A Novel and Fast Adaptive Compressive Sampling Matching Pursuit Algorithm

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Abstract: A weakness of compressive sampling is that it needs the information of sparsity to approximate the compressible signal. In this paper, an fast iterative reconstruction algorithm called Adaptive Compressive Sampling Matching Pursuit is presented to solve the problem mentioned above, which delivers the same guarantees as the best optimization-based approaches and get rid of the dependence on the information of sparsity. Experimental results also demonstrate that the image reconstructed performance of the proposed algorithm is improved in terms of PSNR, SNR and reconstructed time, compared to Orthogonal Matching Pursuit (OMP) and Compressive Sampling Matching Pursuit (CoSaMP).

1 INTRODUCTION

Compressive sensing (CS) (Donoho, 2006) (Candes and Wakin, 2008) (Baraniuk, 2007) is a relatively novel theory in signal sampling, which is based on sparse or compressible signal. Reconstruction algorithm (Candes et al, 2006) is one of the most active and challenging part of compressive sensing, which is of great significance to accurately reconstruct the signal and verify the sampling accuracy.

However, current reconstruction algorithms based on compressive sensing also have drawbacks. Matching Pursuit (MP) algorithm (Mallet and Zhang, 1993) needs to go through multiple iterations to obtain convergence, since the results of each iteration may be sub-optimal due to non-orthogonal projection of the signal on the selected atom sets (measurement matrix column vector). To overcome the drawback of MP, TroPP J et al. proposed Orthogonal Matching Pursuit (OMP) (TroPP and Gilbert, 2007). But OMP's theoretical guarantee which ensures accurate reconstruction is weaker than the minimum l_1-norm approach, so not all signals can be reconstructed accurately. On the basis of OMP, Needell et al. (Needell and Vershynin, 2009) and Donoho et al.(Donoho et al, 2009) proposed Regularized Orthogonal Matching Pursuit (ROMP) and Stagewise Orthogonal Matching Pursuit (StOMP), respectively. Their computing speeds are faster and their reconstruction complexities are lower, compared to OMP; yet their properties are poor. As a result, M must be large enough to obtain better reconstructed performance of the signal. Needen et al. proposed Compressive Sampling Matching Pursuit (CoSaMP) (Needell and TroPPJ, 2009). The algorithm offers rigorous bounds on computational cost and storage. It is likely to be extremely efficient algorithm for practical problems because it requires only matrix vector-multiplies with the sampling matrix. These algorithms are built on the basis of known sparsity K , yet the sparsity K is often unknown in a practical application.

In this paper, we propose a improved reconstruction algorithm named Adaptive-CoSaMP based on CoSaMP, according to the prior information that reconstruction algorithm needs sparsity of sampling signal (Davenport,M.A. and Wakin,M.B., 2010). Property measures, such as PSNR and reconstruct time, are used to evaluate the performance of the proposed algorithm. Simulation results of Adaptive-CoSaMP are compared with that of CoSaMP and OMP.

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2 COMPRESSIVE SAMPLING

To enhance intuition, we focus on sparse and compressible signals. For vectors $\boldsymbol{x}$ in C^N, the ℓ_0 "quasi-norm" (Candes and Romberg, 2007) is defined by

$$\|x\|_0 = |\sup p(x)| = |\{j : x_j \neq 0\}| \tag{2.1}$$

A signal $\boldsymbol{x}$ is called *s-sparse* if $\|x\|_0 \leq s$. Compressible signals are well approximated by sparse signals. In compressive sampling theory, a *sample* is a linear functional applied to a signal. The process of collecting multiple samples is best viewed as the action of a *sampling matrix* Φ on the target signal. If we take m samples or *measurements* of a signal in C^N, then dimension of the sampling matrix Φ is $m \times N$.

The minimum number of measurements satisfies $m \geq 2s$ on account of the following simple argument. The sampling matrix must not map two different *s*-sparse signals to the same set of samples. Therefore, each collection of $2s$ columns from the sampling matrix must be nonsingular. As a result, some sparse signals are mapped to very similar sets of samples, and it is unstable to invert the sampling process numerically. Instead, Candes and Tao proposed the stronger condition that the geometry of sparse signals should be preserved under the action of the measurement matrix (Needell and TroPPJ, 2009). To quantify this idea, they defined the r th restricted isometry constant of a matrix Φ as the least number δ_r for which

$$(1-\delta_r)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1+\delta_r)\|x\|_2^2 \quad \text{whenever } \|x\|_0 \leq r \tag{2.2}$$

We have written $\|\bullet\|_2$ for the ℓ_2 vector norm. When $\delta_r < 1$, these inequalities imply that each collection of r columns from Φ is nonsingular, which is the minimum requirement for acquiring *(r/2)*-sparse signals. When $\delta_r \ll 1$, the sampling operator very nearly maintains the ℓ_2 distance between each pair of *(r/2)*-sparse signals. In consequence, it is possible to invert the sampling process stably.

3 AN ADAPTIVE COMPRESSIVE SAMPLING MATCHING PURSUIT ALGORITHM

Compressive Sampling Matching Pursuit (CoSaMP) is proposed by Candes and Donoho (Needell and Vershynin, 2009). The CoSaMP algorithm selects the reserved atom based on the known sparsity of the approximation to be produced and removes a fixed number of atoms combining backward thought, so computing speed of each iteration of the CoSaMP algorithm is slow to some extent. In this section, an Adaptive-Compressive Sampling Matching Pursuit is proposed based on CoSaMP. The algorithm gets rid of the dependence on sparsity, reconstructs the original signal through adaptively adjusting the step size in iteration, and has less reconstruct time.

The algorithm is initialized with a trivial signal approximation, which means that the initial residual equals the unknown target signal. During each iteration, Adaptive-CoSaMP performs five major steps:

(1) Identification. The algorithm forms a proxy of the residual from the current samples and locates the $3t$ largest components of the proxy.

(2) Support Merger. The set of newly identified components is united with the set of $3t$ largest components that appear in the current approximation.

(3) Estimation. The algorithm solves a least-squares problem to approximate the target signal on the merged set of components.

(4) Pruning. The algorithm produces a new approximation by retaining only the t largest entries in this least-squares signal approximation.

(5) Sample Update. Finally, the samples are updated so that they reflect the residual, the part of the signal that has not been approximated.

The main source code of the proposed algorithm is summarized in Table 1.

4 EXPERIMENTAL RESULTS AND DISCUSSION

In this section, the peak signal-to-noise ratio (PSNR) and SNR (signal-to-noise ratio) are used to evaluate the visual quality of the reconstructed image $\hat{F}$. PSNR is defined as

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$$PSNR = 20\log_{10}(\frac{255}{\sqrt{MSE}})dB \, , \tag{4.1}$$

and SNR is defined as

$$SNR = 20\log_{10}(\frac{\hat{F}(i,j)}{MSE})dB \, , \tag{4.2}$$

where MSE is the mean square error between the original image F and the reconstructed image $\hat{F}$. It is given by

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[F(i,j) - \hat{F}(i,j)]^2 \, . \tag{4.3}$$

To further evaluate the effectiveness of the proposed algorithm, the proposed algorithm is compared with existing algorithm proposed in reference (TroPP and Gilbert, 2007) and in reference (Needell et al, 2009). Figure 2, Figure 3, Figure 3, Figure 4 and Table 2 show the comparative results of two test images：

Lena (256×256) and Barbara(256×256).

Table 1 The main source code of the Adaptive-CoSaMP algorithm

Adaptive-CoSaMP algorithm
Input: Measurement matrix Φ , Sampling vector y , halting criterion ε , iteration k Output: An sparse approximation $\hat{x}$ of the target signal
Residual $r^0 = y$ Initial step $t = 1$ Index set of values A= $\varnothing$, $S = \varnothing$ stage=0 $k = 0$ repeat $k \leftarrow k+1$ $g^{(k)} \leftarrow \Phi^* r^{(k-1)}$ {Form signal proxy} $S \leftarrow \sup p(g_{3t}^{(k)})$ {Identify large components} $A \leftarrow A \cup S$ {Merge supports} $b\|_A \leftarrow ((\Phi^*\Phi)^{-1}\Phi^*)_{\|A}\, y$ {Signal estimation by least-squares} $b\|_{A^C} \leftarrow 0$ $\hat{x}_k \leftarrow b$ {Prune to obtain next approximation} $r^k = r^{(k-1)} - \Phi\hat{x}_k$ {Update current samples} $stage = stage + 1$ $t = stage * t$ until halting criterion true

Table 2 SNR, PSNR and reconstructed time comparison between our proposed method and method in (TroPP and Gilbert, 2007)and(Needell et al, 2009) for host images in the case of compression ratio respectively 0.4 and 0.5

		Compression ratio (0.5)			Compression ratio (0.4)		
		Reference (TroPP and Gilbert , 2007)	Reference (Needell et al, 2009)	Proposed algorithm	Referenc e (TroPP and Gilbert , 2007)	Referenc e(Needell et al, 2009)	Proposed algorithm
Lena	PSNR	31.59	30.85	33.84	25.18	26.45	29.09
	SNR	16.00	17.39	18.99	8.31	9.22	9.5
	Time	69.35	70.72	55.19	18.64	8.67	7.08
Barbara	PSNR	29.79	31.06	31.66	21.24	20.48	21.22
	SNR	14.46	16.56	18.11	6.54	6.54	8.79
	Time	79.24	75.12	57.64	21.94	9.15	7.64

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(a) (b)

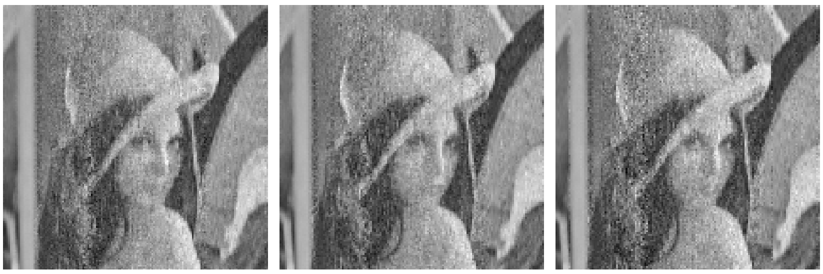
Figure 1.Original images:(a) Lena image; (b) Barbara image

(a) OMP (b) CoSaMP (c)Adaptive-CoSaMP

Figure 2.Reconstructed images of Lena image in the case of compression ratio 0.5

(a) OMP (b) CoSaMP (c)Adaptive-CoSaMP

Figure 3.Reconstructed images of Barbara image in the case of compression ratio 0.5



(a) OMP (b) CoSaMP (c)Adaptive-CoSaMP

Figure 4.Reconstructed images of Lena image in the case of compression ratio 0.4

From the Table 2, we can observe that our algorithm has higher SNR values and PSNR values for all the compression ratio of reconstruction, compared to algorithms in (TroPP and Gilbert , 2007) and. (Davenport,M.A. and Wakin,M.B., 2010) And the reconstructive time of the scheme is less than that of reference (TroPP and Gilbert , 2007) and reference(Needell et al, 2009). From Figure 2, Figure 3, Figure 4 and Figure 5, it is not hard to observe that

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(a) OMP (b) CoSaMP (c)Adaptive-CoSaMP

Figure 5.Reconstructed images of Lena image in the case of compression ratio 0.4

proposed algorithm has better visual qualities of the reconstructed images, compared to algorithms in reference (TroPP and Gilbert, 2007) and reference(Needell et al, 2009). Hence we can conclude that the proposed algorithm-Aaptive-CoSaOMP algorithm is superior to other algorithms for reconstructing signal .

5 CONCLUSION

In this paper, we discuss compressive sampling theory which is one of the most active and challenging subject in signal processing in recent years. Taking advantage of the greedy iterative algorithm often-used in compressive sensing, an improved matching pursuit algorithm—Adaptive-CoSaMP algorithm, which is based on compressive sampling matching pursuit algorithm, is proposed. The proposed algorithm not only allows a accurate reconstruction of signal in the case of unknown sparsity K, but also can gradually update to approximate the original signal by setting the step value. Simulation results also shows that the improved algorithm has significantly improvement in reconstruction effect of the image whether from the visual effects of the reconstructed image or from the PSNR value of the reconstructed image.

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