

# Mathematical Modeling of the Ethno-social Conflicts by Non-linear Dynamics

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**Abstract:** The issue of modeling various kinds of social conflicts (including ethno-social) using diffusion equations is discussed. The main approaches to and methods of mathematical modeling in contemporary humanitarian sciences. The main concepts of social conflicts, ways of their classification, interpretation, including ethnic-social, religious and other conflicts are considered. The notion of a conflict in a social system is defined in terms of mathematical modeling. A model based on Langevin diffusion equation is introduced. The model is based on the idea that all individuals in a society interact by means of a communication field -  $h$ . This field is induced by each individual in the society, modeling informational interaction between individuals. An analytical solution of the system of thus obtained equations in the first approximation for a diverging type of diffusion is given. It is shown that even analyzing a simple example of the interaction of two groups of individuals the developed model makes it possible to discover characteristic laws of a conflict in a social system, to determine the effect of social distance in a society on the conditions of generation of such processes, accounting for external effects or a random factor. Based on the analysis of the phase portraits obtained by modeling, it is concluded that there exists a stability region within which the social system is stable and non-conflictive.

## 1 INTRODUCTION

A social conflict can be defined as a peak stage in the development of contradictions in relations between individuals, groups of individuals, or a society as a whole, characterized by the presence of contradicting interests, objectives and viewpoints of the interacting subjects. Conflicts can be latent or explicit, and are caused by lack of a compromise or sometimes even a dialogue between the two or more parties involved (Petukhov, 2015a).

The English sociologist E. Giddens introduced the following definition of a conflict: "a social conflict is understood as real struggle between interacting people or groups, no matter what its causes, ways and means used by each of the involved parties are".

Works of the foreign scientists that became fundamental in analyzing practical problems of this complex inter-disciplinary science played an important part in the development of general conflictology at the present stage. These are the classical works of L. Coser, R. Darendorf, U. Habermas, G. Bekker, A.S. Ahiezer, who

substantiated a natural and attributive character of ethnic-political conflicts and their functions in the life of a society; K Boulding, L. Kozler, P. Bourdier, who laid the groundwork for developing a general theory of conflicts; J. Burton and his followers, who addressed the issue of effective practical technologies of settling and principal resolution of conflicts as the first-priority one for making the conflictological knowledge effective; P. Schtompke, who absolutized the "western main road" of social salvation; F. Glazl, who introduced modern mechanisms of solving conflicts.

The issues of studying, classifying and, most important, predicting conflicts have always been of importance in fundamental sociology. This issue was addressed in numerous works of the leading sociologists and mathematicians: J. Bernard, R. Bailey, K. Boulding, D. Bucher, J. Duke, L. Coser, L. Krisberg, D. Leidis, R. Makk, A. Rapoport, R. Snamayer, R. Stagner, T. Shelling, T. Bottmore, J. Rex, G. Boutoul, M. Crosieau, A. Touren, K. Darendorf, E. Vyatr, Y. Moukha, Y. Sctumski, Y. Reykovski, L.A. Nechiporenko, I.I. et al. (Davydov, 2008; Kravchenko, 2003; Shabrov, 1996).

Taking into account the important impact of such phenomena on a society and all the processes inside it, any ways of predicting and discovering characteristic laws of social conflicts are certainly of a paramount importance.

One of the directions of searching for possible solutions of this problem is forecasting and describing a social conflict mathematically, i.e., using mathematical modeling (Shabrov, 1996; Mason, 2013; Blauberg, 1973; Saati, 1991; Bloomfield, 1997; Plotnitskiy, 2001).

## 2 MATHEMATICAL MODELING IN SOCIAL SCIENCE

Mathematical modeling based on nonlinear dynamics, which is widely used in natural sciences, is still rather rarely resorted to in sociological studies.

In the recent years, considerable success has been achieved in the field of developing models of social and political processes (Plotnitskiy, 2001).

The already available models can be tentatively subdivided into three groups:

1) models-concepts based on discovering and analyzing general historic laws and representing them in the form of cognitive schemes describing logical relations between various factors affecting historical processes (G. Goldstain, I. Wallerstein, L.N. Goumilyov, N.S. Rozov and others). Such models feature a high degree of generalization, but are of a purely logical, conceptual, rather than mathematical, nature;

2) special mathematical models of the simulation type, designed for describing particular historical events and phenomena (Y.N. Pavlovskiy, L.I. Borodkin, D. Meadows, J. Forrester and others). In such models, the main attention is paid to carefully accounting for and describing factors and processes affecting the studied phenomena. The applicability of such models is, as a rule, limited by a fairly short spatial and time interval; they are ‘tied up’ to a particular historical event and cannot be extrapolated onto longer periods of time;

3) mathematical models, intermediate between the two mentioned types. These models describe a certain class of social processes without giving a detailed description of the features characteristic for each specific historical event. They are designed for discovering basic laws characterizing the processes of the type in question. Accordingly, such mathematical models are called basic (Malkov, 2004).

In the classical models, the dynamics of nonlinear systems is modeled using multidimensional differential equations, difference equations, the mathematical apparatus of cellular automation, the mathematical apparatus of the catastrophe theory, the mathematical apparatus of the self-organized criticality theory, stochastic differential equations of Langevin and Ito-Stratonovich, analysis of systems with chaos and reconstruction of stable states (attractors) along time series (Malkov, 2004; Haken, 1985; Ebeding, 1979).

Holyst J.A., Kacperski K., Schweiter F. presented an effective model of social opinion, based on representing the interaction between individuals in the form of Brownian motion (Holyst, 2000).

There are also other numerous studies in the field of modeling social and political processes published by K. Troitzsch, R. Hegselmann, P. de Vries, D. Gernert, A. Nowak, R. Vallacher and E. Burnstein, H. Ader and I. Bramsen, Y.-F. Yung, W. Chan and P. Bentler, R. Geuze, R. van Ouwwerkerk and L. Mulder, A. Klovdahl and many others (Mikhailov, 2012; Gutz, 2000).

## 3 THE MAIN CONCEPTS OF A SOCIAL CONFLICT

The contemporary literature on sociology abounds in classifications of types of conflicts according to various grounds. Consider some of them from the viewpoint of defining a social conflict as a mathematical notion in our model.

From the viewpoint of subjects involved in a conflict, four types of conflicts can be discerned:

1) intrapersonal conflict (it can appear in the following forms: a role conflict – it appears when contradictory requirements are imposed on a person, regarding what the result of his/her work has to be; intrapersonal – it can result from a mismatch between the working requirements and the individual’s needs and values);

2) interpersonal conflict (it can be manifested in the form of a clash of individuals having different characters, views or values, and is the most common one);

3) conflict between an individual and a group (when an individual assumes a position differing from the position of the group);

4) intergroup conflict.

Conflicts can also be classified according to the spheres of activity as: political, social-economic, national-ethnic and others (Malkov, 2004).

There are quite a few concepts of the theory of social conflict. Some of the best-known of them are:

L. Coser's concepts:

- in any society there exists inevitable inequality, permanent psychological discontent of its members, interpersonal and intergroup tension (emotional, psychic disorder), leading to social conflict;
- social conflict as incongruity between the reality and ideas of various social groups or individuals about what it should be like;
- social conflict as struggle for values and pretensions to a certain status, power and resources, in which the antagonists aim at neutralizing, damaging or eliminating the opponent (Coser, 2000).

Conflict model of society by R. Darendorf:

- permanent social fluctuations in society, suffering social conflict;
- any society is based on making some of its members obey other members = inequality of social positions in the distribution of power;
- difference in the social position of various social groups and individuals leading to reciprocal tensions and contradictions resulting in the alteration of the social structure of the society (Darendorf, 1994);.

General theory of conflict by K. Boulding:

- all conflicts have common development patterns; their detailed study and analysis makes it possible to develop a generalized theory – “the general theory of conflict” which will allow society to control conflicts, manage them and predict their consequences;
- Boulding argues that conflict is an intrinsic part of social life (striving for struggling with the similar is in the human nature);
- a conflict is a situation in which each of the parties tries to adopt an attitude which is incompatible and contrary in respect to the interest of the other party;
- two aspects of social conflict: static and dynamic:

The static aspect is the analysis of the parties (subjects) involved in the conflict (individuals, organizations, groups) and relations between them (classification: ethnic, confessional, professional).

The dynamic aspect studies interests of the parties as stimuli for conflictive behavior of people. The definition of the conflict dynamics is a set of responses to external stimuli (Boulding, 1969).

From the above said, the following important for our model conclusions can be drawn:

1. A large social conflict is initiated mainly by an informational and social distance between individuals or groups of individuals. A basis for such a distance can root in ethnic, cultural, confessional, as well as economic dissimilarities.

2. This distance increases in the process of conflict, especially in its extremal forms (revolutions, civil wars etc.), bringing the opposing parties to the attitude of irreconcilability. Unfortunately, history knows very few examples of short- and medium-term positive scenarios for such situations.

3. Hence, the point of no return in question is somewhere before the initiation of conflict, and this transition of a social system from one state to another is determining.

## 4 MATHEMATICAL MODEL

For mathematical modeling, an important point is that social and political processes cannot be rigorously assigned. They tend to be subjected to minor changes and fluctuations. Using analogy, a social process is similar to a Brownian particle, i.e., a particle moving along a fairly definite trajectory which, on closer examination, is highly winding and broken. These small fluctuations are explained by chaotic motion of other molecules. In social processes, fluctuations can be assumed as manifestations of free will of its individual participants, as well as other random manifestations of the external medium (Gutz, 2000).

In physics, such processes are generally described using Langevin stochastic diffusion equation, which is also, to a certain degree, tested for modeling some social processes. For example, Holyst J.A., Kacperski K. and Schweiter F. developed a model of social opinion (Holyst, 2000).

The model is based on the idea that individuals of a society interact by means of a communication field (similar to (Holyst, 2000)). This field is induced by each individual of the society, modeling informational interaction between individuals. However, it should be kept in mind that society, which is considered here, can hardly be viewed as an object in classical physical spatial topology. Really, in terms of transfer of information from individual to individual, space in society has both classical spatial coordinates and some additional specific characteristics. It is because of the fact that in the contemporary informational world it is not necessary to be near the object to transfer information to him/her.

Thus, society is a multidimensional, social-physical space reflecting a possibility of one individual to “reach” another individual with his/her communication field, that is, to affect him/her, his/her parameters and possibility to move in this space. Accordingly, the position of an individual relative

other individuals also models the level of relations between them and their involvement in the informational exchange. Close positions in this models show that there is regular exchange of information and social connection between them. With such a formulation of the problem, a conflict can be considered to be a type of interaction between individuals or groups thereof, which results in a sharp increase of the distance (i.e., social distance –  $\Delta x = x_i - x_j$ , where  $x$  is a coordinate in social-physical space,  $i, j = [1, N]$ , where  $N$  is number of individuals or consolidated groups of individuals) between them, and a further increase of the distance testifies to increasing conflict.

Thus, a communication field can be represented with a diffusion equation as follows:

$$\begin{aligned} & \frac{\partial}{\partial t} h(x_i, t) \\ &= \sum_{j=1}^N f(x_i, x_j) \vartheta(x_i, x_j) \bar{\delta}_{(k_s^i+k_c^i), (k_s^j+k_c^j)} \\ &+ D(h(x_i, t) - h(x_i, t_0)), \end{aligned} \quad (1)$$

In which a divergent type of diffusion is assigned, and function

$$\vartheta(x_i, x_j) = \frac{1}{\varepsilon\sqrt{\pi}} e^{-\frac{(x_i-x_j)^2}{\varepsilon^2}},$$

is used instead of delta-function; for ( $\varepsilon \rightarrow 0$ ) it asymptotically tends to the latter, considerably simplifying the process of computer modeling.

Function  $f(x_i, x_j)$  characterizes relation between individuals, which is modeled here using classical Gaussian distribution

$$\begin{aligned} f(x_i, x_j) &= \frac{1}{u\sqrt{\pi}} e^{-\frac{(x_i-x_j)^2}{u^2}}, \\ u &= k_c^i k_s^i + k_c^j k_s^j, \end{aligned}$$

fairly widely used in various sociological studies.  $k_s^i$  is coefficient of scientific-technological progress and development of the  $i$ -th individual/group of individuals.  $k_c^i$  is coefficient of social activity of the  $i$ -th individual/group of individual.  $\bar{\delta}$  is inverse Kronecker delta.

Coefficients  $k_s$  and  $k_c$  are used for each separate individual or group in the system, and a total coefficient of the entire system is found by fractal transformation of their values of all individuals and clusters of the system (Petukhov, 2015a; Petukhov, 2015b; Petukhov, 2016a; Petukhov, 2016b).

Translations of an individual are described using Langevin equation:

$$\frac{dx_i}{dt} = k_c^i k_s^i \left( \sum_{j=1, j \neq i}^N \frac{\partial}{\partial x_j} h(x_j, t) \right) + \sqrt{2D} \xi_i(t), \quad (2)$$

where stochastic force  $\xi_i(t)$  is introduced, which models a random factor in society, and, in particular cases, external effects on individuals.

When solving equations (1) and (2), differential equation

$$\frac{dh}{dt} = \frac{\partial h}{\partial x_i} \frac{dx_i}{dt} + \frac{\partial h}{\partial t}$$

should also be taken into account.

In a general case, initial conditions for equations (1) - (3) can be taken as follows:

$$x_i|_{t=0} = x_{0i} \quad h(x_i, t = 0) = h_{0i}.$$

It is also necessary to assign a range of characteristic parameters  $0 < k_c, k_s, D < 1$  (individual distribution).

## 5 APPROXIMATE SOLUTION OF THE SYSTEM

For a simplest model of two interacting individuals or two consolidated groups of individuals (i.e., belonging to the same social, confessional, ethnic etc. group), assumed to be in a state of conflict, accounting for external effects, equations (1) and (2) can be written as:

if

$$\psi = k_c^1 + k_s^1 + k_c^2 + k_s^2, \alpha = \frac{1}{\psi\sqrt{\pi}} \bar{\delta}_{k_c^1+k_s^1, k_c^2+k_s^2},$$

then

$$\begin{cases} \frac{\partial h(x_1, t)}{\partial t} = D [h(x_1, t) - h(x_1, 0)] + \\ + \alpha k_c^2 k_s^1 e^{-\frac{\psi^2+1}{\psi^2}(x_1-x_2)^2}, \\ \frac{\partial h(x_2, t)}{\partial t} = D [h(x_2, t) - h(x_2, 0)] + \\ + \alpha k_c^1 k_s^2 e^{-\frac{\psi^2+1}{\psi^2}(x_1-x_2)^2}, \\ \frac{dx_1}{dt} = k_c^1 k_s^1 \frac{\partial h(x_2, t)}{\partial x_2} + \sqrt{2D} \xi_1(t), \\ \frac{dx_2}{dt} = k_c^2 k_s^2 \frac{\partial h(x_1, t)}{\partial x_1} + \sqrt{2D} \xi_2(t), \end{cases} \quad (3)$$

To obtain approximate analytical solutions of system of equations (3), series expansion is used with the accuracy of up to the quantity of the first order of smallness for  $\Delta x = x_i - x_{0i}, \Delta t = t - 0$  of difference

$$h(x_i, t) - h(x_{0i}, t) \approx \left(\frac{\partial h}{\partial x_i}\right) \Big|_{x_i=x_{0i}} \Delta x + \left(\frac{\partial h}{\partial t}\right) \Big|_{x_i=x_{0i}} \Delta t, \tag{4}$$

Then, assuming the following initial conditions:

$$x_{0i} = 0, h_0(x_{0i}, 0) = \left(\frac{\partial h}{\partial x_i}\right) \Big|_{x_i=x_{0i}} = \left(\frac{\partial h}{\partial t}\right) \Big|_{x_i=x_{0i}} = 1, \tag{5}$$

the first two equations of system (3) are integrated, using (4), (5), after which the following expression results:

$$h(x_i, t) = D \int_0^t x_i(u) du + D \frac{t^2}{2} + \alpha k_c^j k_s^i \int_0^t e^{-\frac{\psi^2+1}{\psi^2}(x_i(u)-x_j(u))^2} du, \tag{6}$$

$j = 3 - i.$

Using expression (6), the last two equations of system (3) can be transformed, based on the continuity of all the functions, into the following form:

$$\begin{cases} \frac{dx_1}{dt} = k_c^1 k_s^1 \left[ D + 2\alpha k_c^1 k_s^2 \frac{\psi^2+1}{\psi^2} \int_0^t (x_1(u) - x_2(u)) e^{-\frac{\psi^2+1}{\psi^2}(x_1(u)-x_2(u))^2} du \right] + \sqrt{2D} \frac{d\xi_1(t)}{dt}, \\ \frac{dx_2}{dt} = k_c^2 k_s^2 \left[ D + 2\alpha k_c^2 k_s^1 \frac{\psi^2+1}{\psi^2} \int_0^t (x_2(u) - x_1(u)) e^{-\frac{\psi^2+1}{\psi^2}(x_1(u)-x_2(u))^2} du \right] + \sqrt{2D} \frac{d\xi_2(t)}{dt}. \end{cases} \tag{7}$$

After time differentiation of (7), the following forms of differential equations are obtained:

$$\begin{cases} \frac{d^2 x_1}{dt^2} = k_c^1 k_s^1 D + \frac{2\alpha(\psi^2+1)k_c^1 k_s^1 k_c^2 k_s^2}{\psi^2} (x_1 - x_2) e^{-\frac{\psi^2+1}{\psi^2}(x_1-x_2)^2} + \sqrt{2D} \frac{d\xi_1(t)}{dt}, \\ \frac{d^2 x_2}{dt^2} = k_c^2 k_s^2 D + \frac{2\alpha(\psi^2+1)k_c^2 k_s^2 k_c^1 k_s^1}{\psi^2} (x_2 - x_1) e^{-\frac{\psi^2+1}{\psi^2}(x_1-x_2)^2} + \sqrt{2D} \frac{d\xi_2(t)}{dt} \end{cases} \tag{8}$$

To further simplify the solution of the problem in question, it is assumed that equality of active stochastic forces for individuals or various groups

$$\xi_1(t) = \xi_2(t)$$

is satisfied.

Then, introducing new designations:

$$\begin{aligned} y &= x_1 - x_2, \\ A &= D(k_c^1 k_s^1 - k_c^2 k_s^2), \\ B &= 2\alpha \frac{(\psi^2+1)}{\psi^2} (k_c^1 k_s^1 k_c^1 k_s^2 + k_c^2 k_s^2 k_c^2 k_s^1), \\ C &= \frac{\psi^2+1}{\psi^2}, \end{aligned}$$

after finding the difference of equations (8), the following equation is obtained:

$$\frac{d^2 y}{dt^2} = A + B y e^{-Cy^2}, B > 0, C > 0. \tag{9}$$

Now, equation (9) is rewritten in Cauchy form:

$$\begin{cases} \frac{dy}{dt} = z, \\ \frac{dz}{dt} = A + B y e^{-Cy^2}. \end{cases} \tag{10}$$

System (10) can be viewed as a dynamic system describing a process of interaction between two individuals or groups thereof.

As is known (see (Goryachenko, 2001; Andronov, 1981)), a dynamic system describes a process of transition from one state to another. The phase picture

of system (10) will be represented by a set of all states; to determine equilibrium states of this set, it is necessary to solve the following system of equations:

$$\begin{cases} z = 0, \\ ye^{-Cy^2} = -\frac{A}{B}. \end{cases} \quad (11)$$

Analysis of system (11) is readily represented graphically (**Fig. 1**):

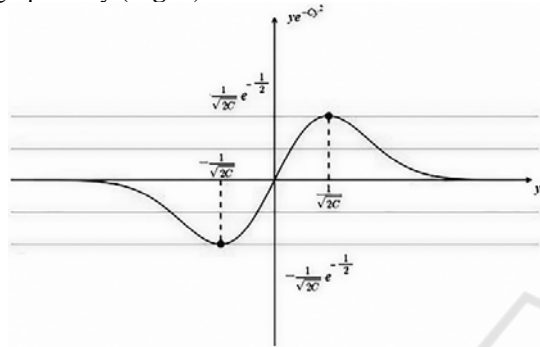


Figure 1.

As follows from **Fig. 1**, two equilibrium conditions are possible if condition

$$-\sqrt{\frac{1}{2C}}e^{-\frac{1}{2}} < -\frac{A}{B} < \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, -\frac{A}{B} \neq 0, \quad (12)$$

is satisfied, and one equilibrium condition if one of the three following equalities are satisfied:

$$\begin{aligned} -\frac{A}{B} &= 0, \quad -\frac{A}{B} = \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, \\ -\frac{A}{B} &= -\sqrt{\frac{1}{2C}}e^{-\frac{1}{2}} \end{aligned} \quad (13)$$

Naturally, if conditions opposite to (12) are satisfied

$$-\frac{A}{B} < -\sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, -\frac{A}{B} > \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, A/B \neq 0 \quad (14)$$

there are no equilibrium conditions.

As system (10) is conservative, the law of conservation of energy holds. Then, knowing the energy integral of system (10), it is possible to find phase trajectories of the system in question. As is known [10], in a conservative system, phase trajectories are lines of the level of the potential energy function, which has the following form:

$$V = -\int_0^y (A + Bue^{-Cu^2}) du = -Ay + \frac{Be^{-Cy^2}}{2C} \quad (15)$$

For social systems the notion of energy is either meaningless or has another definition. However, their dynamic behavior qualitatively coincides with the behavior of conservative mechanical systems, and, in the phase plane, the qualitative behavior of their phase trajectories is similar (Goryachenko, 2001).

As among parameters  $A, B, C$  only  $A$  can invert its sign, only two possible situations are to be considered. First, if conditions

$$-\sqrt{\frac{1}{2C}}e^{-\frac{1}{2}} < -\frac{A}{B} < 0, A > 0 \quad (16)$$

are satisfied, relations  $V(y)$  represented in **Fig.2** and the related phase trajectories are realized, where  $\dot{y} = \frac{dy}{dt}$ .

Second, if conditions

$$0 < -\frac{A}{B} < \sqrt{\frac{1}{2C}}e^{-\frac{1}{2}}, A < 0, \quad (17)$$

are satisfied, relation  $V(y)$  represented in **Fig.3** and the related phase trajectories are realized.

Based on the analysis of the obtained phase pictures (**Fig.2** and **Fig.3**), it can be concluded that there exists a certain stability region (confined in the pictures by a stable loop of the separatrix), i.e. the region within the closed trajectory.

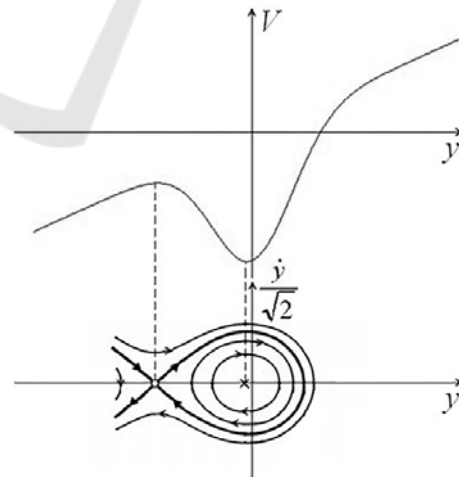


Figure 2.

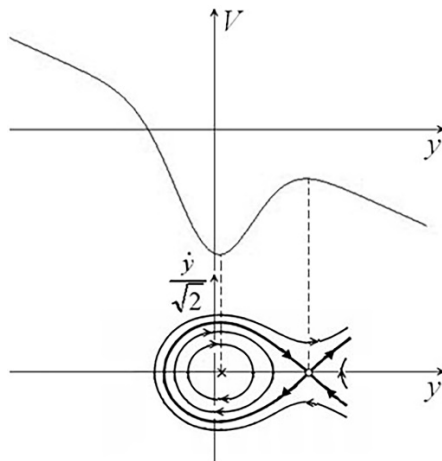


Figure 3.

The boundaries of this region are defined by values of characteristic parameters of individuals, or groups thereof, as well as the society as a whole:  $k_s^i, k_c^i, D$ . These coefficients, strictly speaking, can change in time as a result of the interaction of individuals, thus, affecting the dimensions and position of the stability region. However, in the present study, only a short-term scenario was considered, thus, their possible variation in time was assumed insignificant.

Individuals and groups thereof having parameters necessary for getting into the stability region at an initial time do not move apart from one another to a relatively large social distance as a result of reciprocal interaction. They remain at a distance within which social relations and active informational exchange are possible.

This can be interpreted as existence of an interaction region, parametrization of which makes relatively abrupt fluctuations of social coordinates, i.e., a state of conflict, highly improbable or impossible.

It is true that in a society, where social and informational contact, mutual permeation of different cultures and ethnic groups is sufficient, where particular groups of population do not separate themselves, creating closed subsystems (in which conditions substantially differ from those of the main system), the possibility of initiating ethnic-social, confessional etc. conflicts is relatively minimized.

Outside the stability region, phase trajectories are divergent and not close. Individuals/ groups of individuals that are, at an initial time, outside this region, after some time will find themselves at a relatively large social distance, which corresponds to the increase of social and informational gap between

individuals and/or groups of individuals. It is this state of a social system that can be characterized as a conflict and manifestation of the contradictions existing between individuals and groups thereof. With the introduction of social friction in the system, the picture changes significantly – Fig.4.

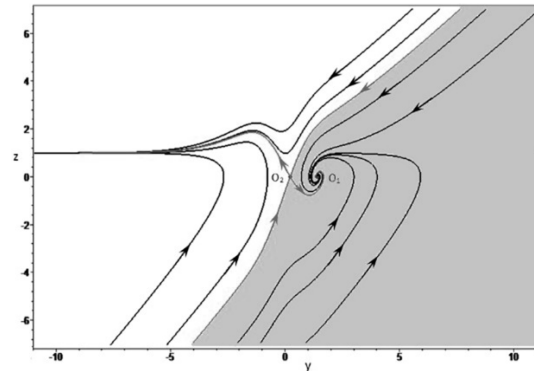


Figure 4.

However, it is a separate task, and it requires a separate article.

Thus, in ethnic-social conflicts, this is manifested in the minimization of social and cultural contacts between different ethnic groups, growth of the social-economic gap, aggravation of contradictions and, as a result, transfer to the phase of explicit confrontation accompanied with the destabilization of the social and political system as a whole.

## 6 CONCLUSIONS

Social hyper-clustering of a society, abrupt distinctions in the informational and social environment of individuals, cultural and inter-ethnic separation create ideal conditions for social conflict. Therefore, prevention of conflicts in society, determination of boundary conditions of their initiation and search for the most effective scenarios of their suppression is a vital issue for contemporary social sciences.

The present article concisely reviews the main approaches to modeling in social sciences, problems of determining social conflict and its main concepts. Conflict in a social system is defined in terms of mathematical modeling.

A mathematical model based on Langevin equation is introduced, an analytical solution in the first approximation for the divergent type of diffusion is given.

It is shown that, even in a simple example of two interacting ethno-groups of individuals, the developed model makes it possible to discover characteristic properties of conflict in a social system, to determine the impact of social distance in society upon the conditions of generation of such processes, accounting for external effects and random factor.

As a result of the modeling, a certain region of stability for a social system is found, within which it is stable and conflict-resistant.

The results of the present investigation will make it possible in future to approach the analysis of general problems for large numbers of individuals.

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