Optimized Modal Research for a Manipulator using Bayesian based Model Updating after Montecarlo-based Sensitivity Analysis

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Abstract: A procedure to identify the dynamic behavior of a 6 degree of freedom (6-DOF) Manipulator based on modal data, has been developed in this paper. A finite element reference model with special emphasis on the modeling of the joints has been built. The most uncertain parameters of the models are updated by minimizing the discrepancies between the analytical and the experimental natural frequencies of the model. The updated models were tested using modal tests according to Monte-carlo based sensitivity analysis and Beyesian based model reduction.

1 INTRODUCTION

In this paper, we investigate the application of the Expectation Maximization (EM) algorithm to operational modal analysis of a 6-DOF manipulator. The mechanical behavior of structures with multi- DOF are idealized in the analysis as interconnected linear elements. However, the response of the zone is more complex and design-dependent, being directly affected by a joint connection. In the past, joints were considered as rigid. This assumption greatly simplifies the analysis, but it does not accurately reflect the true behavior of the joints. In practice, rigid joints exhibit some flexibility. Therefore, adequate modeling and calibration of the joints is essential in the structural design.

The mechanical behavior of the manipulator can be obtained through detailed 3D finite element (FE) models. Analytical models like the component method are also extensively used to characterize the joints. These models should be validated or calibrated through data coming from experimental tests.

Thus, the joints are modeled as linear spring elements for the analysis of the structural serviceability limit state. This approach based only on the moment-rotation stiffness is adequate for static analysis. In the dynamic case, however, it could give inaccurate predictions. This is due to the fact that the geometry and inertia of the joint and the influence of the connection on the mechanical properties of the adjacent elements have a significant effect on the structural response. Therefore, these features should be considered in the joint modeling.

In the present work, a particular manipulator is experimentally and analytically studied. The aim of the paper is a proper modeling of the corresponding joint by means of an FE model comprising beam elements. It is intended to be understood as a whole, considering not only the moment-rotation stiffness of the joint but also its inertia, geometry and its influence on other elements. Once modeled, the aim is to calibrate and validate the proposed models. In this phase, the most uncertain parameters are selected and updated on the basis of experimental data different to those used in testing. For this purpose, two different experimental models and testing procedures are proposed. The first one consists of a cross-like simple supported frame that is dynamically tested in two different support configurations. The second is a semi-portal frame that is statically tested. One of the dynamic tests is used for updating, while the other and the static test are used for testing. The updating is proposed as the minimization of a given fitting function, which accounts for the discrepancies between the analytical and experimental models. A novel adaptive sampling procedure based on values of the fitting function is tried for minimization.

The portal axle is a gearbox that is specially designed for off-road driving conditions. It is installed between the wheel and the axle shaft to give higher ground clearance to the vehicle. The modeling and

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simulation of spur gears in portal axle is important to predict the actual motion behavior. However, gear train design in portal axle is difficult to study comprehensively due to their relatively low cost and short product life cycle. In this study, modal analysis of 6-DOF manipulator is simulated using finite element method (FEM). Modal analysis is simulated on three different combinations of gear train system commonly designed for portal axle. FEM static stress analysis is also simulated on three different gear trains to study the gear teeth bending stress and contact stress behavior of the gear trains in different angular positions from 0° to 18°. The single and double pair gear teeth contact are also considered. This methodology serves as a novel approach for gear train design evaluation, and the study of gear stress behavior in gear train which is needed in the small workshop scale industries.

2 MATHEMATICAL MODELS

The normal structural general equation of motion is denoted by:

$$[M]\{\ddot{x}\}+[C]\{x\}+[K]\{x\}=\{F(t)\} \quad (1)$$

Where $[M]$ is the mass matrix, $[K]$ stiffness matrix, $[C]$ damping matrix, $[F]$ external incentives, $\{x\}$ displacement matrix, $\{\ddot{x}\}$ acceleration matrix.

Modal analysis in ANSYS software is linear (Wei, 2002), and any plastic、large deformation and nonlinear deformation are ignored, while the material of the structure can be linear or non-linear, isotropic or orthotropic, constant or temperature related, so for linear structures, the Eq. (1) can be simplified as:

$$\{x\}=\{\phi_i\}\cos(\omega_i t) \quad (2)$$

Where, ϕ_i is the vibration mode (eigenvector); ω_i is the natural circular frequency for vibration type and the following equations can be obtained:

$$\begin{aligned}&-\omega_i^2[M]\{\phi_i\}\cos(\omega_i t)+[C]\{\phi_i\}\cos(\omega_i t)\\&+[K]\{\phi_i\}\cos(\omega_i t)=0\end{aligned} \quad (3)$$

While $\phi_i=0$ is insignificant, so Eq.(3) can be simplified as follows:

$$[\phi]=[\{\phi\}_1,\{\phi\}_2,\ldots,\{\phi\}_n] \quad (4)$$

Through coordinate transformation for the normal mode matrix, the modal coordinates can be expressed as follows:

$$\{x\}=[\phi]\{\lambda\}=\sum_{i=1}^{n}\lambda_i\{\phi\}_i \quad (5)$$

Where, $\{\lambda\}$ is the weighting factor for the linear superposition of main modes among n-dimensional space, and it can be proved to be orthogonal:

$$\{\lambda\}=\{\lambda_1,\lambda_2,\ldots,\lambda_n\}^T \quad (6)$$

Thus the kinetic energy of the system can be expressed as:

$$\mathrm{E}_e=\frac{1}{2}\left\{\dot{x}\right\}^{\mathrm{T}}[\mathrm{M}]\left\{\dot{x}\right\}=\frac{1}{2}\left\{\dot{\lambda}\right\}^{\mathrm{T}}[\mathrm{M}]\left\{\dot{\lambda}\right\} \quad (7)$$

While $[\overline{\mathrm{M}}]=\{\phi\}^{\mathrm{T}}[\overline{\mathrm{M}}]\{\phi\}$ (8)

And $\overline{m}_r=\phi_r^{\mathrm{T}}\mathrm{M}\phi_r$ is the r-order modal mass, that is its generalized mass for the corresponding coordinates. Similarly, the system potential energy can be expressed as:

$$\mathrm{E}_u=\frac{1}{2}\{x\}^{\mathrm{T}}[\mathrm{K}]\{x\}=\frac{1}{2}\{\lambda\}^{\mathrm{T}}[\mathrm{K}]\{\lambda\} \quad (9)$$

While $[\overline{\mathrm{K}}]=\{\phi\}^{\mathrm{T}}[\mathrm{K}]\{\phi\}$ (10)

Where $\overline{\kappa}_r=\phi_r^{\mathrm{T}}\mathrm{K}\phi_r$ is the r-order modal stiffness, also the generalized stiffness for the corresponding coordinate. Substitute it in the Lagrange equation

$$\eta_i=\sqrt{\kappa_i/m_i}$$

$$\frac{d}{dt}\frac{\partial(\mathrm{E}_e-\mathrm{E}_u)}{\partial\{\lambda\}}-\frac{\partial(\mathrm{E}_e-\mathrm{E}_u)}{\partial\{\lambda\}}=\{\mathrm{N}\} \quad (11)$$

Given generalized the force $\{\mathrm{N}\}=0$, then:

$$[\overline{m_i}]\left\{\ddot{\lambda}\right\}+[\overline{\kappa_i}]\{\lambda\}=0 \quad (12)$$

Converting it to independent expression:

$$\overline{\lambda_i}+\eta_i^2=0 \quad (13)$$

Where is the r-order system natural frequency. Then, one particular solution of the free vibration system would be:

$$\{x\}_i=\{\phi\}_i\sin(\eta_i t+\theta_i) \quad (14)$$

With superposition, the solution of the whole system would be:

$$\{x\}=\sum_{i=1}^{n}\{\phi\}_i\sin(\eta_i t+\theta_i) \quad (15)$$

After transformation coordinate with regular modal matrix, we can get the solution of the whole system:

$$\{x\}=[\phi]\{\lambda\} \quad (16)$$

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The test results for the manipulator using this equator as following:

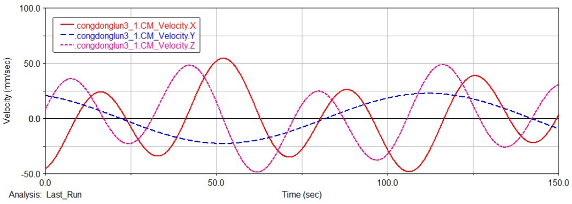

Figure 1: The velocity lines

3 MODEL UPDATING

Several methods of structural model updating have been proposed and the topic is still under active study in various areas. Most of these studies centered on approaches such as the optimal matrix updating, eigen-structure assignment algorithms and neural-networks updating methods. In this paper, the model updating technique was described in detail and updated parameters from the FE model were compared to the original ones. It was presented the theory of bayesian-based model updating with a special focus on the properties of the solution that result from the combination of montecarlo-based sensitivity analysis with model reduction.

It should be attempted to assess the sensitivity which can be attributed to various features of the model. For example, joints and constraints could be considered to be less accurately modeled, and therefore they are in greater need of updating. The parameterization of the inaccurate parts of the model is important. The numerical predictions (e.g. natural frequencies and mode shapes) should be sensitive to small changes in the parameters. Experimental results show that natural frequencies are often significantly affected by small differences in the construction of joints in nominally identical test pieces. However, it can be very difficult to find joint parameters to which the analytical predictions are sensitive. If the analytical response is insensitive to changes in one or several updating parameters, then updating will result in unrealistic values for rest of updating parameters. The result, in this case, will be an updated model which replicates the measurements but lacks physical meaning.

Normally, the numerical model is incompatible with the experimental modal one, therefore, in order to make both of the two models more consistent, it is necessary to modify the model by reducing the finite element one, or by extending the experimental modal one. And the reducing way will be much fast, so here we use the reducing way. So the main goal for the model update is to make the tolerance from the errors between the frequencies obtained experimentally and theoretically equal to zero. But, it is a difficult process because of the uncertainties from the structural parameters such as the elasticity modulus, mass density, boundary conditions, etc.

For this aim, this study denotes updating a finite element model by following a process of following substeps: (i) montecarlo-based sensitivity analysis; (ii) bayesian based model updating.

3.1 Monte-Carlo based Sensitivity Analysis

Sensitivity analysis includes local sensitivity analysis methods and global sensitivity analysis. The first one includes differential method, finite difference method and perturbation method, which has clear concept to facilitate the calculation. It has long been widely used in engineering, but only being applied in linear or non-strong nonlinear systems (Kang, 1990). But Global sensitivity analysis (Yu, 2004), such as Monte Carlo method, also known as stochastic simulation method, is a theory based on statistical sampling, we random sample from probability distribution of an input known model to construct random variables, then we get digital characteristics resulting from its response (Zhou, 1997; Xiao, 2003; Zhang, 2008), and which can be used for more complex models, the analysis principle is outlined as below:

Assuming that in the spatial domain Ω (Wang, 2003; Yan, 2003; Rulka, 2005), the system response function Δ can be expressed as the integral of the function f, and there exists an non-zero probability density function ρ, as following:

$$\Delta = \int_a f(t,x;\lambda)dx = \int_a \frac{f(t,x;\lambda)}{\rho(t,x;\lambda)}\rho(t,x;\lambda)dx = \mathrm{E}\left(\frac{f(t,x;\lambda)}{\rho(t,x;\lambda)}\right) = \mathrm{E}(\Phi) \tag{17}$$

Wherein: $\Phi = \frac{f(t,x;\lambda)}{\rho(t,x;\lambda)}$, t is the time, $X = (x_1, x_2, \cdots, x_n)$ the random input variable vector decided by a probability density function ρ , n the number of input variables, λ a system parameter.

If $\phi_i = \frac{f(t,x;\lambda)}{\rho(t,x;\lambda)}$ let, then Δ can be approximately estimated by the mean ϕ_i generated from N random sample, that is:

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$$\Delta \approx \frac{1}{N}\sum_{i=1}^{N}\phi_i \tag{18}$$

The sensitivity of the system response function Δ for the parameter λ can be expressed as:

$$\partial_\lambda \Delta \approx \frac{1}{N}\sum_{i=1}^{N}\partial_\lambda \phi_i \tag{19}$$

3.2 Bayesian based Model Updating

Model modification is actually a mathematical inverse problem, there are several methods, and Lagrange multiplier method of direct correction matrix has the following deficiencies: ①using experimental modal vectors to correcting mass and stiffness matrix. But Experimental modal vectors and the number of DOF are much less than the calculated model, so they must be extended; ② the error is large normally; ③the sparsity from original mass matrix and stiffness matrix may doesn't exist any longer; ④elements as zero may no longer be zero from the original mass matrix and the stiffness matrix, which may not be accordance with the actual situation; ⑤ false modal (Spurious Modes) may occur.

Therefore, the physical parameter modification method based on sensitivity analysis is commonly used in engineering, and there are two methods generally, such as: direct derivation and ad-joint structure method. Direct derivation, was first proposed by Fox and Kappor. Adjoint structure method first proposed by Van Bell and later improved by Van Bonacker, it is coming from electronics (adjoint network theory), which using the similarity between Lurgan theorem (Tellegen's Theory) from electronics and the virtual work principle from structural mechanics. the structure sensitivity formula can be obtained after analysis the original structure and the accompanying one, through choosing dynamic characteristics of the structural elements from the original structure same as the one which has same topology (structure) and geometry. But the calculation with ad-joint structure method is more complex. So this paper denotes a method combination with direct derivation method.

There are three related requirements between the finite element model and experimental modal model: ①modal frequencies must consistent; ②mode shapes must consistent; ③frequency response must consistent. These three factors can be weighted using Bayesian method when constructing the error function based on the sensitivity analysis.

Bayesian approach lies in that using all known information such as: the prior distribution of the state and contact status, and also using the likelihood function observed and of to construct the posterior probability density for state variables of the system. The main solution steps are as follows:

1) The first step, combined with a first-order Markov process:

$$\mathrm{P}(x_i | x_{i-1}, \mathrm{K}_{1:i-1}) = \mathrm{P}(x_i | x_{i-1}) \tag{20}$$

Priori probability density of the state space of the system model:

$$\mathrm{P}(x_i | \mathrm{K}_{1:i-1}) = \int p(x_i | x_{i-1}) p(x_{i-1} | \mathrm{K}_{1:i-1}) dx_{i-1} \tag{21}$$

2) The second step, using the nearest observation for modifying the formula to obtain the posterior probability density:

$$\mathrm{P}(x_i | \mathrm{K}_{1:i}) = \frac{p(\mathrm{K}_i | x_i) p(x_i | \mathrm{K}_{1:i-1})}{p(x_i | \mathrm{K}_{1:i-1})} \tag{22}$$

The formula above is the optimal Bayesian estimation, wherein, x_i is the state for the system at the moment i, $\mathrm{K}_{1:i}$ the observation sequence from the initial moment to moment i, $\mathrm{P}(x_i | \mathrm{K}_{1:i})$ the likelihood of the posterior probability density function.

Therefore, the linear model updating mathematical expressions:

$$\mathrm{Z}\{\Delta x\} = \{\Delta y\} \tag{23}$$

Wherein, $[\mathrm{Z}]$ sensitivity matrix of $m \times n$ dimensional; $\{\Delta x\}$ the difference between model updating parameters and the initial value; $\{\Delta y\}$ the difference between the eigenvalue tests from experiment model and the calculated values of the finite element model, which containing the test errors $\{\varepsilon\}$ and calculation errors, here ignored the calculation errors ,and assumes $\{\Delta x\},\{\varepsilon\}$ obeys the normal distribution when its mean is υ ,and they are independent on each other, here we have the following formula [24]:

$$\begin{aligned} &\mathrm{E}\left[\{\varepsilon\}^{\mathrm{T}}\{\varepsilon\}\right] = \sigma[U_y] \\ &\mathrm{E}\left[\{\Delta x\}^{\mathrm{T}}\{\Delta x\}\right] = \sigma[U_x] \\ &\mathrm{E}\{\varepsilon\} = \mathrm{E}\{\Delta x\} = 0 \\ &\mathrm{E}\left[\{\Delta x\}^{\mathrm{T}}\{\varepsilon\}\right] = \{0\} \end{aligned} \tag{24}$$

Wherein, $\sigma[U_x]$ the covariance of $\{\Delta x\}$, $\sigma[U_y]$ the covariance of $\{\varepsilon\}$, and the joint probability density:

$$\mathrm{P}(\{\Delta x\},\{\varepsilon\}) = \frac{1}{(2\pi)^{\frac{n+m}{2}}\sqrt{[U_x][U_y]}} \exp\left[-\frac{1}{2}(\{\Delta x\}^{\mathrm{T}}[U_x]^{-1}\{\Delta x\} + \{\varepsilon\}^{\mathrm{T}}[U_y]^{-1}\{\varepsilon\})\right] \tag{25}$$

To have maximal value, first we need to solve the following:

$$\min(\{\Delta x\}^{\mathrm{T}}[U_x]^{-1}\{\Delta x\} + \{\varepsilon\}^{\mathrm{T}}[U_y]^{-1}\{\varepsilon\}) \tag{26}$$

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And substitute formula (24) to formula (26) into it, and differentia it, then we get:

$$\{\hat{x}\} = \{x_0\} + \Lambda\{\varepsilon\} \quad (27)$$

Wherein,

$$\Lambda = (\{Z\}^T [U_y]^{-1}\{Z\} + [U_x]^{-1})^{-1}[Z]^T [U_y]^{-1} \quad (28)$$

Cause the correction model is the same important, we define three indicators to control its quality: the average relative error of the modal frequency $\overline{\Delta f}$, the maximum relative error $\Delta f_{\max}$, the correlation coefficient of average modes $\overline{\Re}$, which are indicated as follows:

$$\overline{\Delta f} = \frac{1}{n}\sum_{k=1}^{n}|\Delta f_k|,$$

$$\Delta f_{\max} = Max(|\Delta f_k|), k = 1,2,\cdots,n, \quad (29)$$

The proposed algorithm is tested on the model of 6-DOF manipulator. The results for both initial model and revised model are shown in Fig 2, Table1 and 2.(here took six modes):

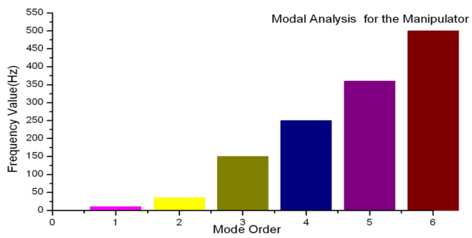

Figure 2: The Frequency Response

Table 1: The finite element model updating Controll index

	order	$\overline{\Delta f}$ (%)	$\Delta f_{\max}$ (%)	$\overline{\Re}$
Before revising	1~6	9.904	26.02	92.06
After revising	1~6	0.817	3.29	89.31

Table 2: The comparison of the natural frequencies of the frame model before and after the update.

No.	initial calculate modal values=a	experimental modal values=b	Difference=a-b (confidence)	Correction modal value=c	Difference=c-b (confidence)
1	2.35	5.68	-3.33(36.2%)	3.39	-2.29(92.3%)
2	37.94	42.57	-4.63(89.6%)	43.94	-1.37(97.1%)
3	173.19	181.23	-7.04(93.6%)	178.35	.2.87(98.6%)
4	273.01	273.01	0(46.6%)	273.02	0.01(82.01%)
5	345.68	340.19	4.49(93.6%)	342.68	1.49(98.9%)
6	437.19	398.22	38.97(93.6%)	400.34	2.12(97.6%)

4 CONCLUSION

This paper addresses a method based on sensitivity which is developed for modal analysis. For this purpose, one example of a 6-DOF manipulator is selected to demonstrate the efficiency of the proposed method.

The model is investigated under five subtitles: analytical modal analysis, experimental measurement, comparison of the experimental and initial analytical natural frequencies, application of the developed modelupdating method using the platform of Grid computing and comparison of the results.

It is observed that there are differences in the natural frequencies obtained from experimental measurement and initial analytical modal analysis of the model because of the uncertain structural parameters. So, the model is updated using the proposed model updating method.

According to the results of the study, the values of each selected parameter are attained to reflect the real condition of the models in terms of the dynamic behavior. The average error in the natural frequencies is decreased from 9.904% to 0.817% for the 6-DOF manipulator by using the developed model updating method. In consequence, the proposed algorithm gives better solutions for model updating compared to the initial values.

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