Langrangian Relaxation of Multi Level Capacitated Lot Sizing Problem with Consideration of Lead Time

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Abstract: Tactical planning consists to develop production plans by determining the quantities of products manufactured by period to best meet customer demand at lower costs. This issue has been widely discussed, according to two criteria: multi level and single level planning. The concept of multi level reflects well the manufacturing structure. For this, we propose in this work a new mathematical model of lot sizing finite capacity (Multi Level Capacitated Lot Sizing Problem) based on Lagrangian relaxation optimization approach. Comparisons of this new model with traditional one demonstrate the efficiency of this new approach as well in simulated case as real situations. The generated production plans are optimal with 68% -98% compared to classical models.

1 INTRODUCTION

We are always looking for a relatively effective solution that can help increase the level of performance of production systems. This problem of planning and scheduling production with limited capacity resources reflects setup times, waiting and production as well as different costs of production.

The formulations of the Big Bucket model (BB) as Capacitated Lot Sizing Problem (CLSP) are considered as a reference model for addressing the plan generation of problematic production manager in a single site environment (Comelli et al. 2008), the Multi Level Capacitated Lot Sizing Problem (MLCLSP), is recognized as a reference model and deals with Manufacturing Resources Planning (MRP and MRPII) issues, can be found in the literature for some studies this problem (Almeder et al.2011, Berretta et al.2005, Chen &Chu2003). If single site issues have been extensively studied in the literature (Nascimento et al.2010) point to the lack of a reference model for multi site issues. This model only determines the production quantities and periods, regardless of the actual production sequence of commands within a period of time. This type of modeling has the advantage that it allows flexible sequencing orders in a period, a significant cost calculation. For small-Bucket models (SB) are known, the Continuous Setup Lot Sizing Problem (PRSP) and the Proportional Lot Sizing and Scheduling Problem (PLSP).

The problem Multi Level Capacitated Lot Sizing Problem (MLCLSP) is one of the most difficult optimization problems known in the production. It arises in any company that uses the sequential approach to planning MRP. The approach was based on the control of quantities on demand, compliance with the BOM structure and the level of stock.

Consideration of production capacity and product classifications pushed the authors to consider the multiplicity of production resources (Buschkuhl et al.2008), and we will find models of mono or multi resource. Multi resource models enable more accurate modeling of the operating range of the various products and better estimate the capacity of the production system. (Bel 1998) demonstrated that to find feasible solutions for MLCLSP is NP-complete, and when there are considered setup time. Therefore, the proposed Lagrangian heuristics include a feasibility strategy to find feasible solutions from the penalization of the constraints of the problem.

Most of the models and algorithms proposed MLCLSP (Tempelmeier et al.2008) rely on one of two possibilities: either the lead time is neglected, and the lead time is taken into account, there is at least one period for each component, forcing the transit time (the number of periods) of finished

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products to be at least equal to the number of levels of the BOM.

According to studies, the hypothesis of zero lead time leads to plans that are not feasible, and the lower level of scheduling problem is infeasible. On the other side, if the lead time is positive, it usually results in significant amounts of work in process, causing subsequently increase the number of levels in the nomenclature. This problem has been studied by different researchers (Buschkühl et al.2010).

A recent study on the different model formulations and solution methods for MLCSP is in (Almeder et al.2014), the authors proposed two models: one is batching and the other is streaming lot. Computations indicate clearly that the solutions obtained by MLCLSP are infeasible, and the lead time excessive work in process from 15% to 60% increase in the overall cost, they deployed the algorithms Benders (Almeder et al.2014) variant with a sanctions significant contribution to improve the computational effort to find satisfactory solutions.

The main contributions of our study are as follows: We show that the solutions obtained with lead time are feasible. We propose a linear programming formulation integer. Regarding the approach of the Lagrangian relaxation is to relax capacity constraints while penalizing their violation in the objective function, our experiences show that calculation variant polarization capacity constraints give the best solution of the problem. The comparison with traditional models, we demonstrate the ability of this new approach with more realistic results (68% -98 %).

In Section 2, we propose the new formulations of the MLCLSP problem with major constraints. In Section 3, we present our optimization approach to problem solving with the new formulation of the model, and in section 4, we apply the standard reference instances and we compare the results with those of classical MLCLSP.

2 PLANNING MODEL GENERIC

2.1 Classic Models

In supply chain management, support for the medium term decision can be considered in the construction of a solution (such a plan) it is to build and generate a solution. The generative of this approach is shown in Figure 1 (Sambasivan et al.2002).

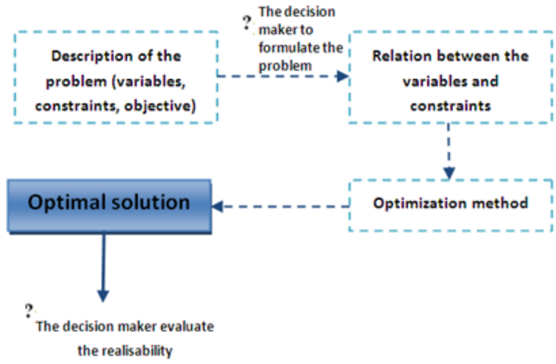

Figure 1: Generative approach (Sambasivan et al.2002).

Production management, where the models were more widely used, the approach described above concerning the planning model. We use planning model into an optical performance evaluation to build an optimal plan.

In this article, we present the classical model of MLCLSP planning aims to generate a production plan that minimizes the sum of setup costs and inventory costs, while respecting the constraints of stock and capacity.

The objective function of MLCLSP model is as follows:

$$\textbf{Min} \sum_{i=1}^{N} \sum_{t=1}^{T} (C_i.Y_{it}+H_i.I_{it}) \qquad (1)$$

Under the constraints:

$$I_{it+1}=I_{it}+X_{i(t-l_i)}-\sum_{j=1}^{N} a_{ij}*X_{jt}-D_{it} \qquad i,t \qquad (2)$$

$$\sum_{i=1}^{N} P_i*X_{it} \leq C_t \qquad i,t \qquad (3)$$

$$X_{it} \leq G*Y_{it} \qquad i,t \qquad (4)$$

$$I_{it} \geq 0 \,,\ X_{it} \geq 0 \qquad i,t \qquad (5)$$

$$Y_{it} \in \{0,1\} \qquad i,t \qquad (6)$$

Model parameters:

i : Product

t: Period

P_i: Time to produce a unit of product i

C_i: Cost of setup of product i

D_{it}: Demand for the product i at time t (external)

G: Arbitrarily large number (e.g., total demand or maximum capacity)

H_i: The cost of stock of product i

C_t: The available capacity (time) at time t

l_i: Lead time of item i (non negative integer corresponding to the number of periods)

a_{ij} : Amount of product i to produce a unit of product j (gozinto-factor).

Decision variables:

I_{it}: Stock level of product at the end of period t

X_{it}: Quantity of product i in period t

$$Y_{it} \begin{cases} 1 \text{ if the product and manufactured in the period} \\ 0 \text{ no production} \end{cases}$$

The objective function (1) minimizes the total cost involved in the production plan, namely the costs of production and storage, as well as fixed setup costs, inventory costs. Constraint (2) expresses the conservation of flux across the horizon with the stress of lead time. Constraint (3) expresses the fact that the plan that we would calculate to be finite capacity. Indeed, for the realization of a plan, we have an amount of resources that will be consumed by the production of one or more references. Total consumption should be less than the available capacity. Constraint (4) to model the following condition: if the setup of production, while the quantity produced must not exceed an upper bound of the output G. This represents the minimum between the maximum amount of the reference can be produced and the total demand on the horizon [t, .., T]. Constraint (5) means that X_{it} and I_{it} variables are continuous no negative for any reference i, for each period t. The last constraint (6) expresses the fact that Y_{it} is a binary variable for any reference i in each period t.

Many researchers are studying this model, assume that the lead time is negligible, to the effect that the predecessors and successors could be produced in the same period (l = 0 for all i). The MLCLSP is a model BB and the periods are supposed to cover the long time intervals with a number of production batches, so it would result in significant amounts of work in production. If we assume that the lead time is positive for at least one period (l = 1 for all i) , we deliver the requested quantity respecting the delivery time , There is always in practice, there is always the possibility of use of overtime to make production in our model the objective function (1) does not account for additional fabrication.

To verify the model, the example of Table 1 represents a lot sizing problem in 2 periods, 4 products and unconstrained capacity, Figure 2 shows the structure of the nomenclature (Almeder et al. 2014).

Table 1: Data example.

Product	D_{i1}	D_{i2}	H_i	C_i	P_i
1	3	0	3	5	0.1
2	0	2	2	5	0.1
3	0	0	2	5	0.1
4	0	0	1	5	0.1

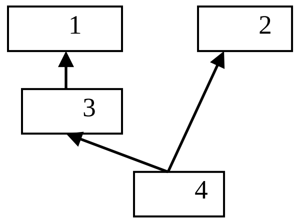

Figure 2: Example of nomenclature.

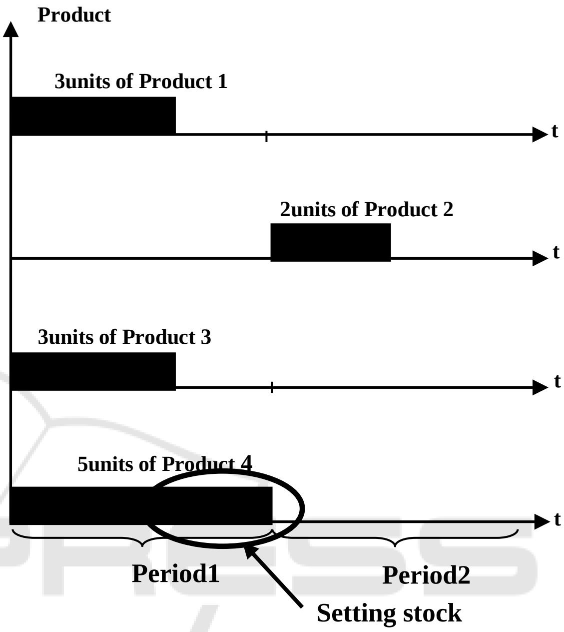

Figure 3: Limitation of the Multi Level Capacitated Lot Sizing Problem.

The configuration presented in Figure 3 provides the solution of the classical model, if we assume that the lead time is negligible, and no setup change to produce the product 4 , but with this solution we do not respond to the objective function to optimize the cost of the stock.

With this plan in Figure 4, we produce only the amount requested for each period, for example in the period 1 we produce an amount of 3 units of product 4, we change the setup to produce product 3 and subsequently the product 1, which results in a delay in customer demand level for product 1 because production will end in period 2. So with this solution we are not optimizing the cost of setup and customer demand.

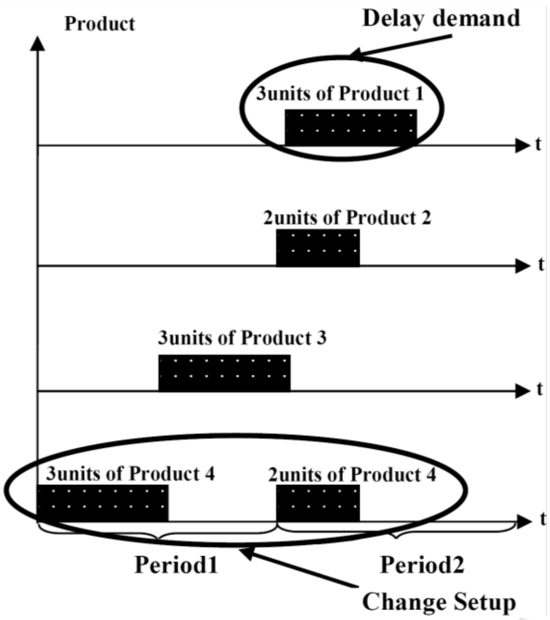

Figure 4: Limitation of the Multi Level Capacitated Lot Sizing Problem.

2.2 Proposed Model

If we consider the two plans proposed above, we notice that the constraint that violates our optimization objective function is the constraint of stock (2), so we propose a new formulation which provides an optimal solution for our model, applying the product of the net principle need i at time t, we produce the necessary need in period t, and our formulation we consider a lead time period to meet customer delivery time.

The proposed new constraint:

$$I_{it+1} = D_{it} - I_{it} - X_{i(t-li)} \quad i,t \qquad (7)$$

This plan in Figure 5, allows us to produce our net needs at time t, if we take our example, our net need for the period 1 of the finished product 1 is 3 units, so for the produce we must have 3 units of subsets 3 and 4, with this solution we gain at the cost of stock, the same principle to produce the finished product 2.

We implemented the proposed model in CPLEX 12.2 (User's Guide Standard Version Including CPLEX Directives.2010), to see if our new formulation is optimized for our objective function. After the simulation, results are optimized with a target of 40.

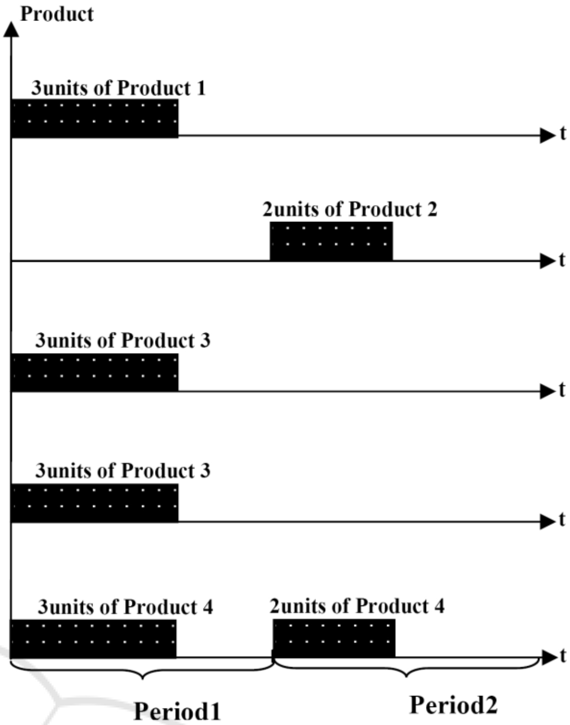

Figure 5: Verification of our new formulation.

```
  // solution (optimal) with objective 40
  plan[1][1]=
<fabrication:1,production:3,stock:0>
  plan[1][2]=
<fabrication:1,production:0,stock:0>
  plan[2][1]=
<fabrication:1,production:0,stock:0>
  plan[2][2]=
<fabrication:1,production:0,stock:0>
  plan[3][1]=
<fabrication:1,production:0,stock:0>
  plan[3][2]=
<fabrication:1,production:0,stock:0>
  plan[4][1]=
<fabrication:1,production:0,stock:0>
  plan[4][2]=
<fabrication:1,production:0,stock:0>
```

Figure 6: Simulation results with CPLEX Solver.

3 OPTIMIZITION METHOD

3.1 Lagrangian Relaxation

The Lagrange relaxation technique has been the subject of several studies (Fisher, M 2004) and was raised in the problems of integration of production.

This method can be used to approximate solutions, seek a lower bound (ZLB) of the problem or to obtain more optimal solutions. (Wu et al.2013) developed a strategy for finding upper bounds (ZUB) on the relaxation using Lagrangian for a lot sizing problem with multiple products. A metaheuristic was proposed in (Toledo et al.2014) to resolve an extension of CLSP with to carry over. (Nascimento et al.2010) proposed a strategy that incorporates a genetic algorithm with a linear program to find approximate solutions to a lot sizing level problem and scheduling.

The approach of the Lagrangian relaxation is to relax a subset of constraints while penalizing their violation in the objective function by associating a Lagrangian multiplier λit.

As shown in section 2, our new formulation of MLCLSP model allows us to achieve an optimal plan, but draws the MLCLSP is a BB model and periods are supposed to cover long intervals with several production batches, so it would lead to a significant production capacity, and our equation (3) expresses that our plan must be calculated with finite capacity. That is why the approach of the Lagrangian relaxation will be based on the penalization of capacity constraints. (Berretta et al.2005) presented a heuristic based on the Lagrangian relaxation of the capacity constraints of the mathematical formulation. To find a lower bound (ZLB) for the problem.

Our news formulation of the objective function is:

$$\textbf{Min}\sum_{i=1}^{N}\sum_{t=1}^{T}(C_i.Y_{it}+H_i.I_{it}) + \sum_{i=1}^{N}\sum_{t=1}^{T}\lambda_{it}(C_t-\sum_{i=1}^{N}P_i*X_{it}) \quad (8)$$

Under the constraints:

$$I_{it+1}= D_{it}-I_{it}-X_{i(t-li)} \quad \forall i,t \quad (9)$$

$$X_{it} \leq G*Y_{it} \quad \forall i,t \quad (10)$$

$$I_{it}\geq 0,\ X_{it}\geq 0 \quad \forall i,t \quad (11)$$

$$Y_{it}\in\{0,1\} \quad \forall i,t \quad (12)$$

(Sambasivan et al.2005) points out three main approaches in his study: the sub-gradient method and the multiplier λ_{it} adjustment method. This last and according to (Fisher, M 2004) proved to be too costly compared to the sub-gradient method. Although the adjustment method has a high potential, exceeding the sub-gradient method in some case studies, but the sub-gradient is the most used to determine the Lagrange multipliers tool.

So to solve this dual problem, the method chosen is the sub-gradient. The sub-gradient algorithm introduced in (8). It updates iteratively multipliers:

$$\lambda_{it}= \max\{0; \lambda_{it}+TG_i\} \quad \forall i,t \quad (13)$$

T is the step of the iteration method, and G_i is the difference between the time required to produce all units of product i in period t and the capacity limit in period t, calculated in Equation (14):

$$G_i= C_t-\sum_{i=1}^{N}P_i*X_{it} \quad \forall i,t \quad (14)$$

It is necessary to initialize the values T and λ_{it} for each iteration. The step T is important to optimize our solution.

The choice of the step size T, is of importance for the convergence of the sub-gradient method for the optimal solution. Thus, the T update is given by the equation below:

$$T =\pi\ (ZUB-ZLB)/G_i^2 \quad (15)$$

Figure 4 shows the algorithm of the principle of our approach.

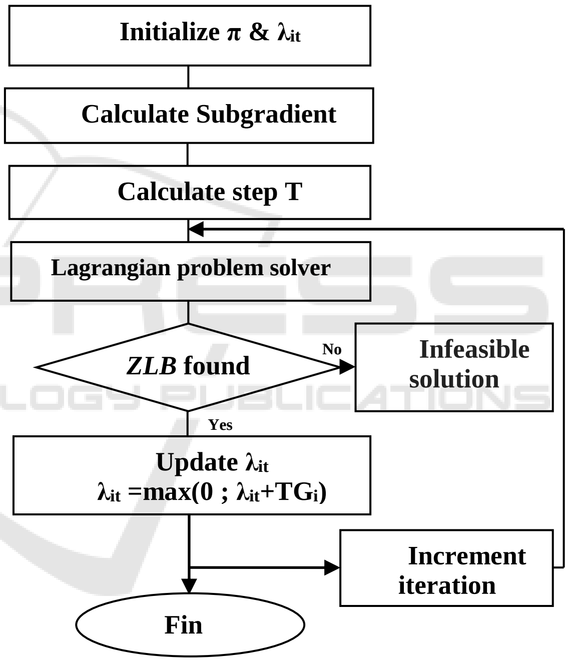

Figure 7: Flowchart of the principle of our approach.

To ensure the convergence of the method, the solutions at each iteration step means that the T tends to 0. According to the equation (15) T depends the upper bound (ZUB) and the lower bound (ZLB), if no lower bound is found to iterate , so the solution is infeasible. (Almeder,C.2010) most BB models provide the best lower bound.

Following algorithm shows a pseudo code of our approach to feasibility of the solution:

```
Data: Approximate dual solution
Result: Either a heuristic solution for
the primal problem or infeasible
solution
```

```
   Repeat
Initialize arrays and variables used in
the loop that follows
   Initialize π ]0; 2]
   Initialize λit values
   Executes Lower Bound and Upper Bound
model
   Calc sub-gradient Gi= Ct-∑Pi*Xit
   Calc T = π(ZUB-ZLB)/Gi²
   Solve the model to get the Upper
Bound
Update λit to pass it as input data to
Lower Bound model in next iteration
   End loop
```

4 SIMULATION

The optimal solution of the lot sizing problem presented in Table 3 is obtained using Lagrangian relaxation. The results are obtained by the linear programming solver integer CPLEX 12.2 (User's Guide Standard Version Including CPLEX Directives.2010).

All tests were implemented in C ++ and run on a PC with 4G HP Core i5 processor.

4.1 Test Instances

We have carried out tests on a series of instances of (C.Almeder et al.2014). The characteristics of bodies are described in Table 2.

Table 2: Instances of C.Almeder et al. (2014).

Class	Instances	Periods	Products
A	1500	4	10
B	600	4	10
C	599	16	40
D	573	16	40

4.2 Analysis of Proposed Method

The model MLCLSP feasibility problem is to consider a minimum period of lead time. As mentioned previously this period may cause amounts of work in production, then a substantial increase in the stock.

(C.Almeder et al.2014) demonstrated the infeasibility of the MLCLSP problem by running the model with conventional test instances without synchronization with a period of lead time. With a run time limit of 10 minutes for CPLEX.

This document puts the Lagrangian strategies for the upper and lower bounds of good quality for cases tested and shows that our proposed approach will guarantee us an optimal solution for the proposed production plans with consideration of one period of lead time.

Table 3 presents results obtained by CPLEX Solver and table 4 presents the resulting solutions for our approach, the columns show the lower bound greater than the percentage of optimality GAP = (ZUB- ZLB) / ZUB for each test class to improve our cost model by considering a period of lead time.

Table 3: Simulation with CPLEX Solver.

Class (Instance)	CPLEX	
	GAP	Time
A(1500)	14%	30s
B(600)	35,25%	9s
C(599)	infeasible	_
D(573)	infeasible	_

Table 4: Simulation with Lagrangian approach.

Class (Instance)	RL			
	ZUB	ZLB	GAP	Time
A(1500)	205,5	26	87,34%	5s
B(600)	206,5	66	68,03%	6s
C(599)	16363	808	95%	25s
D(573)	16362	168	98,97%	26s

Table 4 presents the best values of the proposed method of MLCLSP. So with the relaxation of Lagrangian approach, we find optimal solutions with 87.34% for Class A, 68.03% for Class B, 95% for Class C and 98.97% for Class D.

Table 3 shows that the solutions implemented in CPLEX, are optimal with 14% for Class A and 35.25% for Class B, but for Class C and Class D solutions are infeasible.

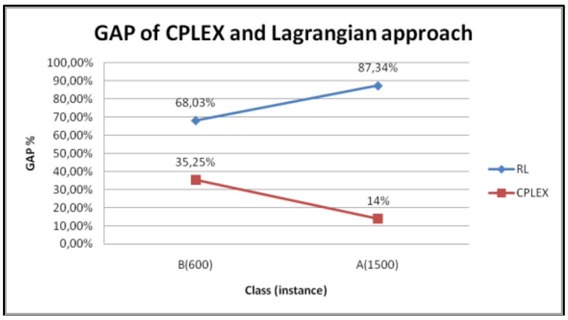

Figure 8: Results obtained for SB.

This graph shows that with the Lagrangian approach we obtain the best optimal solutions for the model

SB, even if we increase the instance. On the other hand, we observe that the results obtained with the CPLEX solver are lower and decrease with the increase of the instance.

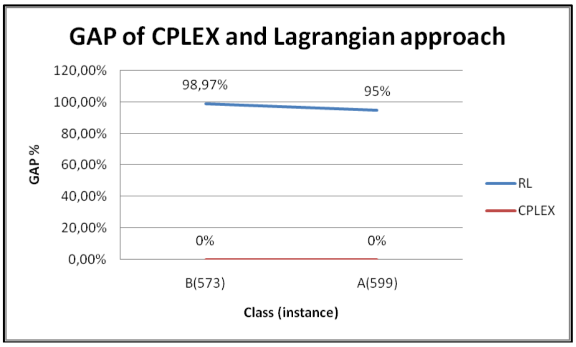

Figure 9: Results obtained for BB.

On this graph, we show that for the model BB our plan remains optimal with the Lagrangian approach, but for CPLEX Solver the solution is infeasible.

So we show with our new formulation, BB or SB model is optimized, resulting in a clear cost estimate from stock and production and in parallel we are responding to the constraint of important quantities of production work, which request the use of overtime to make up for production.

Comparing the results with those obtained by the classical MLCLSP, we realize a 4.5% increase to that produced by the Benders algorithms proposed by (C.Almeder et al.2014).

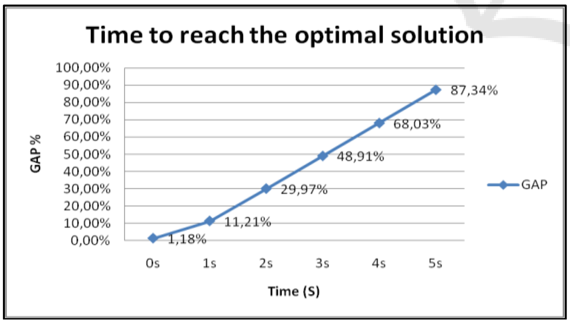

Figure 10: The trend for SB.

The Proposed formulation we guarantee an optimal solution after less 5s for SB and less than 25s for BB. The penalization of capacity constraint provides lower bound to optimize our objective function even if we consider the lead time. Figures 10 and 11, we show the improvement of the objective value of the best solution for the cases studied. Trends show that the improvement is achieved in the initial iterations.

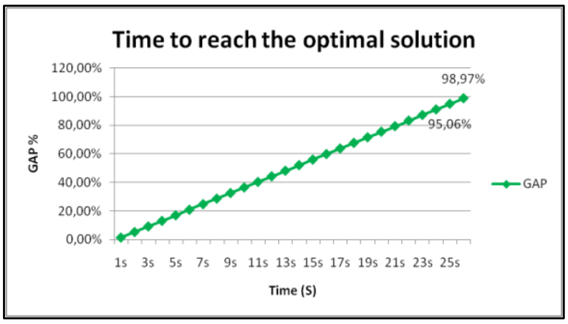

Figure 11: The trend for BB.

5 CONCLUSION

This paper presented a new formulation and solution for the multi level capacitated lot sizing problem (MLCLSP) with considering one period of lead time that requires an important quantity of production work requesting thus the use of overtime to make up for production.

Besides, the proposed optimization method of our problem was Lagrangian relaxation based on the penalization capacity constraints. The performance on several classes with different instances generated was compared with the CPLEX Solver, and the results show the efficiency of 68% to 98.97%. It should be noted that the proposed model is more suitable with the production constraints of planning. We also remember that this approach is now under implementation for real situations.

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