated and aggregate traffic demand scheme is proposed using artificial neural networks. Finally, the authors provide evidence of the efficiency of their scheme in terms of absolute percentage error of downlink and uplink traffic.

In (Pan et al., 2013), the stochastic cell transmission model is extended, in order to include spatiotemporal characteristics of traffic and predict short-term traffic. Initially, the authors utilise a multivariate normal distribution-based best linear predictor as an auxiliary dynamical system predict boundary variables and/or supply functions. Thereafter these variables and functions are input to the stochastic cell transmission model for short-term traffic state prediction. Stochastic cell transmission model is relaxed by utilising the covariance structure calibrated from the spatial correlation analysis for probabilistic traffic state evaluation. Prediction is carried-out in a rolling horizon manner, which is handy for setting the predicted traffic state using real-time measurements.

3 IBS FOR WIRELESS TRAFFIC

We consider a wireless network where nodes communicate with respective APs to transmit their load. We assume that the network is mobile; hence, there is a possibility of loss of connection and transmission in a bursty fashion. This is reflected in the Signal-to-Interference-plus-Noise Ratio (SINR) between the transmitter and the receiver. We employ the SINR model appearing in (Spyrou and Mitrakos, 2015) to construct our case.

We denote as $\gamma_{k,j}$ as the SINR of the transmission for node k to node j and it is given by

$$\gamma_{k,j} = \frac{H_{k,j} p_k}{\sum_{t\neq k, j\neq j} p_t H_{t,j} + N_0} \qquad (1)$$

where $H_{k,j}$ is the channel gain between nodes k and j, $p_k$is the transmission power of node k transmitting, p_t is the interfering node t's transmission power, $H_{t,j}$ is the channel gain between the interferer and the receiver and N_0 is the noise. For a packet to be successfully received the following condition must be satisfied

$$\gamma_{k,j} \geq \gamma_{thr} \qquad (2)$$

Where γ_{thr} is the SINR threshold for successful reception of the packet. In practical scenarios, a packet is successfully received when an acknowledgement is received by the sender. We consider a packet reception series $\{x_1, x_2, \dots, x_N\}$ where x_i is the packet i. We classify each packet into two states that represents the successful reception or not of the packet, as it can be identified by its acknowledgement value, as below

$$I_n = \begin{cases} 1, \gamma \geq \gamma_{thr} \\ 0, otherwise \end{cases} \qquad (3)$$

We map $m + 1$ successive intervals to a binary sequence of length m, called an m-bit "word." Each m-bit word, w_k, therefore, represents a unique pattern of fluctuations in a given time series. By shifting one data point at a time, the algorithm produces a collection of m-bit words over the whole time series. Therefore, it is plausible that the occurrence of these m-bit words reflects the underlying dynamics of the original time series. Different types of dynamics thus produce different distributions of these m-bit words.

The resulting rank-frequency distribution, therefore, represents the statistical hierarchy of symbolic words of the original time series. For example, the first rank word corresponds to one type of fluctuation, which is the most frequent pattern in the time series. In contrast, the last rank word defines the most unlikely pattern in the time series. To define a measurement of similarity between two signals, we plot the rank number of each m-bit word in the first time series against that of the second time series.

If two time series are similar in their rank order of the words, the scattered points will be located near the diagonal line. Therefore, the average deviation of these scattered points away from the diagonal line is a measure of the "distance" between these two time series. Greater distance indicates less similarity and vice versa. In addition, we incorporate the likelihood of each word in the following definition of a weighted distance, D_m, between two symbolic sequences, S_1 and S_2.

$$D_m(S_1, S_2) = \frac{1}{2^m - 1} \sum_{k=1}^{2^m} |R_1(w_k) - R_2(w_k)| F(w_k) \qquad (4)$$

where

$$F(w_k) = \frac{1}{Z}[-c_1(w_k)logc_1(w_k) - c_2(w_k)logc_{2(w_k)}] \quad (5)$$

Here $c_1(w_k)$ and $R_1(w_k)$ represent probability and rank of a specific word, w_k, in time series S_1. Similarly, $c_2(w_k)$ and $R_2(w_k)$stand for probability and rank of the same m-bit word in time series S_2. The absolute difference of ranks is multiplied by the normalized probabilities as a weighted sum by using Shannon entropy as the weighting factor. Finally, the sum is divided by the value $2^m - 1$ to keep the value in the same range of [0, 1]. The normalization factor Z in Equation 5 is given by

$$Z = \sum_k [-c_1(w_k)logc_1(w_k) - c_2(w_k)logc_2(w_k)] \quad (6)$$

Next, we will provide an example using realistic traffic models that will show some initial results of this approach.

We have taken measurements from a wireless network, where we obtained the SINR values of 1000 packet transmissions. We made sure that the configuration of the nodes were as such that there was significant fluctuation on the signal; simulating thus, mobility and bursty traffic. Furthermore, we have obtained packet information from nodes' communication, where the network is fully connected and not connected.

Initially, we have undertaken experiments to show the information similarity index between the three configurations, namely no network, full network and bursty traffic network. The values that we collect from the comparison of the three types of traffic leads to thresholds for the characterisation of traffic; in short, we cluster the data based on these values, as we can see in table 1 below:

Table 1: IBS index for different traffic classes

No – Full	No - Bursty	Full - Bursty
0.140923	0.075158	0.158906

Note that No corresponds to No Network with low SINR, Full Network is a fully connected network with high SINR, and Bursty is a bursty traffic network.

We selected an 8 value word, in order to mimic the bursty traffic that wireless network usually exhibit. We see that the IBS index for the Bursty Traffic - Full Network is similar to the Full Network -No Network configuration. This is the case, due to the existence of the bursts that lead the IBS index method to move the index towards the existence of a fully connected network.

Subsequently, we may include these values as thresholds in our machine learning implementation, in order to identify different states of traffic. However, using IBS to distinguish between these two traffic classes may result in an ambiguity in the state identification. This shows that the IBS may not locate the states that correspond to these different traffic patterns. Moreover, another limitation is that the IBS index method requires to collect at least 9 samples to be able to calculate a distance between the current state and the distance of the new value, in order to compare it to the thresholds. This does not allow us to investigate our problem at the single value granularity that will allow us to see the events that will occur.

4 EUCLIDEAN DISTANCE FOR WIRELESS TRAFFIC

The potential limitations of the IBS index lead us to employ the Euclidean distance for the data clustering. This method allow us to investigate the data at the single value, since it calculates the distance between the current state and the incoming value to compare their distance with our defined threshold. This will give us the opportunity to examine events at the value level, without requiring a set of values to be compared against another set.

The Euclidean distance in terms of machine learning is the distance measure between a pair of samples p and q in an n-dimensional feature space and it is given by

$$d = \sqrt{\sum_{i=1}^{n}(q_i - p_i)^2} \quad (7)$$

The Euclidean is often the default distance used in approaches such as the K-means clustering (MacQueen,1967) to locate the k closest point of a sample point.

5 HMM FOR WIRELESS TRAFFIC

We selected the Hidden Markov Models (Eddy, 1996) to classify and predict traffic in our network due to the fact the predictions may be made using the last recorded value, as opposed to other machine learning techniques, such as neural networks (Kosko, 1992), which requires a plethora of historic data, in order to be trained..

Hidden Markov models (HMMs) are the most popular means of temporal classification. Informally speaking, a hidden Markov model is a variant of a finite state machine. However, unlike finite state machines, they are not deterministic. A normal finite state machine emits a deterministic symbol in a given state. Further, it then deterministically transitions to another state. Hidden Markov models do neither deterministically, rather they both transition and emit under a probabilistic model (Rabiner and Juang, 1986).

Formally, an HMM is essentially a Markov model where a series of observed outputs

$$x = \{x_1, x_2, \dots, x_T\}$$

is available drawn from an alphabet

$$V = \{u_1, u_2, \dots, u_T\}.$$

Furthermore, he have the existence of states

$$z = \{z_1, z_2, \dots, z_T\}$$

provided by a state alphabet

$$S = \{s_1, s_2, , \dots, s_{|S|}\}, z_t \in S, t = 1 \dots T.$$

The transition between states i and j is represented by the respected value in the state transition matrix A_{ij}. Moreover, the probability of generating the output observation is modelled as a hidden state. To this end, we define

$$P(x_t = u_k | z_t = s_j) = \\ p(x_t = u_k | x_1, \dots, x_T, z_1, \dots z_T) = B_{jk}$$

where B_{jk} is the matrix which encodes the probability of the hidden state producing the output u_k provided that the state at the corresponding time was s_j.

We used a similar approach as the one in (Dunham et al., 2004). Essentially, the HMM we developed is a time-varying Markov Chain, which consists of entities that perform tasks, in order to reach a predicted value.

Initially, HMM performs the clustering action. The data that arrives from the wireless devices join a specific cluster labeled by the centroid is calculated using the following equation:

$$\bar{x} = \sum_{i=1}^{n} x_i \tag{8}$$

Where x_i is a set of n points of a dimension

$$dx_i, i = 1,2, \dots n.$$

In order to store an incoming value into a cluster, it is essential to calculate the distance between already existent states and the incoming value. The distance is found using equation (7) described in the previous section.

The present value is declared as a new state if the value of its distance with the already existent states is bigger than the value of a defined threshold. On the other hand, the incoming value is similar to an existent state, whose values are closer to the incoming value. The completion of the clustering initiates the building of the HMM. Given the Markov Chain at time t and the clustering result at t +1 the Markov Chain is updated at time t +1. First, the state transition probability between two successive points is calculated. Thereafter, the time sequence is updated with the state transition probability. Furthermore, the HMM includes a procedure of self-evaluation by calculating certain metrics of its performance such as the Normalised Absolute Ratio Error (NARE) and the Root Means Square (RMS) error.

$$NARE = \frac{\sum_{t=1}^{N} |(O_{(t)} - P_{(t)})|}{\sum_{t=1}^{N} O_{(t)}} \tag{9}$$

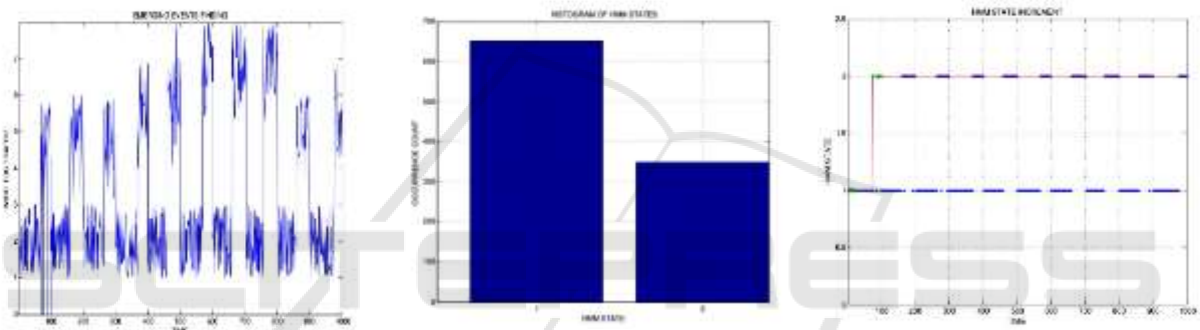
$$RMS = \sqrt{\frac{\sum_{t=1}^{N} \left(O_{(t)} - P_{(t)}\right)^2}{N}} \tag{10}$$

where $O_{(t)}$ is the observed profile, $P_{(t)}$ is the predicted profile, N is the length of the dataset and t is the time variable of the t^{th} tuple in the input dataset.

Then the HMM reaches the prediction phase. Initially, the transition probability of the current state is calculated. The product of the transition probability with the states vector for each sensor recording provides the predicted value of the wireless network traffic. If a node has no connections with another node, then the HMM assumes that the current node is connected to itself.

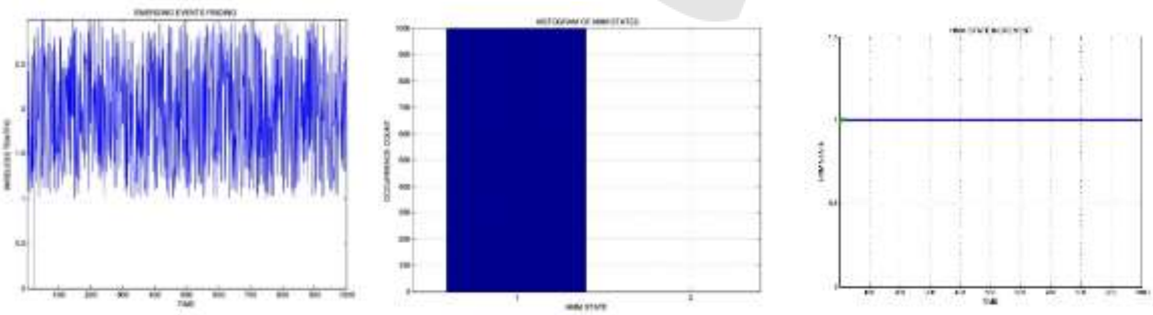
6 RESULTS

We obtained data from a wireless network that consisted of nodes that exhibited bursty traffic and no connectivity due to mobility. Thereafter, we input the data to our HMM to check the identification of different states. We set the threshold that the HMM recognises a new state to be the SINR threshold for successful transmission of a packet.
As we can see in Figure 1, the HMM finds two states, which correspond to the no connection between the nodes and the burst of successful packet transmissions. There is an identification of events that shows the frequency of the identification of the states. Finally, we see a similar result in the increment of the states in the third figure, moving from the state of no connection to the state of connected network. Similarly, in Figure 2 we see the identification of a single state, since the data we put in our HMM does not extend from the value of the SINR threshold. In the events identification subfigure of figure 1 we see that different events are not emerging when a network is not connected; on the contrary, a single state exists dictating that the HMM can identify the presence of a disconnected network. In the same way, we see the next two subfigures of figure 1 that dictate the existence of a single state of a not connected network



Events Identification Bursty   Events Histogram Bursty   Events Increment Bursty

Figure 1: Results for HMM with Bursty Traffic



Events Identification No Net   Events Histogram No Net   Events Increment No Net

Figure 2: Results for HMM with No Net Traffic

SCITEPRESS SCIENCE AND TECHNOLOGY PUBLICATIONS

In terms of performance, we calculated the RMS and NARE for each of the two scenarios. As we can see in table 1, the performance of the HMM is reasonably good with NARE values in the range of 0.20 – 0.26 and RMS from 0.58 – 0.88.

Table 2: RMS and NARE of the two scenarios.

	NARE	RMS
No Net	0.2576	0.5838
Bursty	0.2091	0.8730

Subsequently, we see that the events identification does not show any peaks to indicate that there is a state that has not been found. The above show us that we may have a reasonable mechanism that will be able to classify and predict traffic in a wireless network.

7 CONCLUSIONS

In this paper, we addressed the classification and prediction of wireless traffic using HMMs. We employed two clustering techniques, in order to clarify the states of the data to be input in the HMM.

The first one was the IBS index, which is usually used in physiological signals. We performed three experiments, obtaining the distances between three types of network traffic, namely No Network, Full Network and Bursty Traffic. We have seen that we get a difference in two of the three experiments; thus, resulting in a clear threshold identification for the identification of different traffic states. However, two of the three traffic classes exhibit a very similar distance; hence, the recognition of a new state by the HMM will be ambiguous. Furthermore, the nature of the IBS require bunches of signal values to be examined in order to locate the distance of the data to be evaluated.

Hence, we decided to use the Euclidean distance, which allows us to get a distance between the current state and the incoming value at a single value level; thus, identifying at a greater granularity the traffic patterns.

We put our approach to the test using two traffic files, one of the showing bursty traffic and a second with no connection. The HMM was able to locate the traffic patterns and identify the right number of states and events. We believe that this is an efficient approach for traffic pattern classification and prediction.

For future work, we aim to put our approach to a real network implementation, in order to obtain useful information regarding the operation of the HMM to low-power energy constraint devices.

REFERENCES

Alizai, M. H., Landsiedel, O., Link, J. A´ . B., Go¨tz, S.,and Wehrle, K. (2009). *Bursty traffic over bursty links*. In Proceedings of the 7th ACM Conference on Embedded Networked Sensor Systems, pages 71–84. ACM.

Beran, J. (1994). *Statistics for long-memory processes*, Volume 61. CRC press.

Dunham, M. H., Meng, Y., and Huang, J. (2004).*Extensible markov model*. In Data Mining, 2004. ICDM'04. Fourth IEEE International Conference on, pages 371– 374. IEEE.

Eddy, S. R. (1996*). Hidden markov models*. Current opinion in structural biology, 6(3):361–365.

Jiang, H. and Dovrolis, C. (2005). *Why is the internettraffic bursty in short time scales?* In ACM SIGMETRICS Performance Evaluation Review, volume 33, pages 241–252. ACM. Jiang, M.,

Nikolic, M., Hardy, S., and Trajkovic, L. (2001).*Impact of self-similarity on wireless data network performance*. In Communications, 2001. ICC 2001. IEEE International Conference on, volume 2, pages 477–481. IEEE.

Kosko, B. (1992). *Neural networks and fuzzy systems: a dynamical systems approach to machine intelligence/ book and disk*. Prentice Hall, Upper Saddle River.

Li, R., Zhao, Z., Zhou, X., Palicot, J., and Zhang, H. (2014). *The prediction analysis of cellular radio access network traffic: From entropy theory to networking practice*. IEEE Communications Magazine, 52(6):234–240.

Loumiotis, I., Adamopoulou, E., Demestichas, K.,Kosmides, P., and Theologou, M. (2014). *Artificial neural networks for traffic prediction in 4g networks*. In International Wireless Internet Conference, pages 141–146. Springer.

Maheshwari, S., Mahapatra, S., Kumar, C. S., and Vasu,K. (2013*). A joint parametric prediction model for wireless internet traffic using hidden markov model*. Wireless networks, 19(6):1171–1185.

MacQueen, J. (1967, June). Some methods for classification and analysis of multivariate observations. In *Proceedings of the fifth Berkeley symposium on mathematical statistics and probability* (Vol. 1, No. 14, pp. 281-297).

Ni, F., Zang, Y., and Feng, Z. (2015). *A study on cellular wireless traffic modeling and prediction using elman neural networks*. In 2015 4th International Conference on Computer Science and Network Technology (ICCSNT), volume 1, pages 490–494. IEEE.

Pan, T., Sumalee, A., Zhong, R.-X., and Indra-Payoong,N. (2013). *Short-term traffic state prediction based on temporal–spatial correlation*. IEEE Transactions on Intelligent Transportation Systems, 14(3):1242–1254.

Papadopouli, M., Shen, H., Raftopoulos, E., Ploumidis,M., and Hernandez-Campos, F. (2005*). Short-term traffic forecasting in a campus-wide wireless network*. In 2005 IEEE 16th International Symposium on Personal, Indoor and Mobile Radio Communications, volume 3, pages 1446–1452. IEEE.

Papadopoulos, G. Z., Kotsiou, V., Gallais, A.,Chatzimisios, P., and No¨el, T. (2015). *Wireless medium access control under mobility and bursty traffic assumptions in wsns*. Mobile Networks and Applications, 20(5):649–660.

Park, K. and Willinger, W. (2000). *Self-similar network traffic and performance evaluation.* Wiley Online Library.

Rabiner, L. and Juang, B. (1986). *An introduction to hidden markov models*. ieee assp magazine, 3(1):4–16.

Rutka, G. and Lauks, G. (2015*). Study on internet traffic prediction models.* Elektronika ir Elektrotechnika, 78(6):47–50.

Spyrou, E. D. and Mitrakos, D. K. (2015*). On the Homogeneous transmission power under the sinr model*. In 2015 ICTRS. SCITEPRESS.

Yadav, R. K. and Balakrishnan, M. (2014). *Comparative evaluation of arima and anfis for modeling of wireless network traffic time series.* EURASIP Journal on Wireless Communications and Networking, 2014(1): 1-8.

Yang, A. C.-C., Hseu, S.-S., Yien, H.-W., Goldberger, A.L., and Peng, C.-K. (2003). Linguistic analysis of the human heartbeat using frequency and rank order statistics. Physical review letters, 90(10):108103.

SCITEPRESS
SCIENCE AND TECHNOLOGY PUBLICATIONS