Exact Solution of the Multi-trip Inventory Routing Problem using a Pseudo-polynomial Model

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Keywords: Inventory Routing Problem, Integer Linear Programing, Network Flow Models, Multi-trip.

Abstract: In this paper, we address an inventory routing problem where a vehicle can perform more than one trip in a working day. This problem was denominated multi-trip vehicle routing problem. In this problem a set of customers with demand for the planning horizon must be satisfied by a supplier. The supplier, with a set of vehicles, delivers the demand using pre-calculated valid routes that define the schedule of the delivery of goods on the planning horizon. The problem is solved with a pseudo-polynomial network flow model that is solved exactly in a set of instances adapted from the literature. An extensive set of computational experiments on these instances were conducted varying a set of parameters of the model. The results obtained with this model show that it is possible to solve instances up to 50 customers and with 15 periods in a reasonable computational time.

1 INTRODUCTION

The vehicle routing problem can be applied in real cases on logistics companies in order to reduce transportation costs, which include, among others, costs associated with drivers, vehicles or fuel. The integration of this problem with the inventory management can reflect in considerable savings, since this provides a more efficient management of the resources than the one achieved through the local optimization of the two problems separately.

In the inventory routing problem the goal is to minimize the total transportation cost from the supplier to the customer, so that the customer maintains an inventory level that will satisfy the demand in each period of a given planning horizon, reducing also possible storage costs.

This problem can incorporate a time horizon information, inventory management policies, routes, fleet type and size (Coelho et al., 2014). Routes are considered to be direct or not, whether a single customer or more are visited, respectively (Coelho et al., 2014). The planning horizon is considered finite if it is defined for a short period, or infinite when the schedule of routes is carried out for a long period of time (Coelho et al., 2014; Bertazzi and Speranza, 2013). Typically, the goal is to minimize the overall transportation costs, reducing penalties associated with inventory level, which typically represent storage costs (Bertazzi and Speranza, 2013).

Several practical applications have been implemented in industry, which enable companies to reduce inventory and transportation costs improving the quality of service. A recent study describes the implementation of this problem in a fuel distribution company (Hanczar, 2012). Another study describes an application of the problem in a company with a fleet of ships that delivers chemicals to warehouses located throughout the world (Miller, 1987). The authors describe an integer programming model that was successfully implemented in this company. The inventary routing problem was also considered in the daily strategy of a company that provides calcium carbonate throughout Europe and it allowed to achieve a reduction in millions of dollars of costs per year (Dauzère-Pérès et al., 2007). In addition, different approaches of this problem have also been applied in the maritime industry (Al-Khayyal and Hwang, 2007; Song and Furman, 2013; Persson and Göthe-Lundgren, 2005; Grnhaug et al., 2010).

The problem explored in this paper is the inventory routing problem that allowed the vehicles to carry out more than one route in each period of the planning horizon and therefore it was denominated multi-trip inventory routing problem. The consideration of multiple routes can provide advantages, in the sense that

Braga N., Alves C. and Macedo R.
Exact Solution of the Multi-trip Inventory Routing Problem using a Pseudo-polynomial Model.
DOI: 10.5220/0006118502500257
In *Proceedings of the 6th International Conference on Operations Research and Enterprise Systems (ICORES 2017)*, pages 250-257
ISBN: 978-989-758-218-9

the cost of the vehicle is usually fixed for a period of the planning horizon, therefore reducing the costs at this level. On the other hand, the consideration of this variant with multiple routes makes the problem, which is already difficult to solve, even more difficult.

Since there is an increasing interest in applying this problem in practical cases in industry, several heuristics and exact algorithms have been proposed by several authors in the literature. The difficulty of solving the inventory routing problems has been mostly motivated by the development of heuristics, which in many cases show good results (Herer and Levy, 1997; Archetti et al., 2011; Cordeau et al., 2015; Hemmati et al., 2015). In a recent paper, the inventory routing problem with multiple products and vehicles was solved with an exact model (Coelho and Laporte, 2013). The authors describe an integer programming model, to which are also added valid inequalities with the objective of strengthening it. In the model a branch-and-cut is proposed that is able to solve instances with a maximum of 5 vehicles, 5 products, 7 periods and 50 customers. In another problem with a constant customers demand the authors resorted to a Lagrangian relaxation method that derives lower and upper bounds for the model in order to obtain good quality solutions in acceptable times (Zhong and Aghezzaf, 2012). In another paper it is also proposed a method of Lagrangian relaxation combined with a subgradient method, able to solve instances up to 200 customers (Yu et al., 2008). Two integer programming models were proposed to solve the inventory routing problem when the inventory is managed by the supplier and when it is managed by the customer, comparing the two approaches (Archetti and Speranza, 2016).

The multi-trip inventory routing problem is typically more difficult to resolve as compared with the usual vehicle routing problem. This variant was reviewed elsewhere (Şen and Blbl, 2008). Since this is not a trivial problem, several heuristic methods have been proposed. A tabu search algorithm is also described to solve this problem (Taillard et al., 1996). The same problem was addressed with a heuristic algorithm also using tabu search (Brando and Mercer, 1998). The use of constructive heuristic with three phases was also proposed (Petch and Salhi, 2003). An adaptive memory procedure was described (Olivera and Viera, 2007), and the results were compared with those obtained with others from the literature (Taillard et al., 1996; Brando and Mercer, 1998; Petch and Salhi, 2003). A genetic algorithm was also proposed, for the first time, to solve this problem (Salhi and Petch, 2007). A vehicle routing problem with multiple routes and additional accessibility constraints was studied using a tabu search algorithm that involved instances up to 1000 customers (Alonso et al., 2008).

An exact integer programming method was proposed for the problem of routing with a single vehicle with time windows and multiple routes (Azi et al., 2007). The algorithm is divided in two phases: first, all valid routes are generated and in the second phase routes are affected at different periods of the planning horizon. The authors further generalize the algorithm for the case of multiple vehicles (Azi et al., 2010). The authors resorted to a column generation algorithm able to solve instances with a number of customers between 25 and 50.

A pseudo-polynomial network flow model was used to solve the vehicle routing problem with time windows and multiple routes (Macedo et al., 2011). In the model, the underlying graph vertices correspond to instants of time of the planning horizon, and the arcs define valid routes. It is proposed an exact algorithm that considers an iterative disaggregation of the vertices of the graph, which are first aggregated to obtain a smaller model, and thus easier to solve. The model proposed in this article is similar to the one here described (Macedo et al., 2011) in the sense that it uses a pseudo-polynomial network flow model, the arcs define valid routes and the vertices also correspond to instances of time.

In section 2 we present the definition of the problem, showing also an example. On section 3 it is formally presented the pseudo-polynomial network flow model to solve this problem. In section 4 the computational results are shown and finally, some conclusions are presented in section 5.

2 MULTI-TRIP INVENTORY ROUTING PROBLEM

2.1 Definition

The class of inventory routing problems considers a context in which one or more types of products are shipped from a supplier to a set of customers through a fleet of vehicles.

In this problem the customers demand should be satisfied during several periods of a planning horizon. What differentiates this class of problems, from the vehicle routing problem is the fact that the supplier manages the inventory of the customer, i.e., the product amount supplied to each customer in each period is not necessarily equal to their demands. The products deliveries must be carried out in such a way that the customers have available at each period, the required amount of product.

In this paper, a variant of this problem is addressed that considers the vehicle routing problem with multiple routes, which means that each vehicle can be allocated to more than one route in each period of the planning horizon.

We consider that a fleet of vehicles is located in a warehouse, which supplies a set of customers with a single type of product.

The objective of this problem is to determine the optimal set of routes that minimize the total transportation cost, and any storage costs in the customer. That is, whenever an order is delivered before the set period, incurs in a penalty proportional to the costs of storage of products in the customer. On the other hand, it is considered that customers have an unlimited storage capacity. With regard to anticipated deliveries, they can not be phased. This means that all demand for a period is delivered in a single visit to the corresponding customer, whether made on the same period or in previous periods.

In this problem, the number of available vehicles is limited, as well as the capacity of each vehicle, and the load in each route can not exceed its capacity. It is assumed that each unit of the product transported occupies a unit of volume on the vehicle and the time spent on transportation is equivalent to the distance traveled. Each vehicle can carry out various routes per period, so that, the sum of their lengths does not exceed the duration of a working day.

2.2 Data and Parameters

To clarify the formal presentation of the problem, we provide below an exhaustive list of parameters that characterise it:

- $D = \{0\}$: warehouse;
- $S = \{1, \ldots, N\}$: customers;
- $T = \{1, \ldots, \tau\}$: time periods of the planning horizon.

The warehouse is associated with the index 0. Customers are located within a certain distance from the warehouse, distributed according to their cartesian coordinates. The planning horizon defines the time period for which deliveries to customers will have to be made. This period will subsequently be divided into units of time referred to as work day.

We consider that a customer cannot be visited more than once in each time period and there is a single type of product. Furthermore, we assume that a visit to a customer at a time t requires the delivery of the demand for that period, and eventually the later periods. Stock-outs are not allowed, i.e., all customers must imperatively have at their disposal, in each period, the required quantities of products. Finally, it is considered that there is no initial stock in customers, i.e., at time period 0 of the planning horizon the customers do not have at their disposal any stock.

Below we present the problem data:

- C: vehicle capacity (homogeneous fleet);
- F: number of available vehicles;
- W: duration of a working day;
- d_i^t: demand of customer i on time period t;
- N_{max}: maximum number of customers visited by route.

Some additional settings:

- Ψ_t: set of valid routes in the period t;
- N_r: set of customers visited by route r;
- $\alpha_{irt}^{t'}$: equal to 1 if the route r delivers the demand of the customers i in the period t, or equal to 0 otherwise;
- a route r is characterised by a set of customers (visited by the route) and the periods of demands that are delivered as part of the same route.

The costs considered in this problem are:

- C_v: fixed cost for using a vehicle in a working day;
- C_r: transportation cost associated with the route r;
- C_{h_i}: storage cost of a unit of product on the customers i for a period of time;
- $C_{H_r^t}$: total cost of storage associated with the route r ($C_{H_r^t} = \sum_{i \in N_r} C_{h_i} t_r^i$, being t_r^i the total waiting time until the product is consumed on the customer i delivered through the route r).

2.3 Example of a Problem Instance

Example 1. *Consider the example of an instance for the inventory routing problem.*

The Table 1 indicates all the parameters that define it. Table 1a defines the location of the warehouse, and Table 1b defines the capacity of the vehicles (C), the fleet size (F), the duration of a working day (W), the number os periods of the planning horizon (τ) and the number of customers (N). In Table 1c are represented the customer cartesian coordinates (x, y), as well as the storage costs for each customer (C_{h_i}). Finally, Table 1d defines the demands d_i^t in the period t for the customer i.

The graphical representation of this instance can be observed in Figure 1, showing the warehouse and

Table 1: Example of an instance of the inventory routing problem. (a) Warehouse location data, (b) general data of the problem, (c) customer data and (d) customer demands.

A	10	20

(a)

C	10
C_v	20
F	5
W	120
T	3
N	5

(b)

x	y	C_{h_i}
9	50	24
-9	10	15
-22	22	3
-15	45	9
-3	44	12

(c)

t	i	d_i^t
1	1	4
1	2	2
1	3	2
1	4	2
1	5	2
2	1	2
2	2	2
2	3	2
2	4	2
2	5	2
3	1	4
3	2	2
3	3	2
3	4	2
3	5	3

(d)

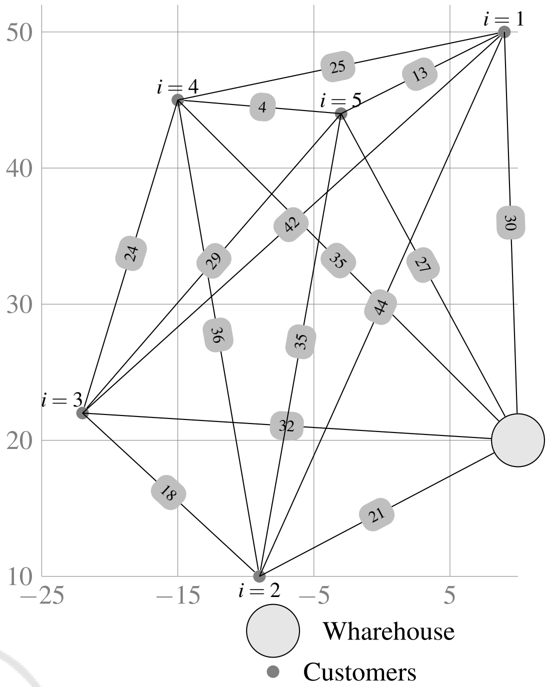

Figure 1: Instance graphical representation with a warehouse, customers and their distances located in midway of the connections.

the customers distributed according to their cartesian coordinates. All connections between customers and warehouse (supplier) are also represented, as well as the corresponding distances to be traveled in this route.

Figure 2 represents a valid solution for this instance, which has in the first period a cost of 214, in the second 198 and in the third and last 131, with a total cost of 543. In this case, it is not possible to use a single vehicle in the first period, since the distance that the vehicle would have to travel to deliver all customer demands is greater than 120 (length of a work day). Thus, two vehicles are scheduled for the routes for the first period, and in each of the routes it is delivered the demand for further periods, in particular customers 3 and 4. These deliveries in later periods incur in a penalty for each unit in storage. In the second period, it may resort to a single vehicle that performs two routes. In the third period, a single vehicle performs the remaining deliveries to customers where the demands have not yet been satisfied, making use of a single route. In this latter period there is no storage costs since all demands are satisfied and consumed in this period. In this figure, the information relative to the demand is denoted by d_i^t, *and can exist for later periods. The information about the distance (l) traveled and the volume (v) occupied on the vehicle in the route is defined in the arcs as* $[l|v]$. *Table 2 discriminates the objective function values for the three periods shown in Figure 1.*

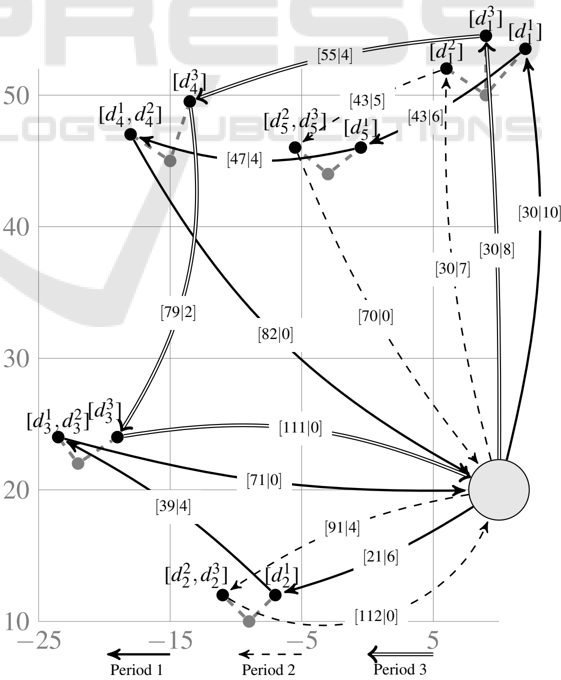

Figure 2: Valid solution to the inventory routing problem instance of Example 1. In the middle of the connection between each customers is shown the distance traveled, as well as the volume of the goods on the vehicle in the route.

Table 2: Objective function values for the three periods.

T	C_r	C_v	C_{h_i}	total
1	82 + 71	20 + 20	$9\times2+3\times2$	214
2	112	20	$12\times3+15\times2$	198
3	111	20	0	131
				543

3 A NETWORK FLOW MODEL FOR THE MULTI-TRIP INVENTORY ROUTING PROBLEM

In this section, we describe a new integer programming model for the multi-trip inventory routing problem.

This network flow model is defined in a set of acyclic and direct graphs, one for each period of the planning horizon, denoted by $G_t = (V, A_t)$, $t \in T$ and V is a set of vertices $V = \{0,\dots,W+1\}$, and A_t is the set of arcs that represent the set of all valid routes in the period $t \in T$, as well as the waiting time in the warehouse. A flow that runs through the graph represents a working day of a vehicle, i.e., the sequence of routes and waiting times this performs from the moment 0 until time instant w of a given planning horizon. A route is defined by a sequence of customers to visit, as well as the respective product amounts to deliver to each customer. In order for a route to be valid, the sum of the quantities of products to be delivered to each customer must not exceed the capacity of the vehicle, and the necessary travel time cannot exceed a working day (a period of the planning horizon). Note that the same route can start at different instants, keeping it valid. The set of all valid routes is generated in advance, and the variable x^t_{uvr} represents the route r that starts at the instant u and ends at time v of period $t \in T$. Routes are generated through a recursive process that will exclude routes violating the vehicle capacity (C) and/or a maximum duration (W) of the route.

$$\min \sum_{t\in T}\sum_{(u,v)^r\in\Psi_t} C_r x^t_{uvr} + C_v \sum_{t\in T}\sum_{(0,v)^r\in\Psi_t} x^t_{0vr} + \sum_{t\in T}\sum_{(u,v)^r\in\Psi_t} C_{H^t_r} x^t_{uvr} \quad (1)$$

$$\text{s.t.} \sum_{t\in T, t\le t'}\sum_{(u,v)^r\in\Psi_t | i\in N_r} \alpha^{t'}_{irt} x^t_{uvr} = 1, \quad \forall i\in S, t'\in T, \quad (2)$$

$$\sum_{(0,v)^r\in\Psi_t} x^t_{0vr} \le F, \quad \forall t\in T, \quad (3)$$

$$-\sum_{(u,v)^r\in\Psi_t} x^t_{uvr} + \sum_{(v,y)^s\in\Psi_t} x^t_{vys} = \begin{cases} 0, & \text{if } v=1,\dots,W-1, \\ -\sum\limits_{(0,v)^r\in\Psi_t} x^t_{0vr}, & \text{if } v=W, \end{cases} \quad \forall t\in T, \quad (4)$$

$$x^t_{uvr}\in\{0,1\}, \forall (u,v)^r\in\Psi_t,\ \forall t\in T. \quad (5)$$

The objective function (1) represents the sum of the transportation costs of the traveled routes C_r, the cost of the vehicles used C_v, and daily storage costs per item C_{H_i}.

Restrictions (2) ensure that deliveries of demands of all periods, for each of the customers are met by one and only one of the traveled routes. This delivery can take place on the same period or in previous periods.

Restrictions (3) impose that more than F vehicles in each period t are not used. Conservation flow is ensured by the restrictions (4).

Example 2. *The graph from Figure 3 represents a valid solution from Figure 2 of Example 1.*

These graphs have a dimension $W = 120$ which represents the duration of a working day, being 0 the beginning and W the end. The set of arcs corresponds to the represented traveled routes. Each vertice defines a time instant and each arch represents a route traveled to visit a series of customers. These flows are the arcs defined for the three periods, in the first one two vehicles perform a route each, in the second period a single vehicle performs two routes and in the third and final period a vehicle performs a single route. This is a valid solution for instance Example 1.

4 COMPUTATIONAL RESULTS

To evaluate the performance of the model, it was used a set of instances adapted from the literature (Moin et al., 2010).

A set of 32 instances was generated, and in all the duration of the planning horizon the working day is $W = 140$. There are two instances for each combination of parameters: number of customers $N \in \{10,20,40,50\}$ and number of periods of the planning horizon $\tau \in \{3,5,10,15\}$.

With this set of instances, different tests were performed varying the capacity of the vehicles. It is considered $C \in \{10,13,20\}$, so that, for all instances all the demands are higher than 20%, 15% and 10% of vehicle capacity, respectively. It was also considered an additional constraint on the maximum number of

Table 3: Summary of instances solved to optimality / instances in which the model found a solution / number of instances to solve.

N_{max}	$C = 10$	$C = 13$	$C = 20$
3	12/32/50	12/30/50	12/20/40
4	12/22/50	8/20/20	8/12/20
5	12/18/40	11/12/20	-
10	12/18/40	10/12/20	-

Table 4: Average values of instances solved with different parameters.

C	N_{max}	$\bar{t_r}$	$\bar{t_m}$	$\bar{t}_{total}$	$\bar{n_r}$	$\bar{var}$	$\bar{Lim}_{inf}$	$\bar{Lim}_{up}$	$\bar{gap}$ %	$\#opt$
10	20	6,28	59,64	65,92	13561,08	21970,00	2680,50	2680,50	0	12
13	20	538,65	196,46	735,17	51020,08	62688,58	2499,22	2507,67	0,17	10
10	3	0,51	31,71	32,21	8479,33	16844,25	2738,50	2738,50	0	12
10	4	2,90	90,23	93,13	12961,50	21370,42	2685,00	2685,00	0	12
10	5	6,20	57,88	64,08	13561,08	21970,00	2680,50	2680,50	0	12
13	3	1,25	45,52	46,78	17398,92	28893,67	2672,17	2672,17	0	12
13	4	12,20	390,00	402,22	38345,67	50014,17	2560,10	2576,83	0,61	8
13	5	66,44	151,35	217,80	49527,67	61196,17	2503,21	2507,33	0,09	11
20	3	4,51	36,00	40,53	53169,92	73678,50	2634,42	2634,42	0	12
20	4	95,99	399,87	495,92	209121,33	230754,58	2480,53	2514,42	1,44	8

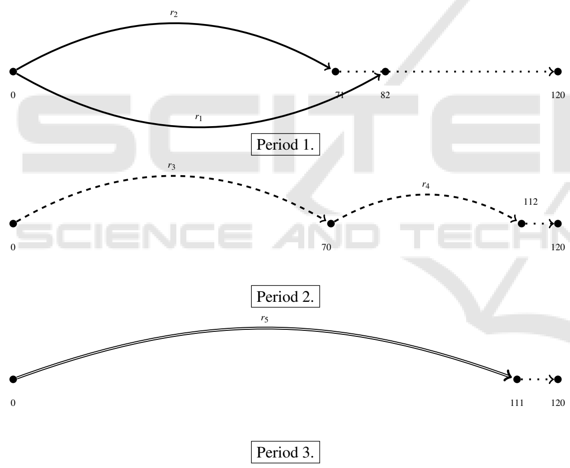

Figure 3: Solution of multi-trip inventory routing problem for three periods.

customers to visit in a route. In Tables 3 and 4, this parameter takes on values that do not restrict, or only limit slightly, the number of customers to consider in a route.

Computational tests were performed using a PC with i7 processor with 3.5 GHz and 32 GB of RAM. The optimization routines resorted to version 12.6.1 of CPLEX. The time limit for resolution of the integer programming model was 900 seconds. For the total time of model generation (including generation of routes) and its resolution it was set a time limit of 9500 seconds.

Tables 3 – 4 report the results obtained and their respective columns have the following meaning:

N_{max}: maximum number of customers to visit on a route;

t_r: generation time of routes;

t_m: execution time of the integer programming model (1) – (4);

t_{total}: total execution time ($t_m + t_m$);

n_r: number of routes generated;

var: number of variables of the integer programming model (1) – (4);

Lim_{inf}: best lower bound;

Lim_{up}: best upper bound;

gap %: gap (percentage);

In tests carried out without limit of customers to visit (N_{max}), where the vehicle capacity is 10 and 13 it was possible to solve, until the optimality, 24 of 18 and 10 of 12 instances, respectively. In this set of tests, where there is no restriction on the maximum number of customers to visit in a route, it was only possible to find a solution for instances with $N \leq 40$.

As expected, increasing the capacity of the vehicles hinders the generation and resolution of the model, since this variation increases the number of valid routes and, consequently, the number of variables. Instances for which it was not possible to find an optimal solution it was however possible to reduce the optimality gap. However, the maximum gap for all the parameters considered was equal to 8.85%.

Table 3 summarises the results obtained for all the instances and aggregates them by the different parameters. Each field in the table reflects the number of instances solved to optimality, the number of instances for which the model found a solution and the number of instances to solve for this combination.

Note that a company that solves this problem will just have to generate all the valid routes once for each set of customers considered, and what will change in practice are the demands.

5 CONCLUSIONS

The multi-trip inventory routing problem has a great practical interest in the industrial field, but on the other hand, it is quite challenging in terms of resolution.

In this paper, we propose a network flow model for multi-trip inventory routing problem, which is solved exactly for a set of adapted instances in the literature. The model was able to solve instances up to 50 customers and 15 time periods in reasonable computational times. Several instances were solved to optimality when set to different parameters. The average gap obtained was relatively low.

ACKNOWLEDGEMENTS

This work was supported by FEDER funding through the Programa Operacional Factores de Competitividade - COMPETE and by national funding through the Portuguese Science and Technology Foundation (FCT) in the scope of the project PTDC/EGE-GES/116676/2010.

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