A Convex Approach for Non-rigid Structure from Motion Via Sparse Representation

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Abstract: This paper presents a convex solution for simultaneously recovering 3D non-rigid structures and camera motions from 2D image sequences based on sparse representation. Most existing methods rely on low rank assumption. However, it will lead to poor reconstruction for objects with strong local deformation. Also, when camera motion is unknown, there is no convex solution for non-rigid structure from motion (NRSfM). In order to solve this problem, we estimate non-rigid structures by sparse representation. In this paper, we estimate camera motions through a sparse spectral-norm minimization approach, and then a fast l1-norm minimization algorithm is introduced to reconstruct 3D structures. Both of them are convex, therefore, our method gives a global optimum. Our method can handle objects with strong local deformation and also doesn't need low rank prior. Experimental results show that our method achieves state-of-the-art reconstruction performance on CMU benchmark dataset.

1 INTRODUCTION

Structure from Motion (SfM) is a well-known technology to simultaneously recover 3D structures and camera motions of a rigid object from 2D corresponding points. Although there are still some open problems such as real-time reconstruction, point matching, large scale and dense reconstructions, the theory has been well established over the past two decades (Carlo and Kanade, 1992). Non-rigid Structure from Motion (NRSfM) is an extension of SfM for non-rigid objects. It's also a fundamental problem in computer vision. During the past decade, it has attracted lots of researches and many different algorithms have been proposed. However, there are still some problems to be unsolved. The difficulty is mainly caused by the inherently high number of degrees of freedom. For rigid objects, the rigidity prior is enough to make the problem well posed because well-known multi-view relations are valid. However, this prior is not valid for non-rigid deformable objects. For time-varying observed 2D points, to obtain the corresponding 3D points becomes ill posed.

Most existing methods have been attempted to solve NRSfM by using additional constraints. For instance, some approaches assume that the 3D non-rigid structures can be modeled as a linear combination of several predefined bases of shapes (Bregler et al., 2000; Torresani et al., 2008; Xiao et al., 2004). Also, other approaches attempt to represent a 3D point trajectory by using a fixed set of discrete cosine transform (DCT) trajectory bases (Akhter et al., 2008; Gotardo and Martinez, 2011; Park et al., 2010). It also has been shown that it is a dual representation to shape representation (Akhter et al., 2011). Gatardo et al. combined these two concepts and proposed an efficient method that recover the trajectory using DCT bases in a linear shape space (Gotardo and Martinez, 2011). Besides, Dai et al. proposed a well-known rank minimization approach which minimizes the rank of 3D structures based on nuclear minimization algorithm and achieves one of the most remarkable performance (Dai et al., 2014). The nuclear minimization based approaches then were also used to reconstruct 3D structures from realistic videos (Fragkiadaki et al., 2014; Garg et al., 2013). These linear representation based methods can achieve better reconstruction performance for some objects with small deformation, but it is unable to handle strong deformations such as complex human motions. Another problem of these methods is that the number of bases must be predefined accurately, because the improper number of bases will largely degrade the algorithm's performance. Unfortunately, the simple way to find

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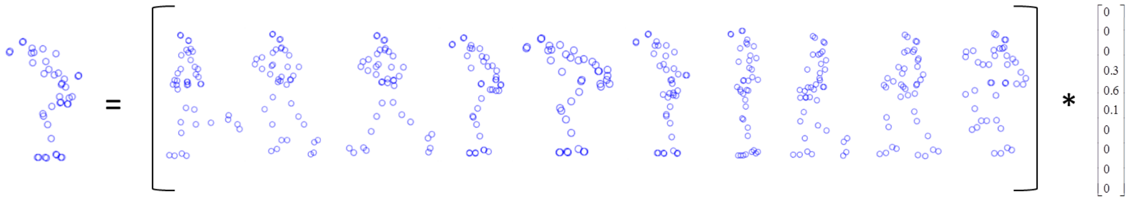


Figure 1: Illustration of the basic idea studied in this paper. The non-rigid 3D shape is represented as a linear combination of some predefined shape bases via sparse representation. We try to find out a convex solution to estimate the sparse coefficients of the shape bases.

the optimal number of bases has not been discovered, so we have to repeat numerous experiments to estimate it.

The above limitations of existing methods prompted us to think in another way. We thought that the objects with complex deformation need to be modeled by using more better bases of shapes or trajectory. In order to avoid the problem of the estimation of basis number, in this paper, we formulate NRSfM as a sparse l1-norm minimization problem. The 3D non-rigid structure is represented as a linear combination of shape bases in the dictionary. One benefit of using shape basis is it allows to recover 3D object in a sequential way which has recently been paid many attentions (Agudo et al., 2014; Agudo and Moreno-Noguer, 2015; Paladini et al., 2010). Due to the fact that camera motion is unknown, the traditional sparse l1-norm minimization approach is nonconvex. Recently, Zhou et al. proposed a convex approach to estimate camera motion based on sparse spectral-norm minimization (Zhou et al., 2015) which encourages us to solve sparse l1-norm minimization problem in a convex way. As the fact that we learn the better shape bases through dictionary learning technique, our method can handle the objects with complex deformation. Comparing to Zhu's trajectory learning method (Zhu and Lucey, 2015), our method gives a convex solution to camera motion and also allows to recover 3D structures in a sequential way. Experiments demonstrate that our method could achieve much more accurate reconstruction performance than several existing well-known algorithms.

2 PREVIOUS WORKS

2.1 Batch Approaches

Batch approaches need to leverage the information of all frames. After tracking 2D points over all frames, these methods recover camera motions and 3D shapes from 2D measurements. It can be represented as:

$$\begin{aligned} &\mathbf{W} = \mathbf{RS} \\ &\text{s.t. } \mathbf{R}_f\mathbf{R}_f^T = \mathbf{I} \end{aligned} \tag{1}$$

where $\mathbf{R} \in \mathbb{R}^{2F\times 3F}$ is the orthographic camera motion, and $\mathbf{R}_f$ denotes the camera motion of f-th frame. $\mathbf{S} \in \mathbb{R}^{3F\times P}$ is the 3D non-rigid shapes matrix. $\mathbf{W} \in \mathbb{R}^{2F\times P}$, is the projections of $\mathbf{S}$ in a set of 2D images. Due to the inherently high number of degrees of freedom, additional constraint is required to recover $\mathbf{S}$, such as modeling the 3D shapes as a linear combination of several predefined bases of shapes:

$$\mathbf{W} = \mathbf{RCB} \tag{2}$$

where $\mathbf{B}$ and $\mathbf{C}$ denote the predefined shape bases and the weight of these bases, respectively. Instead of using predefined shape bases or trajectory bases, Dai et al. (Dai et al., 2014) formulate the following rank minimization problem based on the assumption of representing 3D shapes in a low rank space which is convex and can be solved efficiently by minimizing the nuclear-norm which is its convex approximation:

$$\begin{aligned} &\min \quad \mathbf{rank(S)} \\ &\text{s.t.} \quad\ \mathbf{W} = \mathbf{RS} \end{aligned} \tag{3}$$

By such additional constraint, the non-rigid shape can be recovered exactly. Apparently, it is necessary to recover camera motions firstly. It has been proved that the camera motion can be recovered uniquely and accurately in a batch way only by using orthonormality constraints(Akhter et al., 2008; Dai et al., 2014). But the camera motion can not be recovered for complex deformable objects because the small deformation condition is essential for the recovery of camera motion according to (Yezzi and Soatto, 2003; Zhang and Hung, 2015).

2.2 Sequential Approaches

Sequential approaches recover 3D shape and camera motion per frame. It can be represented as:

$$\begin{aligned} &\sum_{f=1}^{F} \mathbf{W}_f = \mathbf{R}_f\mathbf{S}_f \\ &\text{s.t. } \mathbf{R}_f\mathbf{R}_f^T = \mathbf{I} \end{aligned} \tag{4}$$

where F denotes the total number of frames. $\mathbf{S}_f$, $\mathbf{W}_f$ are 3D shape and its 2D projection points for f-th frame respectively. In order to solve the above ill-posed problem, usually, camera motion and shape deformation are considered to be smooth. Besides, many additional conditions are also put to constrain the deformation (Agudo et al., 2014; Agudo and Moreno-Noguer, 2015; Paladini et al., 2010). The 3D shape and camera motion can be solved by minimizing the following energy function:

$$\min_{\mathbf{R}_f,\mathbf{S}_f} \quad \sum_{f=1}^{F} ||\mathbf{W}_f - \mathbf{R}_f\mathbf{S}_f||_F^2 + \lambda||\mathbf{R}_f - \mathbf{R}_{f-1}||_F^2 + \mu||\mathbf{S}_f - \mathbf{S}_{f-1}||_F^2 \qquad (5)$$

where $||.||_F$ denotes the Frobenius norm of matrix. This optimization function is nonconvex and usually it is solved by an alternating scheme (algorithm 1). To date, in spite of all the efforts, the reconstruction performance of sequential approaches is still not good comparing with batch methods.

Algorithm 1: Sequential non-rigid structure from motion.

```
Input:
    2D observations per frame $W_f$
Output:
    camera motion $R_f$ and 3D non-rigid shape $S_f$ for each frame
Initialize $R_f$ and $S_f$;
while not converged do
    update $S_f$;
    update $R_f$;
end while
```

3 PROPOSED METHOD

Although small deformations can be modeled accurately by a set of predefined shape bases or trajectory bases, more sufficient bases should be prepared for handling complex or strong deformations. It has been shown that complex deformation can be modeled in a nonlinear shape manifold (Tao and Matuszewski, 2013). Such nonlinear shape manifold can be approximated well by sparse representation. Thus, in this paper, we model the 3D non-rigid shape using a overcomplete dictionary. Such dictionary represents lots of shape bases that learned from training data. As the choice of bases is extremely important for NRSfM, good basis should be assigned large weight; on the other hand, bad basis should be abandoned. Thus, a sparse solution is promising. Our method can be formulated as the following optimization problem:

$$\min_{\mathbf{C}} \quad \frac{1}{2}||\mathbf{W}_f - \mathbf{R}_f \sum_{i=1}^{K} \mathbf{C}_i\mathbf{B}_i||_F^2 + \lambda||\mathbf{C}||_1 \qquad (6)$$

where K is the total number of shape basis. $\mathbf{B}_i$,$\mathbf{C}_i$ represent the i-th shape basis in dictionary and its weight, respectively. $\mathbf{C}$ is a vector contains the weight of each shape basis $\mathbf{C}_i$. $||.||_1$ denotes the l1-norm of vector which is a convex relaxation of l0-norm minimization. However, when $\mathbf{R}$ is unknown, the above minimization problem is nonconvex. A common strategy to solve Eq. (6) is to use the alternating scheme as described in algorithm 1. However, the algorithm is not convex, thus it may get stuck at local minimum. In this paper, we aim at solving the above optimization problem in a convex way; the steps of our algorithm are summarized in algorithm 2.

3.1 Camera Motion Estimation

The first task in our method is to estimate camera motion $\mathbf{R}$ accurately. Although Dai et al. introduced a convex approach to recover $\mathbf{R}$ by semi-definite programming (SDP). Their method recovers $\mathbf{R}$ in a batch way and still need to be predefined the number of rank, so it can't be employed to solve sequential NRSfM. Therefore, we have to find an efficient solution to recover camera motion sequentially. As proved in (Zhou et al., 2015), for each $\mathbf{M}_i = \mathbf{R}_f\mathbf{C}_i$, because of the orthonormality of $\mathbf{R}$:

$$\mathbf{M}_i\mathbf{M}_i^T = \mathbf{C}_i^2\mathbf{I} \qquad (7)$$

such that:

$$||\mathbf{M}_i||_2 \leq |\mathbf{C}_i| \qquad (8)$$

where $\mathbf{M}_i$ is a 2×3 matrix which contains camera motion $\mathbf{R}_f$ with weight of i-th shape basis. It's a convex relaxation to the constraint in Eq. (7), where $||.||_2$ denotes the spectral-norm of matrix. Instead of solving Eq. (6), [16] introduced to solve the following spectral-norm minimization problem:

$$\min_{\mathbf{M}_i} \quad \frac{1}{2}||\mathbf{W}_f - \sum_{i=1}^{K} \mathbf{M}_i\mathbf{B}_i||_F^2 + \lambda||\mathbf{M}||_2 \qquad (9)$$

Minimizing the above optimization function gives a convex solution to $\mathbf{M}_i$ and it can be solved efficiently based on the algorithm of (Zhou et al., 2015). Next, we solve the following bilinear factorization problem to get R by the factorization algorithm of (Del Bue et al., 2012).

$$\begin{aligned} &\min_{\mathbf{R},\mathbf{C}_i} \quad ||\mathbf{M} - \mathbf{R}_f\mathbf{C}||_F^2 \\ &\text{s.t. } \mathbf{R}_f\mathbf{R}_f^T = \mathbf{I} \end{aligned} \qquad (10)$$

3.2 3D Shape Estimation

After recovering camera motion $\mathbf{R}$, we will estimate 3D shape. The above spectral-norm minimization approach Eq. (9) and bilinear factorization algorithm

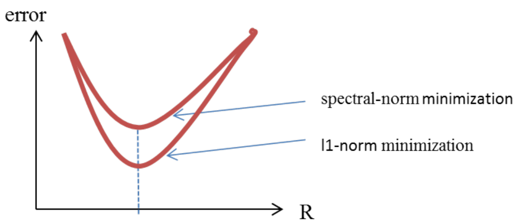


Figure 2: Visualization of global optimum for l1-norm minimization.

Eq. (10) have already given a solution to camera motion **R** as well as the weights of shape bases **C**, it is expected that if we can solve the original convex problem Eq. (6) we can achieve more accurate reconstruction performance. As seen in Figure 2, solving l1-norm convex relaxation problem (spectral minimization) can give a convex solution for camera motion **R**, and when **R** is known, we solve the original sparse l1-norm minimization problem Eq. (6) and find a global optimum for NRSfM by using a fast l1-norm minimization algorithm (Lee et al., 2006). The final deformable 3D shape is represented as:

$$\mathbf{S}_f = \sum_{i=1}^{K} \mathbf{C}_i \mathbf{B}_i \tag{11}$$

3.3 Shape Bases Learning

There are lots of dictionary learning algorithms. A common strategy is to solve the following minimization function:

$$\begin{aligned} \min_{B,L} \quad & \frac{1}{2}\sum_{f=1}^{F} ||\mathbf{S}_f - \sum_{i=1}^{K}\mathbf{L}_i\mathbf{B}_i||_F^2 + \beta||\mathbf{L}||_1 \\ \text{s.t.} \quad & ||\mathbf{B}_i||_F \leq 1 \end{aligned} \tag{12}$$

where $\mathbf{S}_f$, $\mathbf{B}_i$, $\mathbf{L}_i$ denote the f-th shape of training data, a shape basis in the dictionary and its weight respectively. The learned shape bases need to concisely represent the variability of training data. We use the algorithm used in (Zhou et al., 2015) to learn our dictionary.

Algorithm 2 : Convex sparse l1-norm minimization algorithm for NRSfM.

```
Input:
    2D observations per frame $W_f$ and shape bases dictionary B
Output:
    camera motion $R_f$ and 3D non-rigid shape $S_f$ for each frame
1: Calculate $M_f$ by Eq. (9);
2: Recover camera motion $R_f$ by Eq. (10);
3: Estimate the weight $C_i$ of each shape basis in dictionary by Eq. (11);
4: return $S_f = \sum_{i=1}^{K} C_i B_i$
```

4 EXPERIMENTAL RESULTS

In this section, we compare our method against the trajectory basis method (PTA) (Akhter et al., 2008), Dai's rank minimization method (BMM) (Dai et al., 2014), and Zhou's sparse spectral-norm minimization method (Zhou et al., 2015) on the CMU motion capture database (Carnegie mellon university,). This database provides 41 landmark positions corresponding to human motions. We selected five human motions with strong local deformations (Walking, Running, Jumping, Pickup, Marching) from this database. For each motion, we selected three sequences as training data and one sequence for testing from the same motion subject. For each testing data, we generated 2D projections of the 3D markers with the synthesized orthographic camera around the subject for 360 degrees with the angle speed 5 per frame.

The size of the dictionary is set as 300. Since PTA and BMM rely on low rank assumption, we set the number of their low rank parameter from 3 to 13 and reported the best result. To compare the performances, we measured the average 3D reconstruction error using the same error metric as PTA and BMM as follows:

$$e_{3D} = \frac{1}{\sigma F n}\sum_{f=1}^{F}\sum_{n=1}^{N} e_{fn}, \sigma = \frac{1}{3F}\sum_{f=1}^{F}(\sigma_{f}x + \sigma_{f}y + \sigma_{f}z) \tag{13}$$

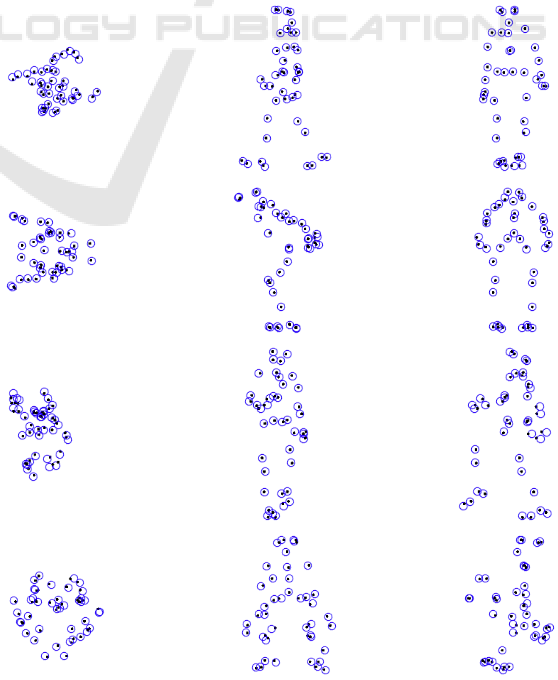


Figure 3: From first row to last row: 3D shape of Walking, Jumping, Marching and Pickup in 3 views recovered by the proposed method, respectively. Recovered shapes are blue circles and ground truth is dark dots.

Table 1: Average 3D reconstruction error of PTA, BMM, Zhou's method and proposed method. (K) denotes the rank number which gave the smallest 3D error.

Dataset	PTA(K)	BMM(K)	Zhou's method	Proposed method
Walking	0.1197(8)	0.1001(4)	0.0666	**0.0394**
Running	0.4212(3)	0.1638(5)	0.1093	**0.0706**
Jumping	0.2086(13)	0.1395(12)	0.1111	**0.0728**
Marching	0.1414(12)	0.1323(9)	0.1385	**0.0731**
Pickup	0.1211(13)	0.1412(12)	0.1163	**0.0706**

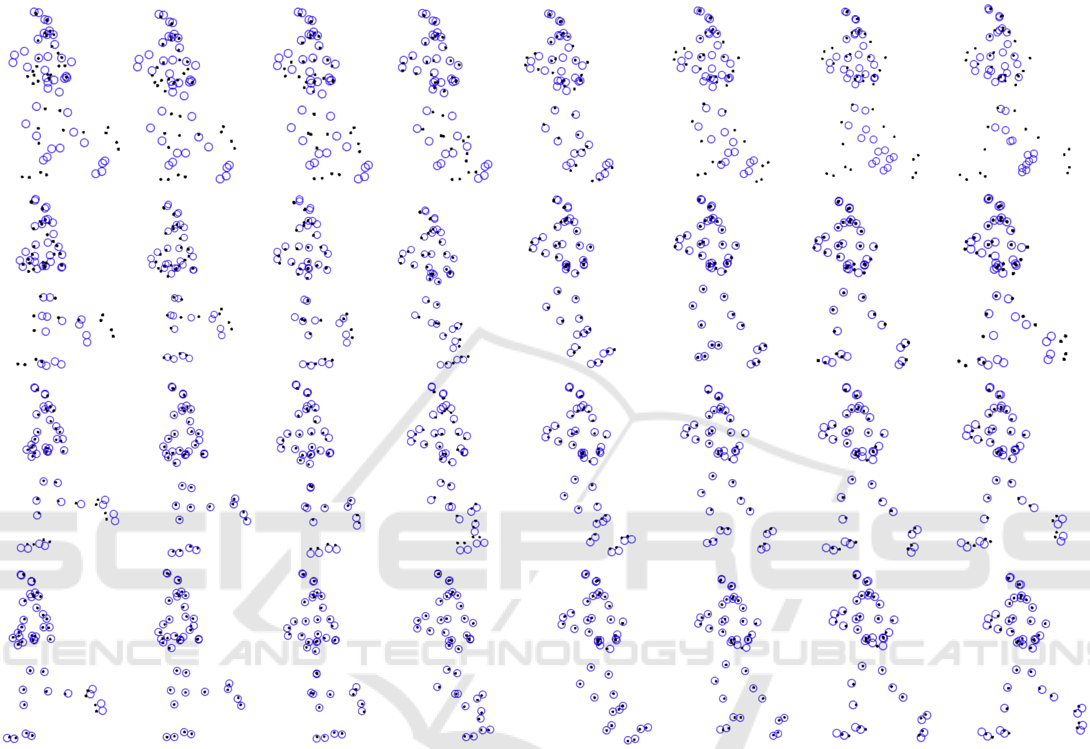


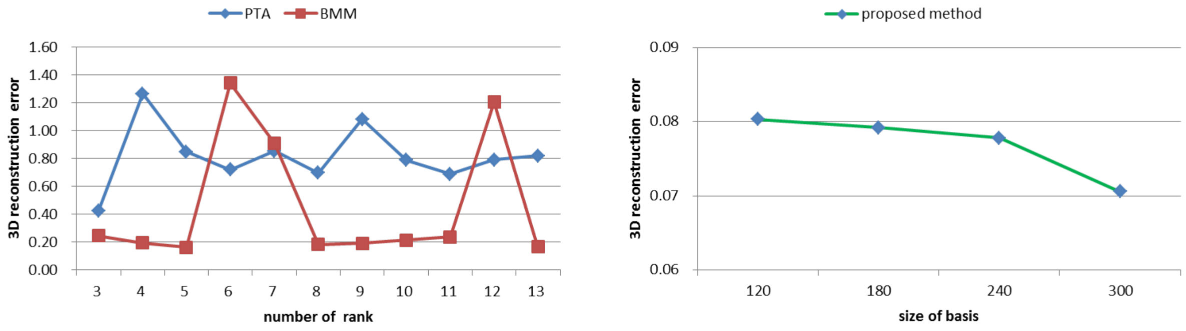
Figure 4: From first row to last row: One view of 3D shapes of Running recovered by PTA, BMM, zhou's method and the proposed method, respectively. Recovered shapes are blue circles and ground truth is dark dots.

where $\sigma_{f}x$, $\sigma_{f}y$ and $\sigma_{f}z$ are the standard deviations of the X, Y and Z coordinates of the original shape in frame f.

Table 1 shows the average 3D reconstruction error of different methods. It is seen that the proposed method achieved the best performance on each data. Figure 3 shows the 3 views of the reconstructed shapes of Walking, Jumping, Marching, Pickup by the proposed method. It is clear that our sparse l1-norm minimization approach achived very accurate reconstruction performance. Figure 4 shows the visual comparison between our method and other approaches on Running. Our method reconstructed the legs (which is strongly deformed part of Running) precisely than others. Also, Figure 5 (a) shows the impact of the low rank to PTA and BMM while varying K on Running, this result proved that the performance of low rank based methods largely rely on the proper number of K. Figure 5 (b) shows the results of the proposed method with different size of dictionary on Running. It can be observed that the 3D error decreases when the size of the dictionary increases. In conclusion, experimental results show that our method can handle non-rigid objects with strong local deformation much better than several current low rank based methods and Zhou's sparse spectral-norm minimization method. Since we don't use the time smoothness constraint, one more benefit of the proposed method is that it can be easily paralleled.

5 CONCLUSION

In this paper, we presented a convex solution for NRSfM based on sparse representation. It does not rely on low rank assumption and can reconstruct 3D



(a)

(b)

Figure 5: The comparison of 3D reconstruction error between low rank based methods PTA , BMM with different rank and our method with different dictionary size on Running.

shapes in a sequential way. As a result, we showed that our method achieved the best reconstruction performance on CMU database against several existing methods. Unlike the low rank based methods such as BMM and PTA, increasing the number of bases will reduce reconstruction error for our method. We believe that our method is a reasonable choice for solving NRSfM when training data is available.

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