Fault Detection for Heating Systems using Tensor Decompositions of Multi-linear Models

Erik Sewe[1], Georg Pangalos[2] and Gerwald Lichtenberg[3]
[1]*PLENUM Ingenieurgesellschaft für Planung Energie Umwelt mbH, Hamburg, Germany*
[2]*Fraunhofer Institute for Silicon Technology ISIT, Application Center Power Electronics for Renewable Energy Systems, Hamburg, Germany*
[3]*Hamburg University of Applied Sciences, Faculty of Life Sciences, Hamburg, Germany*

Keywords: Fault Detection, Boilers, Heating Systems, Multi-linear System, Tensor Representation.

Abstract: A model-based fault detection method for heating systems is proposed. Two examples of heating system units are under investigation. These systems can be represented as multi-linear systems. Subspace identification methods are used to identify linear time-invariant models for each operating regime, resulting in a parameter tensor. In case of missing data and models for some operating regimes, an approximation method is proposed, where the canonical polyadic tensor decomposition method is used. Low rank approximations are found using an algorithm specialized for incomplete tensors. The tensor of these approximations defines the models in operating regimes, where no measurements were available. Fault detection is done using parity equations and application examples using real measurement data of a heat generation unit are given.

1 INTRODUCTION

The analysis of the operation of complex heating systems usually shows potential for optimization, see (Rehault et al., 2013). In many cases failed components, insufficiently tuned controller parameters as well as user intervention result in problems of heat supply and excess consumption. A frequently occurring problem - this shows an investigation of a multitude of malfunctions of heating systems done by PLENUM - are operation errors of combined heat and power plants and gas boilers. This includes faults from manipulated parameters up to complete loss of the heat production. For this reason a continuous fault detection is desirable. Currently this is done on a simple level, meaning threshold value checks and ordinary rules. This is quite simple, but reaches its limits of transferability and expandability, see (Venkatasubramanian et al., 2003). The methods of fault detection used for heating systems are not best practice according to today's technical knowledge and options, (Katipamula and Brambley, 2005).

A model-based fault detection can provide an enhanced detection performance, but is associated with many obstacles, see (Jagpal, 2006). The behavior of heating systems and buildings is difficult to predict. Systems are unique and the creation of exact models is difficult, time consuming and therefore expensive. Detailed building plans and schemes are rarely available, the intended function often poorly documented. Measurement data is intermittent or of poor quality. Some variables are not directly measurable. A high number of disturbances occurs. Additionally, (sub-)system are nonlinear.

The basis of model-based fault detection is a model of the system, that captures the main system dynamics in a precision, that is sufficient for the task. Models of heating systems can be derived by defining heat power balances, see (Pangalos, 2015). The heat power balances have a multilinear property, arising from the multiplication of temperature spread and flow rate. Since both - temperature spread and flow rate - are states of the system or linearly depending on a state of the system (depending on the realization of the model) a direct description as a linear state space model is not possible. Furthermore, discrete signals like boiler switching (on/off) turn the problem into a hybrid one. One possibility to model these hybrid heating systems is to use tensor systems, see e.g. (Lichtenberg, 2011; Pangalos et al., 2015). This approach is quite new, such that so far no methods for system identification exist, only standard nonlinear parameter identification algorithms - which do not take care of the multilinear structure - could be ap-

Sewe, E., Pangalos, G. and Lichtenberg, G.
Fault Detection for Heating Systems using Tensor Decompositions of Multi-linear Models.
DOI: 10.5220/0006401400270035
In *Proceedings of the 7th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH 2017)*, pages 27-35
ISBN: 978-989-758-265-3

plied. For that reason a different approach is used here to model the system. The model of the chosen approach could be represented as a switched system, see (Liberzon, 2003). By quantizing one of the signals - temperature spread or flow rate - no multiplication of these signals is needed, because for each quantized system the signal which was not quantized is multiplied with a constant. Depending on the size of the quantization interval, inaccuracies between the original and the quantized system arise. For each interval of the quantized signal and for each combination of the discrete signals one linear model is estimated using standard techniques, e.g. the N4SID algorithm, (Van Overschee and De Moor, 2012), resulting in several linear time-invariant systems, i.e. a multi-linear time-invariant model. Note, that multi-linear is used to denote multiple linear models, whereas multilinear - without dash - is used to denote models where the multiplication of all states and inputs - and all combinations with all numbers of multiplicants - are admissible. In multilinear systems no square terms are allowed, this could be modeled by polynomial models. Note that for polynomial systems black box identification methods exist, see (Mulders et al., 2009). The heating systems under investigation have the aforementioned multilinear property that could be modeled in the polynomial framework but the hybrid property (the discrete signals) are beyond the scope. In addition, the identification is, opposed to the one step identification of N4SID an iterative identification which also needs to be initialized, (Paduart et al., 2010).

This paper shows how this approach can be used for fault detection in heating systems. In section 2 method and structure for estimating the parameter sets and fault detection are explained. The complete system can be stored in a tensor structure, see section 2.1. Identification of parameter sets is explained in section 2.2. Missing parameter sets for operating points not included in training data can be estimated by tensor completion, see section 2.3. Fault detection is done using parity equations, section 2.4. In section 3 two application examples are given. In section 4 conclusions are drawn.

2 MODEL-BASED FAULT DETECTION METHOD FOR HEATING SYSTEMS

As mentioned before heating systems can be modeled as hybrid multilinear systems. Keeping all but one signals constant in multilinear terms and regarding just one combination of the binary signals will result in a linear behavior. Thus for each combination of quantized and binary signals, which will be denoted as operating regime, see (Murray-Smith and Johansen, 1997), a linear system can be identified around an operating point. For each operating regime, measurement data is used to identify a model. If a large amount of data is available for one operating regime, the measurement data is divided into parts, which will be called operational sections. The set of linear systems for all operating regimes will be denoted as parameter tensor.

2.1 System Definition

The operating regime $\mathbf{I}$ is defined by the combination of binary signals (e.g. on/off, opened/closed) and the center of the intervals of quantized signals (e.g. temperatures, flow rates). The signals will be denoted as partitioning signals, v is the number of partitioning signals. The operating regime is defined by the index vector

$$\mathbf{I} = (i_1, i_2, \dots, i_v) \tag{1}$$

with $i_n = \{1, 2, \dots, I_n\}, \quad n = 1, 2, \dots, v.$ The number of partitions of the corresponding signal is $I_n, n = 1, 2, \dots, v$. Note that $I_n = 2$ for all binary signals, it is assumed that 0 corresponds to index 1 and 1 corresponds to index 2. For the continuous-valued operating regime defining signals, the intervals $J_1, \dots, J_j$ are used for quantizations. The index of these intervals is also the corresponding index in the index vector $\mathbf{I}$. The limits of the intervals are stored. As an example, for a system with one continuous-valued operating regime defining signal divided into five sections and one binary signal, the index vector is $\mathbf{I} = (i_1, i_2)$ with $i_1 \in \{1, \dots, 5\}$ and $i_2 \in \{1, 2\}$.
Around each operating regime a linearization is performed. The result of the linearization is a discrete-time state space model

$$\mathbf{x}(k+1) = \mathbf{A_I}\mathbf{x}(k) + \mathbf{B_I}\mathbf{u}(k) \tag{2}$$

$$\mathbf{y}(k) = \mathbf{C_I}\mathbf{x}(k) + \mathbf{D_I}\mathbf{u}(k) \tag{3}$$

where k denotes the time index, $\mathbf{I}$ the mode of operation, $\mathbf{x} \in \mathbb{R}^n$ the state vector, $\mathbf{u} \in \mathbb{R}^m$ the input vector, $\mathbf{y} \in \mathbb{R}^p$ the output vector, $\mathbf{A_I} \in \mathbb{R}^{n \times n}$ the system matrix, $\mathbf{B_I} \in \mathbb{R}^{n \times m}$ the input matrix, $\mathbf{C_I} \in \mathbb{R}^{p \times n}$ the output matrix and $\mathbf{D_I} \in \mathbb{R}^{p \times m}$ the feed forward matrix. Please note that the $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ matrices must be in a canonical state space representation. The representation as given in (4) is used to store the entire information of the state space model in a single matrix, that has dimension $\mathbb{R}^{(n+p)\times(n+m)}$. Also given in (4) are the elements of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ in state space modal representation. The systems do not have a direct feedthrough, so $\mathbf{D}$ is a zero matrix.

$$\left(\begin{array}{c|c}\mathbf{A} & \mathbf{B}\\ \hline \mathbf{C} & \mathbf{D}\end{array}\right) =$$

$$\left(\begin{array}{cccc|cccc} a_{11} & 0 & \dots & 0 & b_{11} & \dots & \dots & b_{1m}\\ 0 & a_{22} & \ddots & \vdots & \vdots & \ddots & & \vdots\\ \vdots & \ddots & \ddots & 0 & \vdots & & \ddots & \vdots\\ 0 & \dots & 0 & a_{nn} & b_{n1} & \dots & \dots & b_{nm}\\ \hline c_{11} & \dots & \dots & c_{1n} & 0 & \dots & \dots & 0\\ \vdots & \ddots & & \vdots & \vdots & \ddots & & \vdots\\ \vdots & & \ddots & \vdots & \vdots & & \ddots & \vdots\\ c_{p1} & \dots & \dots & c_{pn} & 0 & \dots & \dots & 0 \end{array}\right). \tag{4}$$

To represent the entire multi-linear system, i.e. a linear system for each combination of the v signals, defining the operating regime, the parameter tensor M

$$\mathsf{M} \in \mathbb{R}^{(n+p)\times(n+m)\times I_1\times I_2\times\cdots\times I_v} \tag{5}$$

is created, where the notation of (Cichocki et al., 2009) is adapted. The entries of the third to the $v^{th}+2$ dimension of M correspond to the operating regime $\mathbf{I}$. Keeping all but the first two dimensions constant will give the $\left(\begin{array}{c|c}\mathbf{A} & \mathbf{B}\\ \hline \mathbf{C} & \mathbf{D}\end{array}\right)$ matrix of the linear system that is identified for operating regime $\mathbf{I}$. This matrix is selected despite the amount of zeros in $\mathbf{A}$ and $\mathbf{D}$, to allow a generic system description. The linearization is performed for specific values of the input and output signals. Recall that so far the operating regime was denoted as a point defining a linear system, i.e. basically the switching signal if a switched affine system would have been used. But to find a good approximation also the inputs and outputs of the linear systems need to be at their operating point. These values also need to be stored, but for reasons of size and magnitude they do not fit into M. The input and output operating point values $\bar{\mathbf{u}}_{\mathbf{I}}$ and $\bar{\mathbf{y}}_{\mathbf{I}}$ are stored in the additional tensor O

$$\mathsf{O} \in \mathbb{R}^{(m+p)\times I_1\times I_2\times\cdots\times I_v} \tag{6}$$

with

$$\mathsf{O}(:,\mathbf{I}) = [\bar{\mathbf{u}}_{\mathbf{I}} \quad \bar{\mathbf{y}}_{\mathbf{I}}]^T. \tag{7}$$

2.2 Identify Parameter Tensor Slices

The first step is to read input and output data as well as signals that determine the operating regime. Depending on the data quality some pre-processing such as outlier removal and filling data gaps must be done. By quantization the values of the partitioning signals are partitioned into intervals. The interval edges are stored and used for quantization of the application data. The next step is to detect operating regimes and sort data by operating regimes. For each operating regime the operating point for input and output data is stored in O. For black box estimation of the state-space matrices, MATLAB provides the N4SID algorithm in the System Identification Toolbox (Ljung, 2016). A noise component is not estimated, the value is fixed to zero. The MATLAB command `canon` with option `modal` transforms a linear model into a canonical state-space model in modal representation. As it does not normalize the $\mathbf{B}$ or $\mathbf{C}$ matrix this is done by normalizing the non-zero entries of $\mathbf{C}$ for the first output to f_T. Tensor algorithms are sensitive to scale of data elements (Fanaee-T and Gama, 2016), so the parameter f_T should be chosen in a way that all resulting entries have the same scale. The transformation is performed as in (8) to (12).

$$\mathbf{T_{norm}} = f_T \begin{pmatrix} \frac{1}{c_{11}} & 0 & \dots & 0\\ 0 & \frac{1}{c_{12}} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & \frac{1}{c_{1n}} \end{pmatrix} \tag{8}$$

$$\mathbf{A_{norm}} = \mathbf{T_{norm}^{-1} A T_{norm}} \tag{9}$$

$$\mathbf{B_{norm}} = \mathbf{T_{norm}^{-1} B} \tag{10}$$

$$\mathbf{C_{norm}} = \mathbf{C T_{norm}} \tag{11}$$

$$\mathbf{D_{norm}} = \mathbf{D} \tag{12}$$

For better readability, in this paper it is assumed that the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ stored in and restored from the parameter tensor are normalized without stating the indices $_{norm}$.

2.3 Construct Full Parameter Tensor

For systems where several binary or quantized signals are determining the operating regime, it is very likely for some operating regimes that no measurements are taken. Therefore the tensors (5) and (6) can have unknown entries. In figure 1 a schematic view of an incomplete parameter tensor with one partitioning signal is shown. To cope with this problem a ten-

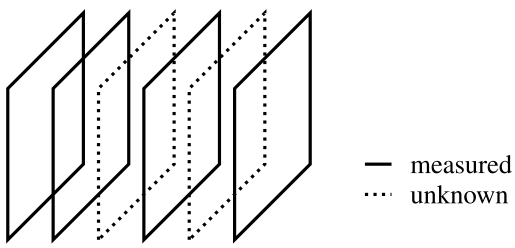

Figure 1: Incomplete parameter tensor, $v = 1$.

sor decomposition approach is chosen. It is assumed, that the operating regimes are not independent of each other but that the system dynamics is influenced by changing one specific signal which is defining the operating regime. The change of one signal will have a similar effect for all operating regimes, or at least can be reconstructed by looking at the change of all operating regime components. To fill up the missing entries in the tensors M and O the tensorlab toolbox, (Vervliet et al., 2016), for MATLAB is used. It is assumed, that if a low rank approximation is found, which can represent the existing entries of the tensors, the system dynamics of all operating regimes are contained in this tensor. By resolving the entire tensor entries for the previously not defined operating regimes can be found. To decompose the tensor the canonical polyadic decomposition algorithm `cpd` of the tensorlab toolbox is used. All entries of the tensor, which are unknown, are defined as non existing, which is done by setting the corresponding entries to the value `NaN` in MATLAB. The used algorithm option `CPDI` is set to true such that a routine is used, which is specialized to cope with incomplete tensors. The general idea to find a decomposed tensor, i.e. the factor matrices $\mathbf{F}^h$ for a tensor T is to solve the minimization problem

$$\min_{\mathbf{F}^{(1)},\ldots,\mathbf{F}^{(H)}} \frac{1}{2}\left|\left|\mathsf{T} - \left[\left[\mathbf{F}^{(1)},\ldots,\mathbf{F}^{(H)}\right]\right]\right|\right|^2 \quad (13)$$

where $\mathbf{F}^{(1)},\ldots,\mathbf{F}^{(H)}$ denote the factor matrices of the decomposed tensor, $[[\ldots]]$ is used to represent a tensor using factor matrices and $||\ldots||$ denotes the Frobenius norm, (Vervliet et al., 2014). The specialized routine solves the problem

$$\min_{\mathbf{F}^{(1)},\ldots,\mathbf{F}^{(H)}} \frac{1}{2}\left|\left|\mathsf{G} \circledast \left(\mathsf{T} - \left[\left[\mathbf{F}^{(1)},\ldots,\mathbf{F}^{(H)}\right]\right]\right)\right|\right|^2 \quad (14)$$

where $\mathsf{G} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_H}$ is a binary observation tensor with the entries set to 1 if the corresponding value in T is known and 0 otherwise and $\circledast$ denotes the Hadamard, i.e. the element-wise product (Vervliet et al., 2014). Solving this minimization problem does not take into account any unknown values.

Obviously this approach can just be used if for a sufficiently large number of operating regimes a linear state space model can be identified. If the entire system dynamic is not captured by the measured operating regimes, resolving the tensor will not yield in a good approximation for the unmeasured operating regimes. Note also that the selection of the rank of the approximation is not trivial. Increasing the rank will introduce freedom in choosing the values of the unidentified tensor entries and thus not result in a good approximation, choosing a rank which is too small results in a tensor which does not capture the system dynamics. In this paper the rank is chosen heuristically. Choosing a high rank results in good approximations for the known parameters in the parameter tensor. The rank is then reduced until the unknown parameters are the approximately same for different initial values in the tensor decomposition. Summing up sections 2.1 to 2.3, the parameter tensor, describing the system behavior in every operating regime, can be created as shown in figure 2.

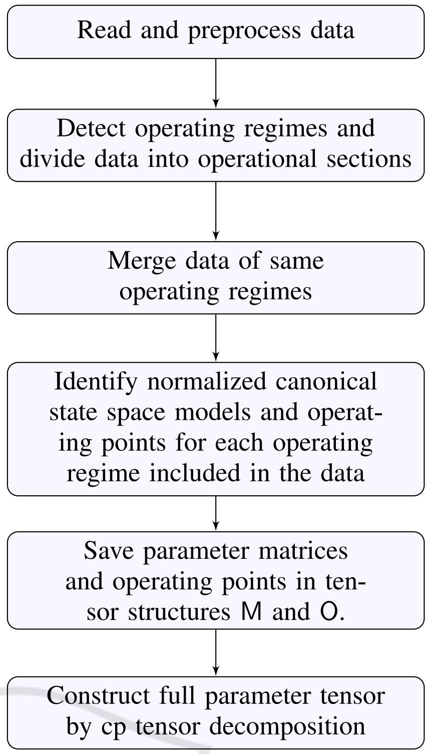

Figure 2: Generating parameter tensor.

2.4 Fault Detection with Parity Equations

The proposed method is used for offline fault detection, using parity equations. See figure 3 for the fault detection procedure. The parameter tensor introduced in section 2.1 describes the nominal, i.e. the non-faulty behavior. Parity equations and observer based designs are a way to detect discrepancies between process and model. Once the design objectives have been selected, both lead to identical or equivalent residual generators (Gertler, 1991). The procedure for parity equations is usually simpler than for observer based designs (Gertler, 1991). One benefit of parity equation is, that no initial value for the state vector $\mathbf{x}$ is needed. Input and output data are com-

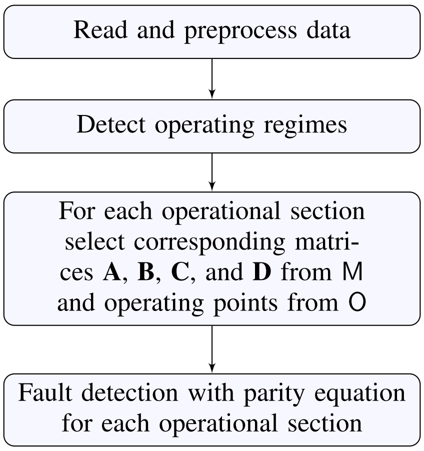

Figure 3: Fault detection.

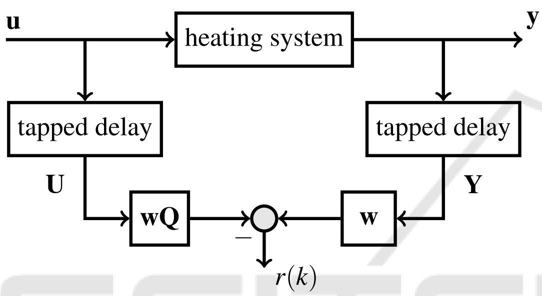

Figure 4: Parity equation.

pared to the model behaviour contained in **w** and **Q**, as seen in figure 4. The tapped delay block delays the input **u** respectively the output **y** by $q+1$ time steps and outputs all the delayed versions, resulting in **U** (15) and **Y** (16).

$$\mathbf{U}(k) = \begin{pmatrix} \mathbf{u}(k-q) \\ \mathbf{u}(k-q+1) \\ \mathbf{u}(k-q+2) \\ \vdots \\ \mathbf{u}(k) \end{pmatrix} \tag{15}$$

$$\mathbf{Y}(k) = \begin{pmatrix} \mathbf{y}(k-q) \\ \mathbf{y}(k-q+1) \\ \mathbf{y}(k-q+2) \\ \vdots \\ \mathbf{y}(k) \end{pmatrix} \tag{16}$$

The residual r at time k for $q+1$ time steps is calculated by

$$r(k) = \mathbf{w_I}\mathbf{Y}(k) - \mathbf{w_I}\mathbf{Q_I}(q)\mathbf{U}(k) \tag{17}$$

with

$$\mathbf{Q_I}(q) = \begin{pmatrix} \mathbf{D_I} & 0 & 0 & \dots & 0 \\ \mathbf{C_I B_I} & \mathbf{D_I} & 0 & \dots & 0 \\ \mathbf{C_I A_I B_I} & \mathbf{C_I B_I} & \mathbf{D_I} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{C_I A_I}^{q-1}\mathbf{B_I} & \mathbf{C_I A_I}^{q-2}\mathbf{B_I} & \mathbf{C_I A_I}^{q-3}\mathbf{B_I} & \dots & \mathbf{D_I} \end{pmatrix}. \tag{18}$$

The derivation for (17) is explained in (Isermann, 2006). The residual generator vector $\mathbf{w_I}$ is determined by solving

$$\mathbf{w_I}\mathbf{R_I} = 0 \tag{19}$$

with

$$\mathbf{R_I}(q) = \begin{pmatrix} \mathbf{C_I} \\ \mathbf{C_I A_I} \\ \mathbf{C_I A_I^2} \\ \vdots \\ \mathbf{C_I A_I^q} \end{pmatrix}. \tag{20}$$

There is more than one solution for (19). The trivial solution must not be chosen. By selecting $\mathbf{w_I}$ properties of fault detection are affected. The fault signal e, where 0 stands for 'no fault' and 1 for 'fault' is calculated by the fault criterion

$$e(k) = \begin{cases} 0 & \text{if} \quad r(k) \in [b_l, b_u] \\ 1 & \text{else} \end{cases}. \tag{21}$$

For every operating section the residual $r(k)$ is compared to upper and lower bound, b_u and b_l for the decision if there is a fault or not. Due to computation time and memory use the length of a operating section is limited to $q+1$ time steps. Operating sections longer than $q+1$ time steps are divided and examined individually. Upper bound b_u and lower bound b_l are determined by calculating the residual $r(k)$ for the data used to create M and are the same for every operating regime **I**. This gives the decision boundary for non-faulty operation. Values of $r(k)$ exceeding the range limited by b_u and b_l are expected to imply faulty operation.

3 APPLICATION TO HEATING SYSTEM FAULT DETECTION

The heat generation unit of the heating system of the State Office for Nature, Environment and Consumerism in Düsseldorf, Germany is investigated. The heat generation unit consists of three hydraulically balanced boilers with attached gas burners, two of them are condensing boilers, one is a low temperature boiler. The three boilers together have a total power of about 1870 kW and supply six consumer circuits. The consumer circuits are not investigated in this paper and therefore no model of them is given. The heating system was already under investigation in (Sewe et al., 2012; Pangalos and Lichtenberg, 2012), where a model of the entire system is given and the controller design and implementation is described. A scheme of the heat generation unit is given in figure 5. Each boiler is equipped with a lid such that the flow rate through the boiler can be stopped when the boiler is switched off.

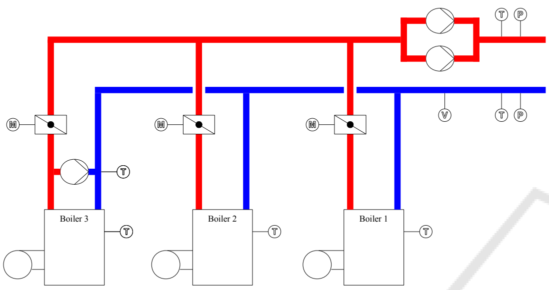

Figure 5: Scheme of the heat generation unit.

The inputs of the model of the heat generation unit are the return temperature T_r and the power to be produced by the boilers P_i, $i = 1,2,3$. The output is the supply temperature T_s. The states of the heat generation model are the supply temperatures of the boilers. The operating regime of the heat generation unit is defined by position of the boilers lids (opened/closed) and the burners switching signals (on/off). The flow rate is divided in four intervals. For each of these intervals and each of the boiler lid and burner switching setting, a model of the system is estimated. Resulting in a tensor M of dimension $\mathbb{R}^{4\times7\times4\times2\times2\times2\times2\times2\times2}$, the operating regime index is $\mathbf{I} = (i_1,i_2,i_3,i_4,i_5,i_6,i_7)$ with $i_1 \in \{1,\ldots,4\}$ and $i_n \in \{1,2\}$ for $n = 2,\ldots,7$. The heat generation unit can be modelled, with some simplifications, as described in (Kruppa, 2014) by

$$T_s = \frac{T_{s1}\dot{V}_{s1} + T_{s2}\dot{V}_{s2} + T_{s3}\dot{V}_{s3}}{\dot{V}}, \tag{22}$$

$$\dot{T}_{si} = \frac{P_i + c_w\rho\dot{V}_{si}(T_r - T_{si})}{c_w\rho V_{si}} \tag{23}$$

and

$$\dot{V} = \dot{V}_{s1} + \dot{V}_{s2} + \dot{V}_{s3}, \tag{24}$$

where T_s is the supply temperature of the heat generation unit, T_r is the return temperature, $\dot{V}$ is the flow rate out of the heat generation unit and back into it, T_{si} are the supply temperatures, V_i the volumes, P_i the input powers of the boilers 1, 2 and 3 respectively, for $i = 1,2,3$, $\dot{V}_i$ are the flow rates through the boilers, c_w is the specific heat capacity of water and ρ the water density.

3.1 Fault Detection on a Multi-boiler System

As stated in section 2.3, tensor completion of the parameter tensor only yields in correct approximations if a sufficiently large number of operating regimes can be identified. For that reason this first example demonstrates the proposed method using a MATLAB SIMULINK model as shown in figure 6. The model is created with slightly simplified components of the HeatLib (Kruppa, 2014). Its parameters are adapted to fit the heat generation unit described above.

Instead of using the N4SID algorithm, a discrete-time linear state-space model around the operating point is extracted, using MATLAB SIMULINK . A complete parameter tensor is created and can be manipulated by deleting several operating regimes. Using the introduced tensor decomposition and reconstruction method, the parameter tensor is completed again. In (25) and (26) the **A** matrix for one operating regime **I** is shown, where (25) is the original and (26) the restored matrix.

$$\mathsf{A}_{\mathbf{I}} = \begin{bmatrix} 0.8532 & 0 & 0 \\ 0 & 0.8157 & 0 \\ 0 & 0 & 0.7731 \end{bmatrix} \tag{25}$$

$$\mathsf{A}_{\mathbf{I}} = \begin{bmatrix} 0.8532 & -0.001 & -0.000 \\ -0.001 & 0.8156 & -0.000 \\ -0.001 & -0.001 & 0.7730 \end{bmatrix} \tag{26}$$

The incomplete parameter tensor has stored sufficient information of the entire system dynamic so that resolving the tensor yields to a good approximation. The tensors M and O are used for fault detection on real input and output data. Figure 7 shows the measured supply temperature and the resulting residual $r(k)$ for each operational section. In case of of fault, i.e. $e(k) = 1$, $r(k)$ is marked red, for non-faulty sections it is marked green. One summer month is tested. After a complete system breakdown on June 18$^{\text{th}}$ the supply temperature drops to temperatures below 35°C. No fault is detected because input data is consistent to output data, boilers are not turned on in this period. As a consequence, on June 20$^{\text{th}}$, boiler 2 has been turned on in manual mode. It is working on full power and ignoring controller signals, thus heating up the system until it is stopped by an emergency switch, now working as a two point controller. This does not match to expected system behavior. The fault is detected correctly. On

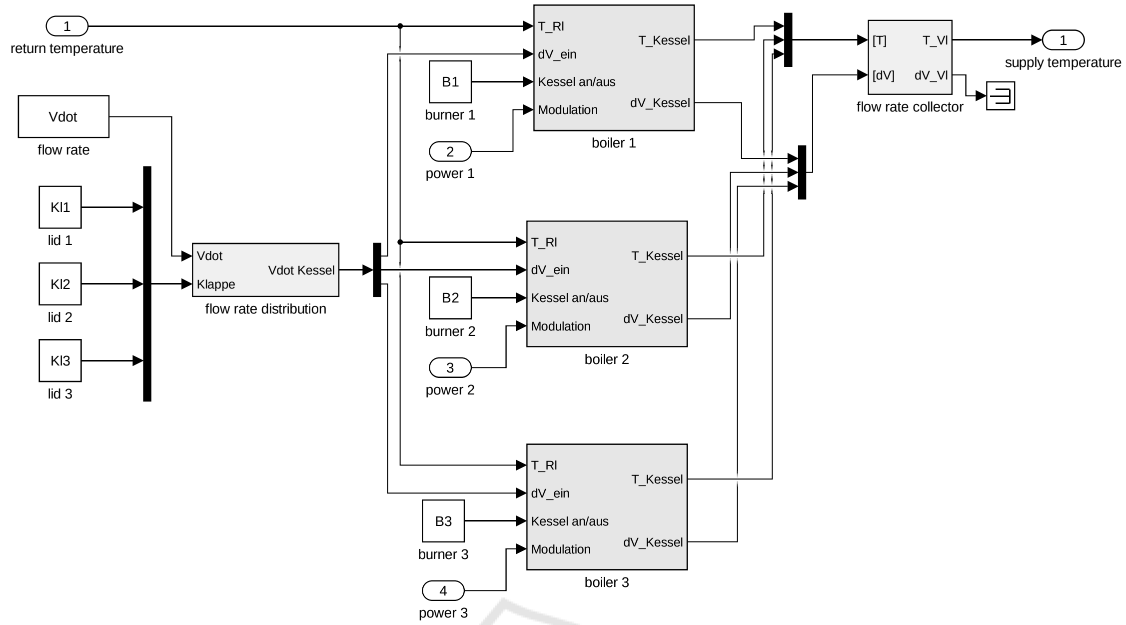

Figure 6: Heat generation unit modelled in SIMULINK .

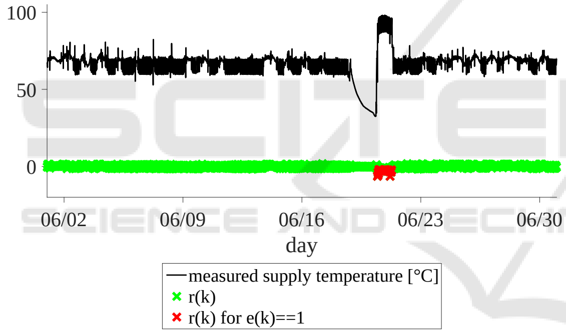

Figure 7: Fault detection - heat generation unit.

June 21 st the boiler has been set back to normal operating mode.

3.2 Fault Detection on a Single Boiler

The method explained in chapter 2 is performed on boiler 2, using black box identification (N4SID, MATLAB System Identification Toolbox) to estimate the state space matrices. It is a first order system. The inputs of the model of boiler 2 are the return temperature T_r and the power to be produced by the boiler P_2. The output is the supply temperature T_{s2}. The state is also the supply temperature of the boiler. The operating regime of the boiler is defined by the burner switching signal (on/off) and the flow rate. As boiler 2 is the main supplier of the heating system, no training data with the lid closed is available and therefore it cannot be used as an operating regime.

The flow rate is divided in seven intervals. For each of these intervals and each of the burner's switching setting a model of the system is estimated. Resulting in a tensor M of dimension $\mathbb{R}^{2\times3\times7\times2}$, the operating regime index is $I_b = (i_1, i_2)$ with $i_1 \in \{1, \dots, 7\}$ and $i_2 \in \{1, 2\}$. The parameter tensor is created with measurement data of six month. Thirteen of fourteen operating regimes appear in this data set. For improved illustration of the proposed method, parameters of two known operating regimes are deleted. The entries of the parameter tensor corresponding to the **A** matrices - which are scalars in this case, since a first order system is under investigation - in the different operating regimes are shown in (27), where $*$ marks the missing modes. The seven entries in the first dimension correspond to the seven intervals of the flow rate and the two entries in the second dimension correspond to the burner switching signal.

$$\mathsf{M}(1,1,:,:) = \begin{bmatrix} 1.00 & 0.89 \\ 0.81 & 0.76 \\ 0.82 & * \\ 0.68 & 0.75 \\ * & 0.69 \\ 0.57 & 0.64 \\ * & 0.62 \end{bmatrix} \quad (27)$$

As the system can be described by (23), for all operating regimes including the missing ones, it can be assumed, that $\mathbf{C} = 1$ and $\mathbf{D} = [0\ 0]$. To fill up the missing entries a tensor decomposition of rank 2 is approximated. Unfolding this tensor to a full tensor gives the result

$$\mathrm{M}(1,1,:,:) = \begin{bmatrix} 1.00 & 0.88 \\ 0.80 & 0.77 \\ 0.82 & 0.79 \\ 0.70 & 0.73 \\ 0.63 & 0.69 \\ 0.57 & 0.64 \\ 0.50 & 0.62 \end{bmatrix}. \tag{28}$$

The parameter tensor partially shown in (28) is validated on the six month of training data, see figure 8. The six errors detected in the training data are in operational sections where the lid of boiler 3 has been opened or closed. So it is assumed, that the flow rate $\dot{V}_{s2}$, calculated by the flow rate $\dot{V}$, hydraulic parameters and the position of the boilers lids, in these sections is faulty due to inaccuracy in data logging. The parameter tensor is tested on data of the same period as used for the simulation in figure 7. As can be seen in figure 9, the fault is detected correctly. Another fault can be observed in a winter month as seen in figure 10. Till December 14$^{\text{th}}$, boiler 2 is not processing the controller power signal, thus not producing the correct amount of heat, but the burner switching signal (on/off) is configured correctly. So the boiler is turned on at the correct periods but does not vary the power output. As a result, the supply temperature T_{s2} exceeds the set point temperature and the controller can not keep the multi-boiler system stable. On December 14$^{\text{th}}$, a service engineer adjusted the parameters of boiler 2 and for the remaining time of the data set the system worked as expected. For the faulty part of the data set the residual $r(k)$ has values exceeding the range limited by b_u and b_l, so the fault is detected correctly. The two errors detected after December 15$^{\text{th}}$ are in operational sections where the lid of boiler 3 has been opened or closed, so as well as in figure 8, faulty input data for calculation of $\dot{V}_{s2}$ is assumed.

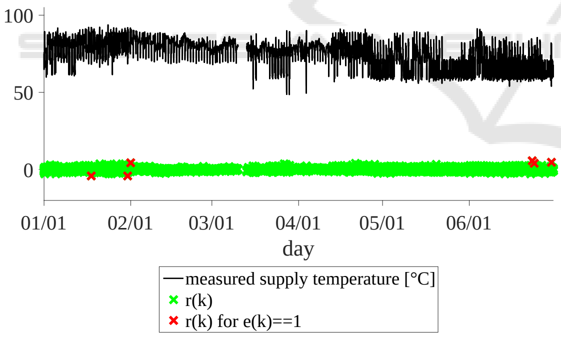

Figure 8: Fault detection boiler 2, model validation.

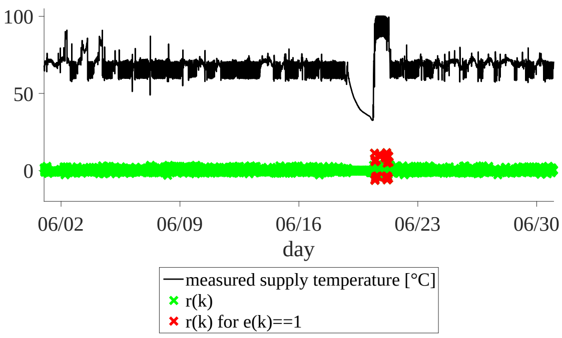

Figure 9: Fault detection boiler 2, manual operation.

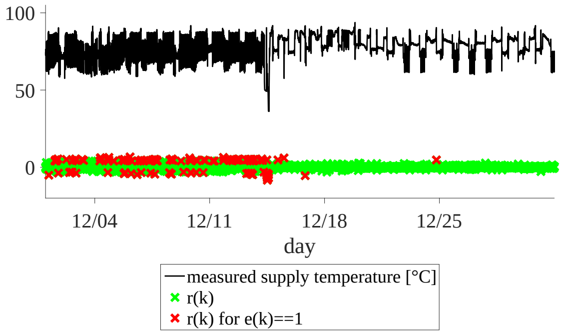

Figure 10: Fault detection boiler 2, failure in power control.

4 CONCLUSIONS

A method to detect faults in heating systems has been proposed. The fault detection is model-based such that models of heating systems were investigated. A multi-linear time-invariant system is constructed by identifying multiple linear time-invariant systems for different operating regimes. To define the operating regime, signals, which are multiplied with inputs or states, are quantized. This multiplication arises from the heat power balances, which are the basis of the models. There the multiplication of temperature (which is a state) and flow rate (which is an input) makes the system nonlinear. By quantizing the flow rate, multiple linear systems can be defined. Furthermore, different binary signals extend the operating regime. For each interval of the flow rate and each binary signal, a linear system is identified. All these systems build the parameter tensor. If for some operating regimes no measurement data is available, an approximation method is proposed. By estimating low rank approximations of the parameter tensor with specialized algorithms for incomplete tensors, good approximations of the systems, where no measurement data was available could be found. The general functionality is shown in two application examples. The fault in the measurement data was identified in both examples. Work will be continued on improved estimation of system matrices on operating regimes with missing measurement data.

ACKNOWLEDGEMENTS

This work was partly supported by the project OBSERVE of the Federal Ministry for Economic Affairs and Energy, Germany (Grant-No.: 03ET1225C) and partly supported by the Free and Hanseatic City of Hamburg (Hamburg City Parliament publication 20/11568).

REFERENCES

Cichocki, A., Zdunek, R., Phan, A., and Amari, S. (2009). *Nonnegative matrix and tensor factorizations*. Wiley, Chichester.

Fanaee-T, H. and Gama, J. (2016). Tensor-based anomaly detection: An interdisciplinary survey. *Knowledge-Based Systems*, 98:130–147.

Gertler, J. (1991). Analytical redundancy methods in fault detection and isolation. In *Preprints of IFAC/IMACS Symposium on Fault Detection, Supervision and Safety for Technical Processes SAFEPROCESS91*, pages 9–21.

Isermann, R. (2006). *Fault-diagnosis systems: an introduction from fault detection to fault tolerance*. Springer Science & Business Media.

Jagpal, R. (2006). EBC Annex 34 - computer aided evaluation of HVAC system performance - technical synthesis report: Computer aided evaluation of HVAC system performance.

Katipamula, S. and Brambley, M. (2005). Methods for fault detection, diagnostics, and prognostics for building systems - a review, part I. *HVAC&R Research*, 11(1):3–25.

Kruppa, K. (2014). Dokumentation Matlab Heatlib - Zur Verwendung mit Matlab. Dokumentation, Institut für Regelungstechnik, TU Hamburg-Harburg.

Liberzon, D. (2003). *Switching in systems and control, ser. Systems & Control: Foundations & Applications*. Boston: Birkhäuser.

Lichtenberg, G. (2011). *Hybrid Tensor Systems*. Habilitation, Hamburg University of Technology.

Ljung, L. (2016). *System identification toolbox: user's guide*.

Mulders, A. V., Schoukens, J., Volckaert, M., and Diehl, M. (2009). Two nonlinear optimization methods for black box identification compared. *IFAC Proceedings Volumes*, 42(10):1086 – 1091.

Murray-Smith, R. and Johansen, T. A. (1997). *Multiple Model Approaches to Modelling and Control*. Taylor and Francis, London.

Paduart, J., Lauwers, L., Swevers, J., Smolders, K., Schoukens, J., and Pintelon, R. (2010). Identification of nonlinear systems using polynomial nonlinear state space models. *Automatica*, 46(4):647 – 656.

Pangalos, G. (2015). *Model-based Controller Design Methods for Heating Systems*. dissertation, Hamburg University of Technology.

Pangalos, G., Eichler, A., and Lichtenberg, G. (2015). *Hybrid Multilinear Modeling and Applications*, pages 71–85. Springer International Publishing, Cham.

Pangalos, G. and Lichtenberg, G. (2012). Approach to boolean controller design by algebraic relaxation for heating systems. *IFAC Proceedings Volumes*, 45(9):210–215.

Rehault, N., Lichtenberg, G., Schmidt, F., and Harmsen, A. (2013). Modellbasierte Qualitätssicherung des energetischen Gebäudebetriebs (ModQS). Abschlussbericht. Technical report.

Sewe, E., Harmsen, A., Pangalos, G., and Lichtenberg, G. (2012). Umsetzung eines neuen Konzepts zur Mehrkesselregelung mit Durchflusssensoren. *HLH Lüftung/Klima - Heizung/Sanitär - Gebäudetechnik*, 1:37 –42.

Van Overschee, P. and De Moor, B. (2012). *Subspace identification for linear systems: Theory - Implementation - Applications*. Springer Science & Business Media.

Venkatasubramanian, V., Rengaswamy, R., Kavuri, S. N., and Yin, K. (2003). A review of process fault detection and diagnosis: Part iii: Process history based methods. *Computers & chemical engineering*, 27(3):327–346.

Vervliet, N., Debals, O., Sorber, L., and Lathauwer, L. D. (2014). Breaking the curse of dimensionality using decompositions of incomplete tensors: Tensor-based scientific computing in big data analysis. *IEEE Signal Processing Magazine*, 31(5):71–79.

Vervliet, N., Debals, O., Sorber, L., Van Barel, M., and De Lathauwer, L. (2016). Tensorlab user guide.