

Optimizing Spare Battery Allocation in an Electric Vehicle Battery Swapping System

Michael Dreyfuss and Yahel Giat

Department of Industrial Engineering, Jerusalem College of Technology, HaVaad HaLeumi 21, Jerusalem, Israel

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Abstract: Electric vehicle battery swapping stations are suggested as an alternative to vehicle owners recharging their batteries themselves. To maximize the network's performance spare batteries must be optimally allocated in these stations. In this paper, we consider the battery allocation problem where the criterion for optimality is the window fill rate, i.e., the probability that a customer that enters the swapping station will exit it within a certain time window. This time is set as the customer's tolerable wait in the swapping station. In our derivation of the window fill rate formulae, we differ from previous research in that we assume that the swapping time itself is not negligible. We numerically analyse the battery allocation problem for a hypothetical countrywide application in Israel and demonstrate the importance of estimating correctly customers' tolerable wait, the value of reducing battery swapping time and the unique features of the optimal battery allocation.

1 INTRODUCTION

Electric vehicles' batteries need to be recharged frequently with inconveniently long recharging time. The US-based corporation Better Place suggested to overcome this problem by separating battery ownership from the vehicle's ownership so that customers purchase the vehicle from the auto-maker and lease battery services from a third party ("the firm"). The firm will construct a network of battery swapping stations in which car owners replace their depleted batteries for charged ones from the station's stock. Separately, the depleted batteries are recharged and put back in the stock to be given to future customers. To improve the network's performance, the firm may decide to place spare batteries in each station. Therefore, given a total budget of spare batteries, the firm must decide how to allocate the spare batteries among the battery swapping stations with the goal of optimizing a predetermined service measure.

The service measure that we consider in this paper is a generalization of the fill rate. With the fill rate, the firm will allocate batteries to maximize the fraction of customers who are served upon arrival. In reality, however, the fill rate is rarely an accurate proxy for the firm's costs. For example, if the battery provider is obliged by contractual commitment to provide service within a certain time then it does not need to have a

battery ready for the customer immediately when she arrives. From the customers' standpoint, too, there is a certain tolerable or acceptable period of wait, which may depend on their level of patience or expectation. If a customer entering the station expects being served within the ten minutes it would take to fill a tank of a conventional car, then the firm experiences reputation and contractual losses only if the customer waits more than ten minutes. Thus, the firm's objective should be to maximize the *window fill rate*, i.e., the probability that a customer is served *within* the tolerable wait.

To address this problem, we use research in the field of exchangeable-item repair systems. Customers arrive to these systems with a failed item and exchange it for an operable item in a manner similar to the battery swapping scheme. Furthermore, since battery charging docks are relatively inexpensive, one may assume that there are ample servers in each location so that each location can be modelled as an $M/G/\infty$ queue. (Dreyfuss and Giat, 2016) develop an algorithm for finding a near-optimal solution in such multi-location systems assuming that the item's assembly and disassembly times are negligible. This assumption is clearly unrealistic for the battery swapping problem since battery removal and installation times are significant compared to the customer's tolerable wait.

In this paper, we develop the window fill rate for-

mula for the case of positive item assembly and disassembly times and show that a Δ increase in the assembly and disassembly time is equivalent to a Δ decrease in the tolerable wait. Using this finding we apply the (Dreyfuss and Giat, 2016) algorithm to find a solution to the battery swapping problem, i.e., how to allocate spare batteries in the network, and make the following contributions.

We estimate the battery allocation problem of the Better Place corporation if it had succeeded going widespread in Israel and derive the optimal solution for different service criteria. This example provides valuable insight into the critical importance of assessing correctly the tolerable wait time and the significant losses that the firm incurs if it neglects to do so. Second, we estimate the savings attained by reducing battery swapping time. Third, we show how customer arrival rate creates two classes of battery exchange stations and therefore managers should develop two different policies with respect to their service time to customers. Although the Better Place adventure has ended with bankruptcy in 2013, the battery swapping idea is either applied or considered by other companies (e.g. XJ Group Corporation in Qingdao, Tesla in California and Gogoro in Taipei). The model presented in this paper, therefore, may yet be applied in real-life large-scale situations.

2 LITERATURE REVIEW

Electric vehicles are considered an environmentally-friendly alternative to internal combustion engine cars and are projected to eventually replace these fuel-burning cars (Dijk et al., 2013). Drivers, however, are still wary of these vehicles and therefore, many governments provide substantial tax incentives to encourage their widespread adoption. Notable examples are West European countries, the United States, China and Japan (Zhou et al., 2015). Despite these efforts, many drivers are wary of purchasing these cars and most governments adoption goals have not been met (Coffman et al., 2016). The major customer concern is the “range anxiety”, namely, the fact that batteries have limited range and their recharging time is very long compared to internal combustion engine cars. An innovative idea to overcome these issues was introduced by the US-based company Better Place who proposed to separate the vehicle ownership from the battery ownership. In lieu of owning the battery, car owners will purchase battery services from a firm that will establish a network of battery swapping stations. Researcher are examining many aspects of this proposition such as the station design, the battery removal

and installation times, the required number of spare batteries and the network layout and managing the loads on the power grid (Mak et al., 2013; Yang and Sun, 2015; Sarker et al., 2015). We contribute to this research by solving the spare battery allocation problem and demonstrating a large-scale application of this problem.

The assumption that customers will tolerate a certain wait is at the core of this paper, which lies at the intersection of inventory and customer service models. While the concept of a tolerable wait is hardly ever considered in inventory models, it is quite common in the service industry and is associated with numerous terms such as “expectation” (Durrande-Moreau, 1999), “reasonable duration” (Katz et al., 1991), “maximal tolerable wait” (Smidts and Pruyn, 1999) and “wait acceptability” (Demoulin and Djelassi, 2013). From a service-oriented approach, the customer’s attitude to wait is mainly subjective and has cognitive and affective aspects (Demoulin and Djelassi, 2013). From a logistics point of view this wait is more objective and usually stated in the service contract. Indeed, researchers have observed that most inventory models fail “to capture the time-based aspects of service agreements as they are actually written” (Caggiano et al., 2009, p.744). Our paper fills this void by incorporating the tolerable wait into the optimization criterion.

Our battery swapping network may be modelled as an exchangeable-item repair system (Avci et al., 2014). These inventory systems have been investigated by researchers in different contexts (Basten and van Houtum, 2014). A common performance measure for such systems is the fill rate, which measures the fraction of customers who are served upon arrival (Shtub and Simon, 1994; Caggiano et al., 2007). These papers, however, do not develop explicit formulas for the window fill rate but use numerical techniques. In contrast, (Berg and Posner, 1990) develop a formula for the window fill rate in a single location when item assembly and disassembly is zero, and (Dreyfuss and Giat, 2016) find that the window fill rate is generally S-shaped with number spares in the location and exploit this property to develop an efficient near-optimal algorithm for finding the optimal spare allocation. We extend these papers by considering the case of positive assembly and disassembly times.

3 THE MODEL

Customers arrive with a depleted battery to a battery swapping network that comprises L stations. Upon ar-

rival, the battery is removed and placed in a charging dock and once it is fully recharged it is added to the station's stock. To reduce customer waiting time, the network keeps a number of spare batteries, so that if there is a spare battery available on stock it is installed in the client's vehicle in exchange of the depleted battery that she had brought. Customers are served according to a first-come, first-serve policy and leave the swapping station once their battery is replaced.

For each station l , $l = 1, \dots, L$, we assume that customer arrival rate follows an independent Poisson process with parameter λ_l (see (Avci et al., 2014) for a justification of this assumption). We assume that there are ample charging docks in each station and that charging time at each dock is i.i.d. The combination of these two assumptions is that once the battery is removed from the vehicle, charging commences immediately and that recharging times are independent. Let $R_l(t)$ denote the cumulative probability of a battery to be recharged by time t and let r_l denote the mean recharging time. Battery removal and battery installation times are t_1 and t_2 , respectively. The battery swapping time, $t_1 + t_2$, is assumed to be no more than the tolerable wait.

3.1 Single Station

Consider a random customer, Jane, that arrives at time s to station l that was allocated b spares. The non-stationary window fill rate, $F_l^{NS}(s, t, b)$ is the probability that Jane will exit the station by time $s + t$. This happens if and only if by date $s + t - t_2$ Jane is at the head of the queue and there is at least one charged battery available at the station's stock. By "head of the queue" we mean that all the customers who arrived before Jane ("Pre-Jane customers") have either exited the station or are in the process of installing batteries in their vehicles. We can ensure this by verifying a supply and demand equation for recharged batteries. On the supply side, we consider the initial number of spare batteries in the station, b , plus all the batteries whose recharging was completed during the time segment $[0, s + t - t_2]$. On the demand side we consider the number of Pre-Jane customer plus Jane herself. Let:

- N_1 denote the number of batteries brought by Pre-Jane customers who were recharged before $s + t - t_2$.
- N_2 denote the number of batteries brought by Pre-Jane customers who were recharged after $s + t - t_2$.
- N_3 denote the number of batteries brought by customers who arrived after Jane ("Post-Jane customers") and were recharged before $s + t - t_2$.
- Z denote a Binary variable that is equal to one if

Jane's battery is recharged by $s + t - t_2$ and zero otherwise.

The probability that Jane will exit the station by $s + t$ is the probability that the supply is greater than the demand as follows

$$\begin{aligned} F_l^{NS}(s, t, b) &= Pr[Supply \geq Demand] \\ &= Pr[b + N_1 + Z + N_3 \geq N_1 + N_2 + 1] \\ &= Pr[b + Z + N_3 \geq N_2 + 1] \end{aligned} \quad (1)$$

Since the battery brought by Jane begins recharging at $s + t_1$, the probability for $Z = 1$ is the probability that a battery completes recharging during the interval $[s + t_1, s + t - t_2]$, which is equal to $R_l(t - t_1 - t_2)$. Therefore, we can condition on the value of Z and rewrite (1) as

$$\begin{aligned} F_l^{NS}(s, t, b) &= R_l(t - t_1 - t_2)Pr[b + 1 + N_3 \geq N_2 + 1] \\ &\quad + (1 - R_l(t - t_1 - t_2))Pr[b + N_3 \geq N_2 + 1] \\ &= Pr[N_2 - N_3 \leq b - 1] + R_l(\hat{t})Pr[N_2 - N_3 = b], \end{aligned} \quad (2)$$

where $\hat{t} := t - t_1 - t_2$.

Our assumption that batteries arrive according to a Poisson process and the ample server assumption guarantee that N_2 and N_3 are independent Poisson random variables that are also independent of Z . Recall, N_2 is the number of batteries who arrived between $[0, s]$ and were not repaired by $s + t - t_2$. Of these customers, consider a customer that arrives during the time interval $[u, u + du]$ in $[0, s]$. Due to the conditional uniform distribution property of the Poisson process, the probability for this is du/s (Ross, 1981, Chapter 3.5). This customer's battery is removed and begins to be recharged at $u + t_1$. The probability that recharging is completed only after $s + t - t_2$ is $1 - R_l(s + t - t_2 - (u + t_1))$. Thus,

$$\begin{aligned} N_2 &\sim \text{Poisson}\left(\lambda_l s \int_{u=0}^s (1 - R_l(s + t - t_2 - u - t_1)) \frac{du}{s}\right) \\ &\sim \text{Poisson}\left(\lambda_l \int_{u=\hat{t}}^{s+\hat{t}} (1 - R_l(u)) du\right). \end{aligned} \quad (3)$$

To derive the distribution of N_3 we consider the customers who arrived between $[s, s + t - t_2]$ and whose batteries were recharged by $s + t - t_2$. Of these customers, consider a customer that arrives during the time interval $[u, u + du]$. The probability for this is $du/(t - t_2)$. This customer's battery is removed and begins to be recharged at $u + t_1$. The probability that recharging is completed by $s + t - t_2$ is, therefore

$R_l(s+t-t_2-(u+t_1))$. Thus,

$$\begin{aligned} N_3 &\sim \text{Poisson}\left(\lambda_l(t-t_2)\int_{u=s}^{s+t-t_2} R_l(s+t-t_2-u-t_1)\frac{du}{t-t_2}\right) \\ &\sim \text{Poisson}\left(\lambda_l\int_{u=-t_1}^{\hat{t}} R_l(u)du = \lambda_l\int_{u=0}^{\hat{t}} R_l(u)du\right). \end{aligned} \quad (4)$$

The stationary window fill rate is obtained by taking the limit of s in (3) to infinity, which results with the following proposition:

Proposition 1. *The stationary window fill rate for station l with b spares is given by*

$$F_l(t, b) = \Pr[N \leq b-1] + R_l(\hat{t})\Pr[N = b] \quad (5)$$

where $N := N_2 - N_3$ and where N_3 is defined in (4) and $N_2 \sim \text{Poisson}\left(\lambda_l \int_{u=\hat{t}}^{\infty} (1-R_l(u))du\right)$.

3.2 The Network

Let $\vec{b} = (b_1, \dots, b_L)$ be a network battery allocation and let $\lambda := \sum \lambda_l$ denote the (total) arrival rate to the network. The network's window fill rate, $F(t, \vec{b})$, is the weighted average of the local window fill rates. Therefore, given a budget of B spare batteries, the battery allocation problem is:

$$\max_{\vec{b} \geq 0} F(t, \vec{b}) := \sum_{l=1}^L \frac{\lambda_l}{\lambda} F_l(t, b_l) \quad \text{s.t.} \quad \sum_{l=1}^L b_l = B. \quad (6)$$

Since the window fill rate depends only on $\hat{t} = t - t_1 - t_2$ we can instead assume that the battery removal and installment times are zero and use the adjusted tolerable wait, \hat{t} , in lieu of the true tolerable wait t . The implication of this observation is that we can use the results of (Dreyfuss and Giat, 2016) who assume zero swapping time. In the remainder of this section, we apply the results of (Dreyfuss and Giat, 2016) to our model with positive swapping time. We state only the results that are necessary for understanding the battery allocation application.

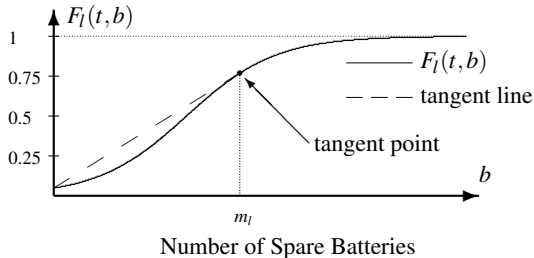


Figure 1: The window fill rate for an S-shaped station.

Result 1: The Shape of the Window Fill Rate

$F_l(t, b)$ is strictly increasing in b . If $t \geq r_l + t_1 + t_2$ then $F_l(t, b)$ is concave in b . Otherwise, $F_l(t, b)$ is either concave in b or initially convex and then concave (S-shaped) in b . The tangent point decreases with t and increases with $t_1 + t_2$.

Result 2: The Optimization Algorithm

By (6), the window fill rate is a separable sum of either concave or S-shaped functions. For each S-shaped station we define the tangent point, m_l , as the first integer such that $(F_l(t, m_l) - F_l(t, 0))/m_l > \Delta F_l(t, m_l)$, where $\Delta F_l(t, b)$ is the first difference of $F_l(t, b)$ and is given by

$$\begin{aligned} \Delta F_l(t, b) &:= F_l(t, b+1) - F_l(t, b) \\ &= (1-R_l(t))\Pr[N=b] + R_l(t)\Pr[N=b+1]. \end{aligned} \quad (7)$$

For concave stations we set the tangent point to zero. For each station, let $H_l(t, b)$ denote the concave covering function of $F_l(t, b)$ in the following manner:

$$H_l(t, b) = \begin{cases} F_l(t, 0) + \frac{F_l(t, m_l) - F_l(t, 0)}{m_l} b & \text{if } 0 \leq b \leq m_l - 1 \\ F_l(t, b) & \text{if } b \geq m_l. \end{cases}$$

That is, for any b smaller than the tangent point, we replace $F_l(t, b)$ with the straight line connecting the point $(0, F_l(t, 0))$ and the point $(m_l, F_l(t, m_l))$. By construction, for all $b \geq 0$, $H_l(t, b)$ is concave and $H_l(t, b) \geq F_l(t, b)$. Finally, we define $H(t, \vec{b})$ as the weighted sums of all the stations' functions $H_l(t, b_l)$ similarly to (6).

Since $H(t, \vec{b})$ is a separable sum of concave functions we can use a greedy algorithm to maximize it. This algorithm will choose the "best for the buck" station and since H_l is initially linear, it will stay with this station until it has reached the station's tangent point. It then continues with the next best station and so forth. Before switching to the next linear slope it is possible that stations that have reached their tangent will get additional spares (as long as their current slope is steeper than the next best linear slope). However, once a region begins receiving slopes in its linear region, it will be the only one to receive spares until it has reached its tangent point. Consequently, the algorithm produces an allocation with properties stated in the following result.

Result 3: The Optimal Allocation

\vec{b}^H satisfies one of the following two cases:

1. For every $l = 1 \dots L$, either $b_l^H \geq m_l$ or $b_l^H = 0$ and the optimal solution to (6), $\vec{b}^F = \vec{b}^H$.

2. There exists a single station, denoted by \hat{l} such that $0 < b_{\hat{l}}^H < m_{\hat{l}}$. For every other station $l \neq \hat{l}$, either $b_l^H \geq m_l$ or $b_l^H = 0$. In this case:
 - (a) The optimal value of F is bounded above by $H(t, \vec{b}^H)$.
 - (b) The distance from optimum is bounded by $\frac{\lambda_{\hat{l}}}{\lambda} (H_{\hat{l}}(t, b_{\hat{l}}^H) - F_{\hat{l}}(t, b_{\hat{l}}^H))$.

4 NUMERICAL APPLICATION

The US-based corporation Better Place was founded in 2007 with the ambitious goal of a large-scale adoption of fully electric cars. Since battery-related issues are the greatest obstacle to achieving this goal, Better Place developed a unique business model in which it retained battery ownership. Customers were to purchase the car absent the battery and Better Place was to provide battery swapping and recharging services and to assume all the battery-related risks (Dijk et al., 2013).

Although Better Place has filed for bankruptcy in 2013, its business model is still considered a promising solution to solving the battery problem in the electric car industry (Avci et al., 2014). Most of the cars produced for Better Place’s customers were sold in Israel, in which Better Place even completed the construction of fifty battery swapping stations before it filed for bankruptcy. The following section is a hypothetical full scale application of the Better Place model in a country with geographical and demographic characteristics similar to Israel.

Each of the three largest gas companies in Israel operate approximately two hundred fifty gas stations and accordingly we assume that the battery service firm (“the firm”) operates a network of two hundred fifty battery swapping stations distributed throughout the country, with a total arrival of approximately 14,000 customers per hour. The population density in Israel is such that the center region is the densest, followed by the northern region. The south of Israel, which constitutes more than half of Israel’s land area, is sparsely populated. Therefore, the number of stations per customer in the south is higher than the number of stations in the center, reflecting the large geographical size that must be serviced. To model the differences between the different stations in Israel, as well as differences between small neighborhood stations and busy major stations we assume that the arrival rates to the stations are equally spaced between 6.4 and 106 customers per hour.

An empty battery can be recharged to 50% of capacity within twenty minutes (Bullis, 2013). Since

there are many factors that affect recharging time we assume that the recharging time is distributed normally with mean forty minutes and standard deviation ten minutes. Battery swapping time, i.e., the battery removal and battery installation, is considerably shorter than recharging time and with state-of-the-art design, battery swapping can be done in less than two minutes (Mak et al., 2013). Each station is assumed to have ample battery rechargers since recharging docks are relatively inexpensive. Since the electric vehicle cars are poised as an alternative to the traditional gasoline-fueled cars, we assume that the tolerable wait for refueling is similar for both cars. Anecdotal evidence suggests that a ten minute wait for battery swapping is tolerated by customers. Finally, we use a baseline budget of nine thousand spare batteries.

To summarize, the baseline parameters of the example are: $L = 250$ stations; $N = 9000$ spare batteries; $\lambda_l = 6 + 0.4 \cdot l$ customers per hour, $t_1 + t_2 = 2$ minutes, $R_l \sim Normal(40, 10^2)$ minutes and the optimization criterion is the window fill rate for a tolerable wait $t = 10$ minutes (F_{10}).

4.1 The Baseline Scenario

Figure 2 describes the near-optimal spare allocation for the baseline case, \vec{b}^H , and Figure 3 displays the window fill rate for the optimal allocation as a function of t , $F(\vec{b}^H, t)$. Recall, that the optimization algorithm supplies spares to the station with the steepest slope until it reaches its tangent point and only then proceeds to the next station. The tangent line’s slope and the tangent point are increasing with the arrival rate and therefore the bigger the station index, the greater the tangent point. The near-optimal allocation dictates that the 50 slowest-moving stations will have no spares, whereas each of the busier stations will receive at least its tangent point. Station 51 is the exception; it has only two spares although its tangent point is 19 (see case 1 of Result 3). This implies that the solution $F(\vec{b}^H, 10) = 88.5\%$ is a lower bound that it not necessarily optimal. However, the distance between the bounds is a mere 0.02% (see notes to Table 1).

The low arrival rate stations are not given any spares and consequently, their window fill rate is almost zero. To compensate customers for the longer wait, the system’s managers could offer customers, for example, discounted meals or drinks. Behavioral research about customer waiting experience may be used to incentivize customers to agree to longer than usual waiting times (Maister, 1985).

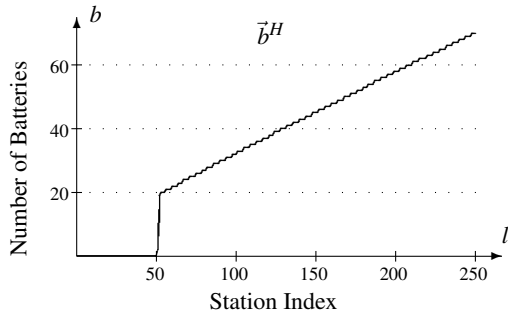
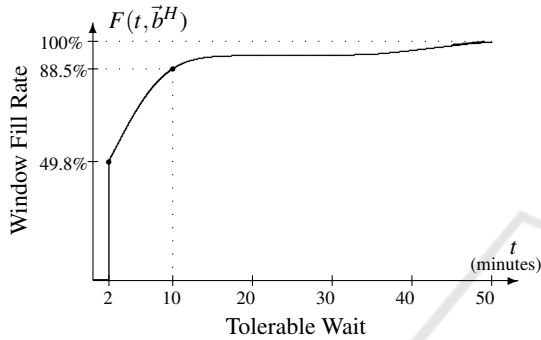


Figure 2: The spare battery allocation for the baseline case.


 Figure 3: The window fill rate as a function of t for the baseline optimal allocation.

4.2 The Effect of the Tolerable Wait

Table 1 details performance statistics for the baseline criterion, F_{10} , and three other optimization criteria; the window fill rate for tolerable waits of two (F_2), five (F_5) and fifteen (F_{15}) minutes. For the performance statistics, we use the same measures that we use for the optimality criteria, i.e., the window fill rates for two, five, ten and fifteen minutes. We see that different criteria lead to significantly different optimal values of the objective function. As is discussed below, the near-optimal spare allocations also differ dramatically. These observations stress the importance of defining the criterion for optimality correctly. For example, if the firm optimizes F_2 instead of the “correct” criterion F_{10} , then the percentage of satisfied customers (i.e., customers who were serviced within ten minutes) decreases from 88.5% to 77.5%. Similarly, if the firm errs to the other side and optimizes F_{15} then the percent of satisfied customers decreases from 88.5% to 84.9%.

Figure 4 compares the near-optimal allocations for three different criteria, F_2 , F_{10} and F_{15} . When $t = 15$, the tangent points are appreciably less than the baseline case and therefore the 9000 batteries are enough to supply all the stations with their tangent points. At this point, all the stations are in their con-

Table 1: The network’s performance for different optimization criteria.

		Performance Statistic			
		F_2	F_5	F_{10}	F_{15}
Criterion	F_2	73.5% ¹	76.5%	77.5%	77.6%
	F_5	70.6%	78.6% ²	82.8%	83.2%
	F_{10}	49.8%	68.5%	88.5% ³	93.5%
	F_{15}	35.0%	54.2%	84.9%	97.9% ⁴

Notes:

¹ Lower bound displayed. The distance between bounds is 0.12%.

² Lower bound displayed. The distance between bounds is 0.05%.

³ Lower bound displayed. The distance between bounds is 0.02%.

⁴ Optimal value displayed.

case region and the residual batteries are distributed among all the stations. Conversely, when $t = 2$ the tangent points are higher than in the baseline case. Now, the busy stations will demand more batteries to reach their tangent point and so the budget is depleted after allocating spares to fewer stations compared to the baseline case.

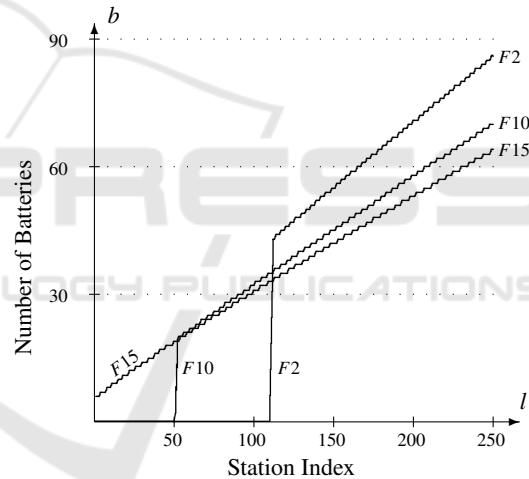


Figure 4: The spare battery allocation for different optimization criteria.

4.3 The Budget Effect

Figure 5 describes the near-optimal spare allocation for the baseline case and for spare budgets of 7000 and 11000 batteries. In the baseline scenario, the number of spare batteries in the network is 9000. If we increase the budget then the lower-rate stations will receive batteries one by one according to their tangent point. Eventually, all the stations will reach their tangent point. Now, any additional batteries will be distributed among all the stations instead of given to only particular stations. In contrast, if the budget is decreased then some slowest-moving stations will forfeit all their batteries. The busiest stations, how-

ever, will lose at most the few (if any) batteries they received beyond their tangent point.

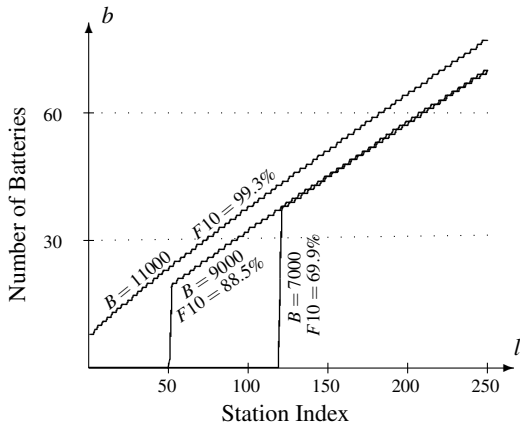


Figure 5: The window fill rate and spare battery allocation for different values of B .

4.4 The Swapping Time Effect

A corollary of Proposition 1 is that a $+\Delta$ change to the swapping time, t_1+t_2 is equivalent to a $-\Delta$ change to the tolerable wait, t . Figure 6 shows how the near-optimal allocation changes with the swapping time. As the swapping time increases, the tangent points increase too and therefore the busiest stations require more spares. As a consequence, more and more slow-moving stations will remain with zero spares.

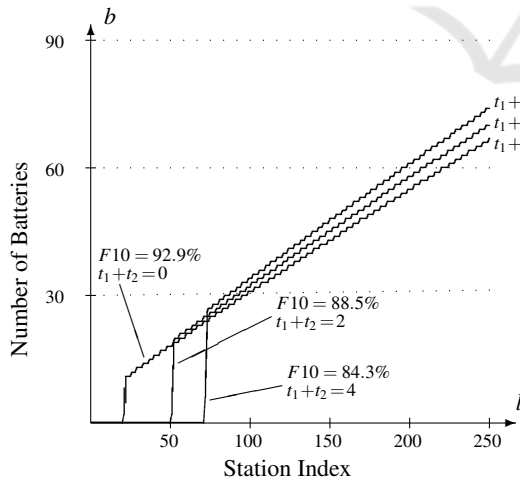


Figure 6: The spare battery allocation for different battery swapping times.

Thus far, we assume that the budget of spares is fixed. Consider, now the dual problem of (6).

$$\min_{\vec{b} \geq 0} \sum_{l=1}^L b_l \quad s.t. \quad F(t, \vec{b}) \geq \alpha. \quad (8)$$

where α is the network's required performance.

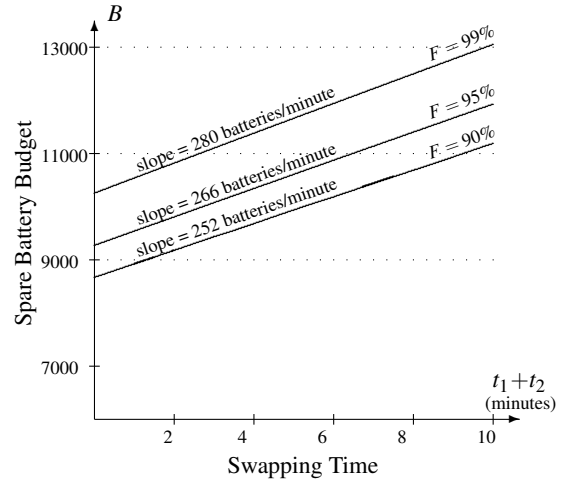


Figure 7: The spares budget for different performance target levels.

We may easily solve (8) using the optimization algorithm. The allocation of spares is done in an identical manner, but now the stopping condition is that we have reached the required level of service. Figure 7 displays the budget required to reach a 90%, 95% and 99% window fill rates for different swapping times. The slope of the graph reveals the savings obtained by reducing swapping time. For the performance levels of 90%, 95% and 99% the graph is almost linear and a minute reduction in the swapping time saves the network approximately 252, 266 and 280 batteries, respectively.

4.5 Optimizing Total Inventory Costs

The problems (6) and (8) assume that either the budget or the service level is predetermined. We now consider the problem of minimizing the total inventory costs. Let c_B denote the cost of a battery and let c_p denote the penalty cost each time a customer is not served within the tolerable wait. Since batteries have a limited lifetime, T , the planning horizon is T and the total number of customers arriving into the network during the planning horizon is λT where, recall, $\lambda = \sum \lambda_l$. The problem is given by

$$\min_{\vec{b} \geq 0} TC(\vec{b}) := c_b \sum_{l=1}^L b_l + c_p \lambda T (1 - F(t, \vec{b})). \quad (9)$$

We can easily adjust the optimization algorithm to find the optimal solution to (9). Each time we consider adding a battery we measure its contribution to the window fill rate, δ . As long as $\delta c_p \lambda T \geq c_b$ we increase the number of batteries in the network.

Batteries for family sized electric vehicles range between 20kWh and 80kWh with current prices reaching as low as US\$200 per-kWh (Sarker et al., 2015). We therefore examine battery prices of up to US\$25,000. We conservatively estimate battery life to be four years, and since we assume 12 daily hours of operation $T = 17520$ hours (Arcus, 2016). Finally, the arrival rate to the network is the sum of the arrivals to all the stations, $\lambda = 14,050$ per hour.

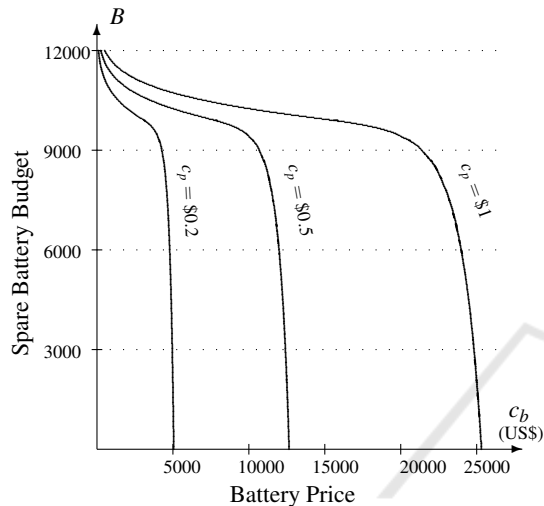


Figure 8: Spares Budget for different penalty costs and battery price.

In Figure 8 we depict the near-optimal spares budget depending on battery prices for different penalty costs. Interestingly, the budget may be very sensitive or very insensitive to battery prices. For example, when the penalty is \$1, then in the battery price range of \$3261 and \$21378 then budget size will only change between 11000 and 9000 batteries. In stark contrast, if the price increases from \$21378 to only \$25321, then the optimal budget will decrease from 9000 to 0 batteries.

5 CONCLUSIONS

In this paper, we suggest a model to solve the spare battery allocation problem. Since customers will tolerate a certain wait when they enter the station we claim that to minimize its penalty costs the network should maximize the fraction of customers who are served within the tolerable wait, the window fill rate.

We show that the relationship between the window fill rate when item removal and installment times are positive to the window fill rate with zero removal and installment times. Using this relationship, we build on (Dreyfuss and Giat, 2016) to solve our prob-

lem. To illustrate the application of the model we estimate a hypothetical application of a full-scale battery swapping network in Israel, similar to the network envisioned by the Better Place corporation. Our numerical analysis of the problem reveals interesting findings such as the value of better battery swapping design, the creation of different classes of stations and the critical importance of estimating the tolerable wait correctly.

The model assumes that battery swapping time is deterministic and that the customer arrival rate is constant over time. While the first assumption is reasonable, the second assumption is clearly unrealistic. We leave addressing these issues to future research.

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