Spatially Constrained Clustering to Define Geographical Rating Territories

Shengkun Xie[1,3], Zizhen Wang[2] and Anna Lawniczak[2]

[1]*Department of Mathematical & Computational Sciences, University of Toronto Mississauga, Mississauga, Canada*

[2]*Department of Mathematics and Statistics, University of Guelph, Guelph, Canada*

[3]*Ted Rogers School of Management, Ryerson University, Toronto, Canada*

shengkun.xie@utoronto.ca, shengkun.xie@ryerson.ca, zizhen@uoguelph.ca, alawnicz@uoguelph.ca

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Abstract: In this work, spatially constrained clustering of insurance loss cost is studied. The study has demonstrated that spatially constrained clustering is a promising technique for defining geographical rating territories using auto insurance loss data as it is able to satisfy the contiguity constraint while implementing clustering. In the presented work, to ensure statistically sound clustering, advanced statistical approaches, including average silhouette statistic and Gap statistic, were used to determine the number of clusters. The proposed method can also be applied to demographical data analysis and real estate data clustering due to the nature of spatial constraint.

1 INTRODUCTION

Clustering analysis has been now widely used for business analytic including automobile insurance pricing as a machine learning tool (A.C. Yeo and Brooks, 2001; Grize, 2015). It aims to partition a set of multi-dimensional data to a limited size of groups. It has also been used for territory analysis in many states of USA where zip codes were treated as an atomic geographical rating unit (Peck and Kuan, 1983). The aim of such analysis is to balance the group homogeneity and the number of clusters desired in order to ensure that insurance premium is fair and credible. This is particularly important when insurance premium is regulated. The main focus of this type of clustering is to determine an optimal partition of spatially constrained data into a set of groups, based on some distance measures such as Euclidean distance. The optimality is in the sense of being statistically sound as well as being able to satisfy insurance regulation. In clustering, the distance measures are applied to each data dimension first and then the overall distance measure of each data point is compared to each other to create different clusters or groups. However, how to handle the spatially constrained data in clustering become a challenging task.

In determining a suitable insurance classification of territory, average loss cost (or loss cost in short), i.e. pure premium, is often used as one of the key variables to differentiate levels of loss for each designed territory. Loss cost per geographical rating unit is calculated by dividing the total loss per year (in terms of dollars amount) within a given rating unit by the total number of risk exposures, i.e., the number of vehicles per year. The spatially constrained loss cost clustering is not only of particular interest to insurance regulators, who are mainly focusing on studying high level statistics estimates, but also it is important for auto insurance companies, where accurate pricing based on different territories are needed for the success of business to avoid the adverse selection.

In this work, we aim for an optimal grouping strategy for average loss costs at a Forward Sorting Area (FSA) level. In Canada, a FSA consists of first three letters of a postal code and it covers a much bigger area than a single postal code does. This allows a more reliable estimate of pure premium in a given region of interest as it includes more risk exposures (i.e., number of vehicles). Within insurance area, geographical information using postal codes has been seriously considered for flood insurance pricing because the nature of insurance coverage is heavily determined by geographical location of insureds. To our best knowledge, the territory design using geo-coding of FSAs has not appeared in the literature. This work is considered as the first attempt on discussing this

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topic.

In many countries including United States and Canada, auto insurance rates are heavily regulated. This implies that any rate-making methodology being used to analyze data must be both statistically and actuarially sound. Ensuring statistical soundness, it means that the approach being used must convey meaningful statistical information and the obtained results must be optimal in the statistical sense. From the actuarial perspective, it requires that any proposed rate-making methodology must take insurance regulation and actuarial practice into consideration. For example, loss cost must be at the similar level within a given cluster and the total number of territories used for insurance classification should be within a certain range. Also, the number of exposures should be sufficiently large to ensure that an estimate of statistics from the given group is credible. Because of these, it is critical to quantify the clustering effect and balance the results by taking both statistical soundness and actuarial rate & class regulation requirements into consideration.

The main contribution of this work is to propose a spatially constrained clustering approach, which is suitable for regional based business decision making using analytical approach. The proposed method has been applied to auto insurance pricing problem. Due to the nature of this work, it is possible to apply this method to other types of problems such as real estate pattern analysis.

This paper is organized as follows. In Section 2, we discuss the proposed methods including rate-making, clustering algorithms, and the choice of number of clusters. In Section 3, analysis of spatial loss cost data and summary of main results are presented. Finally, we conclude our findings and provide further remarks in Section 4.

2 METHODS

In rate-making methodologies for auto insurance pricing, territory design and analysis based on loss cost of a geographical rating unit is one of the key aspects. Loss cost is defined as a ratio of total loss to total number of exposures. It is an average cost to cover an exposure of risk for a given period (i.e., policy term) and it is often called pure premium or theoretical premium. The need of territory design is to ensure that the number of exposures in each territory is sufficiently large so that the estimate of statistic within a territory is credible. Also, the loss cost of basic rating units within a territory must be at a similar level, i.e. it must consider a suitable number of rating territories that satisfy contiguity constraint which ensure the homogeneity and credibility for each territory. Often large sizes of rating territories or a small total number of rating territories easily satisfy the full credibility requirement, but often not the homogeneity requirement. How to balance these two sides becomes the major focus of this type of research. Also, each territory should contain only their neighbors, and cannot include any rating units acrossing the boundaries between territories. This contiguity constraint inspires us to consider a clustering with geocoding. Often we refer this as a spatially constrained clustering.

2.1 Geocoding and Weighted Clustering

Auto insurance loss data contains residential information of policy holders, i.e., postal codes, reported claim information and others. The loss amount and exposure of risk are then aggregated by postal codes to derive loss cost. Often loss cost at postal code level is less credible as it may not cover sufficient number of reported claims for accurate analysis. Therefore, in order to better reflect the nature of loss level, we have to consider a rating unit that includes a larger size of exposures, so that the loss cost estimate becomes more credible. In this work, we define FSA as a basic geographical rating unit. This geographical information is then coded into latitude and longitude. The geocoding is then combined with other loss information to become an input of a clustering algorithm. We then consider an optimization problem that essentially leads to a clustering algorithm, which can be described as follows when taking K-mean as an example.

Given a set of high dimensional observations $\{X_1,\ X_2,\ \ldots,\ X_n\}$, where each observation is a d-dimensional real vector, i.e. $X_i \in R^d$, a weighted K-mean clustering aims to partition the n observations into K sets ($K \leq n$), $S = \{S_1, S_2, \ldots, S_K\}$, so that it minimizes the within-cluster sum of squares (WCSS):

$$\arg\min_{S} \sum_{i=1}^{K} \sum_{X_j \in S_i} \|X_j - \mu_j\|_2^2, \tag{1}$$

where μ_i is the mean point of the cluster S_i. The weighted sum of squares is defined as follows:

$$\|X_j - \mu_j\|_2^2 = \sum_{l=1}^{d} w_l (x_{jl} - \mu_{il})^2, \tag{2}$$

where w_d is used to specify the importanceness of each dimension of data variable X_i. In auto insurance pricing, a typical focus on determining w_d is to evaluate the importanceness of each pricing factor.

Often each dimension of data variable X_i needs to be scaled, i.e., a normalization procedure needs to be applied before clustering. We assume that X_i has been normalized. Specifically, in our case (i.e., when $d = 3$), $\mu_i = (\mu_{i1}, \mu_{i2}, \mu_{i3})^\top$ corresponds to mean value of ith center of designed territory and $X_j = (x_{j1}, x_{j2}, x_{j3})^\top$ is the vector consisting of the standardized loss cost x_{j1}, latitude x_{j2} and longitude x_{j3} of the jth FSA, and w_d is the weight applied to dth dimension of data variable. In this work, without loss of generality, we take w_2=w_3=1 and we allow w_1 to take different values. The idea is to define a relativity measure between loss cost and geographical location as w_1. When w_1=1, the loss cost is deemed to be as important as geographical information, while when w_1 takes a value greater (less) than 1 the loss cost is more (less) important than geographical information in a clustering.

One can also use K-medoid clustering instead of K-mean. The major difference between these two approaches is estimate of the center of each cluster. The K-mean clustering determines each cluster's center using the arithmetics means of each data characteristic, while the K-medoid clustering uses the actually data points in a given cluster as the center. For our clustering problem, it does not make any essential difference, which clustering method is selected, as we aim for grouping only. Similarly, the hierarchical clustering, which seeks to build a hierarchy of clusters, can also be considered.

2.2 Spatially Constrained Clustering

The K-mean or K-medoid clustering does not necessarily lead to clustering results that satisfy the cluster contiguity requirement. In this case, spatially constrained clustering is needed as all clusters are required to be spatially contiguous. We start from an initial clustering. We assume that each cluster from the initial clustering will contain only a few non-contiguous points, and we just need to re-allocate these points following an initial clustering. To re-allocate these non-contiguous points, we first identify them, and then re-allocate them to the closest (minimal-distance) point within a contiguous cluster. In order to implement this allocation of non-contiguous points, we propose an approach that is based on Delaunay triangulation (Recchia, 2010; Renka, 1996). In mathematics, a Delaunay triangulation for a set P of points in a plane is a triangulation, denoted by DT(P), such that no point in P is inside the circumcircle of any triangle in DT(P). If a cluster P is in DT(P) and DT(P) forms a convex hull (Preparata and Hong, 1977), the clustering then satisfies the contiguity constraint. In order to construct a DT, we propose the following procedure:

1. We first do K-mean clustering as an initial clustering.
2. Based on the obtained clustering results from the previous step, we find all points that are entirely surrounded by points from other clusters.
3. We then find the neighboring point at minimal distance to the point that has no neighbors in the same cluster. We called the associated cluster as a new cluster.
4. The points that have no neighbors are then reallocated to new clusters.

It is possible that the reallocated points may still be isolated, thus this entire routine should be iterated until we find that no such isolated point exists. Note that this implementation is purely based on algorithm we develop and the boundary created for each cluster is often not corresponding to the geographical boundary of each basic rating unit. However, based on this results, one should be able to further refine them to ensure that the boundary of cluster is determined by the boundary of FSAs.

2.3 Choice of the Number of Clusters

In data clustering, the number of clusters needs to be determined first. In this work, the number of clusters represents the number of territories. Finding optimal number of clusters becomes especially challenging in high dimensional scenarios where visualization of data is difficult. In order to be statistically sound, several methods including average silhouette (Rousseeuw, 1987) and gap statistic (R. Tibshirani and Hastie, 2001) have been proposed for estimating the number of clusters. The silhouette width of an observation i is defined as

$$s(i) = \frac{b(i) - a(i)}{\max\{a(i), b(i)\}}, \tag{3}$$

where $a(i)$ is the average distance between i and all other observations in the same cluster and $b(i)$ is the minimum average distance between i to other observations in different clusters. Observations with large $s(i)$ (almost 1) are well-clustered, observations with small $s(i)$ (around 0) tend to lie between two clusters and observations with negative $s(i)$ are probably placed in a wrong cluster.

Varying the total number of clusters from 1 to the maximum total number of clusters K_{max}, the observed data can be clustered using any algorithm including K-mean. Next average silhouette can be used to estimate the number of components. For a given number of clusters K, the overall average silhouette width for clustering can be calculated as

$$\bar{s} = \sum_{i=1}^{n} \frac{s(i)}{n}. \tag{4}$$

The number of clusters which gives the largest average silhouette width is used to estimate the optimal number of clusters. Note that, the optimal number of clusters may not be used eventually in practice, given the fact that the real-world data is complex and contains high level of variation. Often it is the case that the optimal number of clusters provides a starting point for clustering work and each cluster from optimal clustering may be partitioned further in order to improve the results.

Gap statistics, proposed by Tibshirani et al. (2001), is another resampling-based approach for determining the optimal number of clusters. This method compares the observed distribution of the data samples to a null reference distribution (such as uniform distribution). The number K of clusters is selected such that K is the smallest value whose difference (i.e., gap) is statistically significant. The gap at selected number K of clusters is defined as follows

$$\mathrm{Gap}(K) = E_N^*[\log(W_K)] - \log(W_K) \tag{5}$$

where W_K is the within-cluster sum of squares and E_N^* is the expectation under a sample size of N from the reference null distribution.

In order to compute the gap statistics, $E_N^*[\log(W_K)]$ needs to be determined first. This is done by resampling from a given reference null distribution and using A different reference distributions, as null distributions. Varying the total number of clusters from 1 to K_{max}, both the observed data and the reference data are clustered. $E_N^*[\log(W_K)]$ and the standard deviation σ_K are estimated as follows

$$E_N^*[\log(W_K)] = \frac{1}{A}\sum_{a=1}^{A} \log(W_{Ka}) \tag{6}$$

and

$$\sigma_K = \left[\frac{1}{A}\sum_{a=1}^{A} \{\log(W_{Ka}) - E_N^*[\log(W_K)]\}^2\right]^{1/2}. \tag{7}$$

The clustering result from the selected K clusters is said to be statistically significantly different from the null reference if

$$\mathrm{Gap}(K) \geq \mathrm{Gap}(K+1) - \sigma_{K+1}\sqrt{1+1/A}. \tag{8}$$

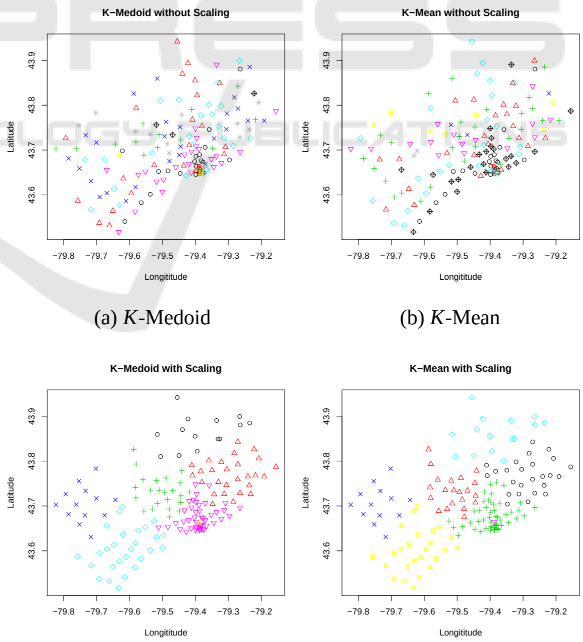
The optimal K is the smallest value of K that achieves this statistical significance. Similarly to the silhouette statistic, due to the natural complexity and high level of variation from real data as well as lack of the certainty in selecting null distributions, the mean and standard deviation computed by using A reference distributions may lead to a significant bias. So in practice, the optimal number of clusters determined by (8) only provides a starting point for further clustering analysis of the data. In order to more systematically select the suitable number of clusters, we propose a refined approach based on (8). From (8), we derive the following expression.

$$\Delta G(K) = \mathrm{Gap}(K+1) - \mathrm{Gap}(K) \leq \sigma_{K+1}\sqrt{1+1/A}. \tag{9}$$

Often A is large, therefore (10) suggest that an optimal number is the smallest K which satisfies the corresponding incremental Gap statistics which is less than one standard deviation. This leads to the following simplified version of (10)

$$\Delta G(K) = \mathrm{Gap}(K+1) - \mathrm{Gap}(K) \leq \sigma_{K+1}. \tag{10}$$

Furthermore, $\Delta G(K)$ fluctuates with K and the fluctuation will distort the estimate of the optimal number of clusters. Instead of looking for the smallest K that satisfies the equation (11) based on the empirical pattern, one can estimate the signal component of $\Delta G(K)$ by imposing a power-law relationship.



(a) *K*-Medoid (b) *K*-Mean

(c) *K*-Medoid (d) *K*-Mean

Figure 1: The clustering results using equal weight (i.e., w_1=1) for the data with and without scaling. Each cluster has the same color, and there are 10 clusters determined by average silhouette method.

Table 1: The summary of model performance when the clustering results fitted to a linear model. An insurance claim frequency of 2% and full credibility of 1082 claims are assumed for computing credibility of the minimum size of clusters, denoted by $\min(E_i)$.

w_1	K	Std.Dev	adjusted R^2	$\min(E_i)$	$\min(\sqrt{\frac{\min(E_i)*2\%}{1082}}, 1)$
0.5	4	672.8	0.5159	30,041	0.7452
0.6	4	672.8	0.5159	30,041	0.7452
0.7	3	867.6	0.1951	97,359	1
0.8	4	800.2	0.3152	54,092	0.9999
0.9	4	672.8	0.5159	30,041	0.7452
1.0	4	672.8	0.5159	30,041	0.7452
1.1	2	815.5	0.2889	159,966	1
1.2	2	815.5	0.2889	159,966	1
1.3	2	815.5	0.2889	159,966	1
1.4	2	815.5	0.2889	159,966	1
1.5	2	815.5	0.2889	159,966	1
1.6	7	**408.3**	**0.8217**	9,252	0.4135

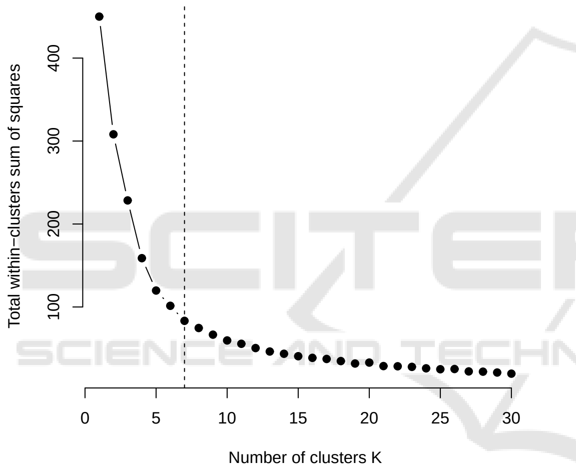


Figure 2: The selection of number of cluster based on elbow method using total within-clusters sum of squares. The vertical dotted line is at $K = 7$, which is suggested by the method.

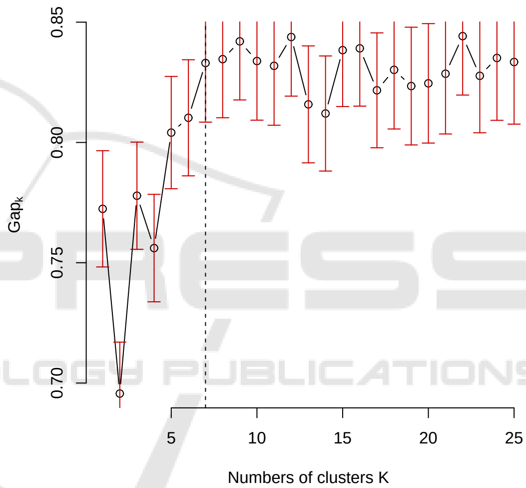


Figure 3: The selection of number of clusters based on the gap statistics. The vertical dotted line is at $K = 7$. The vertical red bounded line indicates one standard deviation region.

3 RESULTS

In this section, we present the results of analysis using a real data set from an auto insurance regulator. The data consists of geographical information in terms of FSAs, loss cost for each FSA, and exposures of risk for each FSA. The number of exposures will be used for credibility weighted and is not passed to a clustering algorithm. Since the geo-coder takes only the input of zip codes or postal codes, we first collect all postal codes that are associated with each FSA. Within each FSA, the postal codes are geo-coded. We then use the geo-coding of postal codes within each FSA to estimate the geo-coding of the given FSA simply by taking the average of geo-coding along each dimension. The obtained Latitude, Longitude and its associated loss cost for each FSA becomes the input of clustering algorithm.

Scaling of input data is an important procedure for clustering. High data scale is not necessarily more important than low scale of data. Here we demonstrate the impact of scaling on clustering by comparing results obtained from both with and without scaling methods. In the case of without scaling of data method, the clustering results shown in Figure 1 in which K-mean and K-medoid methods are used, respectively, suggest that the partitioning of input data is not successful because the contiguity constraint of clustering is not met. The interior of a cluster con-

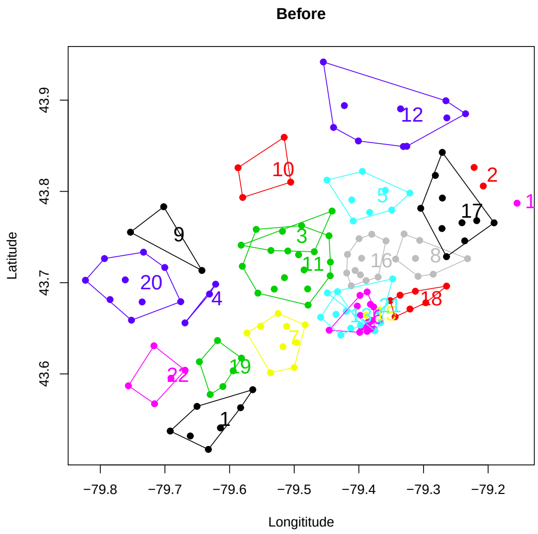


Figure 4: Convex hull plot of clusters obtained from the K-mean clustering without re-allocation of isolated points.

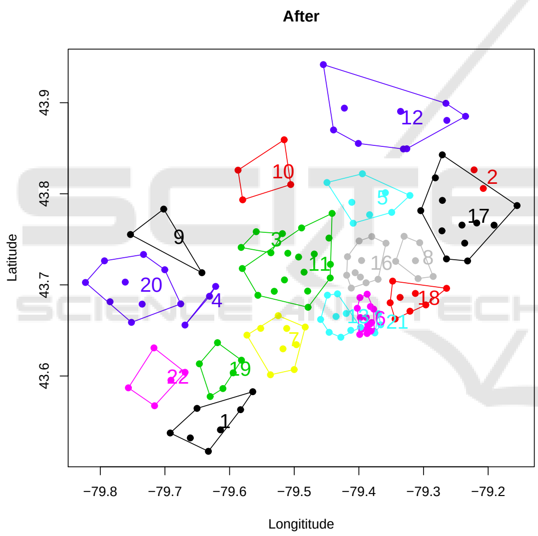


Figure 5: Convex hull plot of clusters obtained from the K-mean clustering with re-allocation of isolated points.

tains elements from the other clusters. Even though the sum of squares between the groups has explained 98.9% of the total sum of squares for a partition of 10 clusters, this partition of data does not satisfy the insurance pricing regulation because of the contiguity issue. When the input data is scaled, the clustering results are more promising because they better satisfy the contiguity constraint requirement. The cost of having this better result is a lower proportion of between groups variation, which is decreased from 98.9% to 81.5%. However, the contiguity constraint is still not fully satisfied, because there exists some points are isolated and within another cluster.

In order to produce the results of Figure 1, a choice of the number of clusters was required. In this paper, three methods including elbow method, average silhouette and gap statistic were used. For the input data with scaling, 7 clusters were suggested by both elbow method (qualitatively determined) and average silhouette method. The result of using elbow method is shown in Figure 2. The choice of the number of clusters is based on the smallest K that is associated with insignificant decrease of within cluster sum of squares. The selection of the number of clusters using gap statistic is presented in Figure 3. When applying either gap statistic and elbow method, the optimal number of clusters should be determined with care. For instance, gap statistic suggests the choice of minimum K when the increment ΔG is larger than σ_{K+1}. When this choice is applied to our data, the gap statistic suggests the number of clusters to be one, which does not make sense as it suggest that no clustering is required. This result is apparently due to the heavy distortion caused by the underlying uncertainty of the estimate of gap statistic and the standard deviation. Therefore, a more suitable choice of K should be made based on the overall pattern of gap statistic with respect to K number of clusters. One should focus on the signal component of $G(K)$ and select the one that first approaches the stable state of gap statistic. When this rule is applied, the similar number is obtained to the ones obtained by the elbow method or average silhouette method.

The clustering results are further analyzed by fitting the data to ANOVA linear model. The ANOVA model standard deviation and adjusted R^2 are obtained from comparing the performance of model fitting, in terms of predictive power (through adjusted R^2) and the model reliability (by looking at the model standard deviation). From these results one can see that the best performance is obtained when w_1=1.6, which means that the loss cost needs to be given more weight. In this case, 7 clusters are suggested by the algorithm in order to achieve the statistical soundness, however this leads to a lower credibility. In calculating credibility, a 2% car insurance claim frequency and full credibility of 1082 claims are assumed. This further confirms that when the loss cost is given more weight, the clustering is done mainly based on the loss cost, and to satisfy the contiguity constraint, more clusters may be needed.

To demonstrate the improvement of using spatially constrained clustering that we proposed, we first apply the K-mean clustering with w_1=1 (i.e., being equally important between the geophysical location and the loss cost) using 22 clusters. The 22 clusters were used by the regulator who owns the data to come up with a design of territory for regulation purpose. The result is shown in Figure 4. From this result one can see that there are still many clusters, such as 3, 13, 15, 21 and 18 (indicated within convex hull), which do not satisfy the contiguity constraint completely. Thus, we apply the proposed procedure discussed in the methodology section to further refine the results. From the output shown in Figure 5, all the clusters form convex hull. Thus, the contiguity constraint is satisfied.

4 DISCUSSIONS AND CONCLUDING REMARKS

In this work, spatially constrained clustering of the insurance loss cost was studied. The FSAs represented by their computed geocoding, and their associated insurance loss costs are the input of clustering algorithms. The geocoding does not require a big extra effort as it can be easily obtained from some geo-coders using Global Positioning System (GPS). In geocoding of an FSA, each co-ordinate of centroid is determined by using the mean value of either latitude or longitude value of the total postal codes within each FSA. The geocoding and the loss cost values must be standardized before using them in the clustering algorithm. The standardization procedure is just a relocation and re-scaling of each variable, i.e. loss cost, latitude and longitude. The method of Delaunay triangulation is used to ensure that the contiguity constraint is satisfied. In fact, the contiguity constraint has many other applications in the earth and social sciences and in image processing (Recchia, 2010). It has been demonstrated that the spatially constrained clustering is a promising approach for clustering insurance loss costs as it is able to satisfy the contiguity constraint while implementing clustering. In the presented work, to ensure clustering to be statistically sound, advanced statistical approaches including average silhouette statistic and Gap statistic were used to determine the number of clusters. The presented work is based on data for all insurance coverage, it may be interesting to see how the loss cost change from one sub-coverage to another one. This needs to be done by investigating sub-coverage data. Also, in order to quantify the homogeneity of clustering, entropy based method may be considered in future research as it can measure how uniformity of the distribution of loss cost is. The uniformity is what insurance company expect to ensure that policyholders are responsible for extract their cost.

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