

# A Practical Approach for the Auto-tuning of PD Controllers for Robotic Manipulators using Particle Swarm Optimization

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**Keywords:** Robotic Manipulators, Particle Swarm Optimization, PD Control, Automatic Tuning.

**Abstract:** An auto-tuning method of PD controllers for robotic manipulators is proposed. This method suggests a practical implementation of the particle swarm optimization technique in order to find optimal gain values achieving the best tracking of a predefined position trajectory. For this purpose, The integral of the absolute error *IAE* is used as a cost function for the optimization algorithm. The optimization is achieved by performing the desired movement of the robot iteratively and evaluating the cost function for every iteration. Therefor, the necessary constraints that guarantee a safe and stable movement of the robot are defined, which are: a maximum joint torque constraint, a maximum position error constraint and an oscillation constraint. A constraint handling approach is suggested for the optimization algorithm in order to adapt it to the problem in hand. Finally, the efficiency of the proposed method is verified through a practical experiment on a real robot.

## 1 INTRODUCTION

PID control schemes provide simple and effective solutions for most applications of control engineering. However, the effectiveness of PID controllers is conditioned by accurate tuning of the controller gains.

Robotic manipulators are highly non linear, highly coupled, Multi-Input Multi-Output (MIMO) dynamic systems. Using PID controller to control robotic manipulators can be a desired choice because of its simplicity and effectiveness. However, the conventional tuning methods of the gains depending on manual or experimental approaches do not necessarily give satisfactory results for such complex systems (Johnson and Moradi, 2005).

The difficulty of using experimental and manual tuning methods rises in the application fields, where the assigned task of a robot might constantly change or where a robot is of variable configuration or geometry (e. g. modular robots). In such cases, the need for an auto-tuning method is urgent.

Recently, after the rapid increase of computing power, auto-tuning methods based on optimization techniques has been applied to non linear systems in order to obtain an increased performance with respect to predefined fitness functions.

In the field of robotic manipulators, a number of optimization methods (e. g. Genetic Algorithms (GA) (Kim et al., 2012), Particle Swarm Optimization

(PSO) (Kapoor and Ohri, 2015)) has been used to automatically tune the PID controllers of robot manipulators. Also, comparative studies between different algorithms have been done in (Ouyang and Pano, 2015) and (Kwok and Sheng, 1994) in order to find the most effective tuning method.

In (Ayala and dos Santos Coelho, 2012), more than one single objective were considered within the controller design. The proposed optimization method was based on a multi-objective evolutionary algorithm (MOEA), which aimed to tune the PID controller gains by taking two conflicting objective functions into consideration: minimization of position errors and minimization of the control signal variation (joint torques). In (Pierezan et al., 2014), a comparative study between different multi-objective optimization techniques has been introduced and an improved multi-objective particle swarm optimization (I-MOPSO) has been proposed.

Artificial intelligence techniques like fuzzy logic and neural networks have also been implemented to build PID tuning systems for robotic manipulators. Examples for those systems can be found in (Bekit et al., 1998), (Llama et al., 2001), and (Melek and Goldenberg, 2003). Those systems gave the PID gains variable values depending on the online measurements of the robot joint positions and, therefore, turned the traditional controller into an adaptive controller.

Another approach has been proposed in (Nahapetian

et al., 2009). Here a combination of both the GA technique and the fuzzy logic was used to form a hybrid tuning method for a PID regulator.

Regarding the optimization methods, the known and previously mentioned research focuses on analyzing and testing the optimization algorithm itself. For this purpose, the validation of the proposed methods has only be tested on simple simulations of the robots. However, from a practical point of view, more attention needs to be diverted to the problem in hand, i. e. the necessary constraints, which guarantee a safe movement of the robot during the search for optimal controller parameters. Otherwise, these optimization algorithms will not be practicable.

## 2 ROBOT CONTROLLER OPTIMIZATION

This work considers a robot manipulator controlled by an independent PD controller for every joint of the robot. The controller gains are usually tuned by using manual tuning methods which are unable to obtain critical damping behavior (Craig, 2005) and , therefore, settle for an overdamped one. Recently, it became possible to use heuristic optimization methods to solve the practical problems of complicated mechatronics systems such as robotic manipulators. The principle of these optimization methods is based on defining a search algorithm aiming at finding the optimal solution of the problem after a number of iterations. To do an auto-tuning of PD controllers using an optimization method, it is required to define the optimization problem and its parameters. In the considered problem, it is desired to find the control parameters that lead to the best trajectory tracking accuracy of the robot. The optimization parameters are the PD gains  $K_p$  and  $K_d$ . The cost function is the integral of the absolute error *IAE*:

$$IAE = \int_0^T |e(t)| dt = \int_0^T |(q_d(t) - q(t))| dt. \quad (1)$$

the aim of this work is to produce a practical application of an optimization method for a robotic manipulator, i. e. the evaluation of the cost function will be depending on a real movement of the robot along the desired trajectory. Therefore, it is inevitable to define necessary constraints that guarantee a safe and stable movement of the robot while searching for the optimal gain values.

### 2.1 The Optimization Problem Constraints

Usually by the tuning procedure of the controller's gains in robotic manipulators, one pays attention to three possible dangerous situations. The gains should not be too high and result in high torques from the actuators, they should also be high enough to make the tracking error lower than a maximum limit, and finally, one should be careful not to excite high oscillations by the chosen gains. These oscillations occur mostly as a result of the potential flexibilities in the joints and/or the links of the robot.

The proposed approach in this work considers these same conditions without assuming any knowledge of the robot dynamics. That is why the problem will be addressed only from a practical viewpoint by combining the evaluation of the cost function with an observation system, which will stop the movement immediately if one of the constraints is violated and inform the optimization algorithm about the occurred situation. After that, the optimization algorithm must generate new parameter values based on these information.

Detecting the violations of the error and the torque constraints can be done simply by comparing the absolute values of the positions and the motor torques to the maximum limits  $e_{max}$  and  $\tau_{max}$  respectively. A bigger challenge by choosing the controller parameters, however, is to detect any excited oscillations in the robot movement. This can not be done analytically because no knowledge of the robot dynamics is assumed. Therefore, the most suitable option here is to do an on-line detection of the oscillations in real time. It is important to do the task on-line so the movement can be stopped as soon as the oscillations are detected in order to prevent any unsafe movement of the robot. A simple -but effective- way to detect oscillations was suggested in (Hägglund, 1995), where the integral of the absolute error *IAE* between successive zero-crossings of the error signal was calculated. This value can distinguish between oscillations and random signals taking into consideration the fact that an oscillating signal will have between the zero-crossings relatively larger *IAE* values and longer time periods in comparison to random signal. As a threshold, the *IAE* of a sinusoidal signal was used, which has an amplitude equals 1% of the control range and a frequency equals the ultimate frequency of the closed loop.

Another approach was introduced in (Forsman and Stättin, 1999), which also was capable of detecting oscillations on-line in the time domain. In this method, an index was defined to estimate the similar-

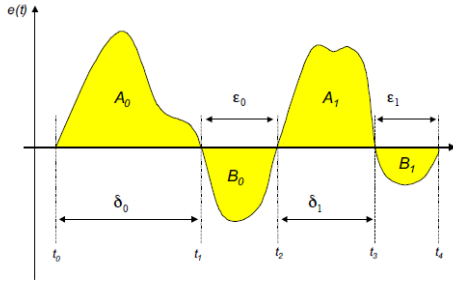


Figure 1: The Oscillation Detection Method from (Forsman and Stattin, 1999).

ity between every two successive zero-crossing IAEs of the error signal. However, This method handled the positive error IAEs and the negative error IAEs separately. Accordingly, the similarity was examined between every two successive areas which had the same sign (positive or negative). This separation enabled the defined criterion to detect asymmetric oscillations. The proposed index was defined as follows:

$$h_A(N_{zc}) = \# \left\{ i < \frac{N_{zc}}{2}; \alpha < \frac{A_{i+1}}{A_i} < \frac{1}{\alpha} \wedge \gamma < \frac{\delta_{i+1}}{\delta_i} < \frac{1}{\gamma} \right\} \quad (2)$$

$$h_B(N_{zc}) = \# \left\{ i < \frac{N_{zc}}{2}; \alpha < \frac{B_{i+1}}{B_i} < \frac{1}{\alpha} \wedge \gamma < \frac{\epsilon_{i+1}}{\epsilon_i} < \frac{1}{\gamma} \right\} \quad (3)$$

$$h(N_{zc}) = \frac{h_A(N_{zc}) + h_B(N_{zc})}{N_{zc}} \quad (4)$$

where  $\#S$  denotes the number of elements in the set  $S$ .  $A_i$  is the IAE of the positive area  $i$  of the error signal,  $B_i$  is the IAE of the negative area  $i$  of the error signal.  $\delta_i$  and  $\epsilon_i$  are the time durations of the positive and the negative areas  $i$  respectively.  $N_{zc}$  is the number of the considered successive zero-crossings.  $0 < \alpha < 1$  and  $0 < \gamma < 1$  are tuning parameters define the degree of similarity. It was suggested in (Forsman and Stattin, 1999) to chose  $\alpha = 0.5 - 0.7$  and  $\gamma = 0.7 - 0.8$ .

In this work, the two previously mentioned indexes will be combined. i. e. both the similarity and the maximum area of the zero-crossing regions will be considered to detect oscillation.

This is done simply by adding another condition to the calculation of the index  $h$  of the second method where the values of  $h_A$  and  $h_B$  increases only if  $A$  and  $B$  are bigger than maximum limits  $A_{max}$  and  $B_{max}$  respectively. This modification on the second method will increase its robustness against low amplitude noises, which might be detected as potential oscillations. The maximum limits are defined as follows:

$$A_{max}, B_{max} = \int_0^{\Delta t_{zc}} 0.01 e_{max} \sin\left(\frac{\pi t}{\Delta t_{zc}}\right) dt = \frac{0.02 \Delta t_{zc} e_{max}}{\pi} \quad (5)$$

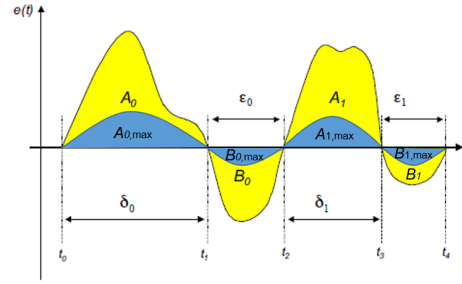


Figure 2: The Proposed Oscillation Detection Method.

With  $\Delta t_{zc}$  being the time between the two corresponding zero-crossings and  $e_{max}$  is the previously defined limit of the position error.

Based on the foregoing, the modified index for oscillations detection becomes:

$$h_A(N_{zc}) = \# \left\{ i < \frac{N_{zc}}{2}; A_i < A_{i,max} \wedge \alpha < \frac{A_{i+1}}{A_i} < \frac{1}{\alpha} \wedge \gamma < \frac{\delta_{i+1}}{\delta_i} < \frac{1}{\gamma} \right\} \quad (6)$$

$$h_B(N_{zc}) = \# \left\{ i < \frac{N_{zc}}{2}; B_i < B_{i,max} \wedge \alpha < \frac{B_{i+1}}{B_i} < \frac{1}{\alpha} \wedge \gamma < \frac{\epsilon_{i+1}}{\epsilon_i} < \frac{1}{\gamma} \right\} \quad (7)$$

$$h(N_{zc}) = \frac{h_A(N_{zc}) + h_B(N_{zc})}{N_{zc}} \quad (8)$$

Because the used controller here is only a PD controller and no integral gain is applied, it is not guaranteed that the error signal will oscillate around the zero value. Therefore, instead of using the error signal, it is chosen to use the difference between the error signal and its mean value  $e(t) - mean(e(t))$ , which will rescale the oscillating signal around the zero value. Finally, the proposed procedure to detect oscillations is achieved by the following steps:

1. After the beginning of the robot movement, record the values of the position error through a long enough period  $\Delta T$ . A reasonable value of  $\Delta T$  is  $0.1T_{tot} - 0.2T_{tot}$ , where  $T_{tot}$  is the total duration of the robot movement.
2. Calculate the function  $e_{zc}(t) = e(t) - mean(e(t))$  along the period  $\Delta T$ .
3. Determine and count the zero-crossing points, and calculate the values  $A$ ,  $B$ ,  $\delta$ ,  $\epsilon$  between these points.
4. For every  $N_{zc}$  successive zero-crossings, calculate  $h_A$ ,  $h_B$  and  $h$ .

5. If  $h > h_{\max}$ , oscillations exist and the movement must be stopped immediately. Otherwise, record the values of the position error for the next period  $\Delta T$  and repeat the previous steps until reaching the end position of the robot.

It was suggested in (Forsman and Stattin, 1999) to choose  $h_{\max} \in [0.4 - 0.8]$  and  $N_{zc} \geq 20$ .

### 3 PARTICLE SWARM OPTIMIZATION

After defining the optimization problem and the necessary constraints, It is now possible to choose one of the optimization methods to find the optimal gains. The chosen method in this work is the particle swarm optimization (PSO).

The PSO, first introduced in (Eberhart and Kennedy, 1995), is a population-based algorithm simulating the movement of a swarm of particles in a predefined space. After a number of iterations, the particles are attracted towards the optimal location and gathered around it. This location defines the optimal solution of the problem.

The PSO has many features that make it efficient in solving optimization problems, e. g. it has less parameters to be identified in comparison with other optimization methods and has a very high success rate in finding the global minimum (Rezaee Jordehi and Jasni, 2013), and it has also a relatively high convergence speed to a near optima (Angeline, 1998).

In our case, each particle position represents a vector with a number of elements equals to the total number of gains. The main contribution of this research is to propose a practical implementation of the PSO method for robotic manipulators.

#### 3.1 Defining the PSO Parameters

For every iteration of the PSO algorithm, every particle of the swarm will have a new position and velocity assigned to it based on the following equations:

$$\mathbf{V}_j(i+1) = \omega(i)\mathbf{V}_j(i) + c_1\gamma_1(\mathbf{P}_j(i) - \mathbf{X}_j(i)) + c_2\gamma_2(\mathbf{G}(i) - \mathbf{X}_j(i)) \quad (9)$$

$$\mathbf{X}_j(i+1) = \mathbf{X}_j(i) + \mathbf{V}_j(i+1) \quad (10)$$

Where  $i$  indicates the current iteration,  $j$  indicates a particle of the swarm,  $\mathbf{X}_j(i)$  is the position vector of the particle  $j$ ,  $\mathbf{V}_j(i)$  is the velocity vector of the particle  $j$ ,  $c_1$  and  $c_2$  are the cognitive and the social acceleration coefficients respectively,  $\omega(i)$  is the inertia factor and  $\gamma_1$  and  $\gamma_2 \in [0 \ 1]$  are random variables

with uniformly distributed values.

There is no standard way to choose the swarm size and the maximum number of iterations. However, both parameters must be high enough in order to guarantee a convergence of the objective value toward the global minimum.

Regarding the inertia weight, a dynamically changing inertia leads to better results than a constant one, and defining  $\omega$  as a linear decreased function is an effective and reasonable choice as it was proven in (Shi and Eberhart, 1999) and (Bansal et al., 2011). In this work, the inertia weight is given as follows:

$$\omega = \omega_{\max} - \frac{(\omega_{\max} - \omega_{\min})N_i}{N_{\max}} \quad (11)$$

where  $N_{\max}$  is the maximum number of iterations,  $N_i$  is the current number of iterations,  $\omega_{\max}$  and  $\omega_{\min}$  are the maximum and the minimum values of the inertia weight respectively. The chosen values in this work are  $\omega_{\max} = 0.9$  and  $\omega_{\min} = 0.4$  as it was suggested in (Shi and Eberhart, 1998).

Based on the stability conditions that was introduced in (Perez and Behdinan, 2007), it is possible to define the parameters  $c_1$  and  $c_2$  in a way that guarantees a convergence toward an equilibrium point eventually. These conditions are:

$$0 < c_1 + c_2 < 4 \quad (12)$$

$$\frac{(c_1 + c_2)}{2} - 1 < \omega < 1 \quad (13)$$

Considering that  $\omega \in [0.4 - 0.9]$ , choosing  $c_1 = c_2 = 1$  is suitable.

#### 3.2 Constraints Handling

After defining the PSO parameters, one step is only needed before applying the method on the robot, which is finding a suitable way to handle the constraints that were defined in the last section. The mostly used methods to handle constraints in PSO are either penalty-based methods, where a penalty value is added to the cost function if the particle position violates any constraints, or methods that try to define the feasible regions in the search space and restrict the particle positions to be always inside these regions. Unfortunately, the practical nature of the optimization problem in this work makes both methods unsuitable. As it was mentioned before, if one of the constraints is violated, the movement of the robot must be stopped immediately, which means that the cost function  $IAE$  cannot be calculated for this movement and, therefore, no penalty-based method can be used. Besides, the feasible region cannot be defined theoretically in advance and the only way to detect a violation of constraints is by performing the movement that results

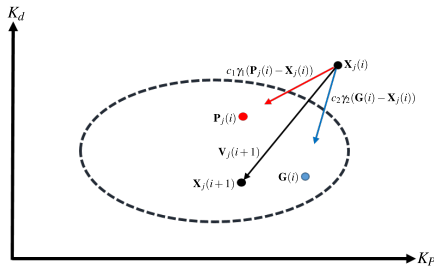


Figure 3: Constraints Handling Method.

according to the particle position.

The proposed method here to handle the constraints was inspired by the work in (Venter and Sobieszczanski-Sobieski, 2003), where modifications of the optimization method were suggested. The Handling method is done after considering the following simplifications:

- Assume that one constraint corresponding to the link  $l$  is violated, if we ignored the coupling effects between the robot links, one can state that the gains  $K_p^l$  and  $K_d^l$  of the corresponding controller are only responsible for this violation and must be modified, i. e. handling the constraints can be done by modifying the particle position in only two dimensions.
- There is only one continuous feasible region inside the search space, i. e. if a particle moves in a direction that leads to a constraint violation, continuing to move the particle in the same direction will also lead to a constraint violation.
- Most of the initial swarm positions are located inside the feasible region.

based on these simplifications, a proper method to keep the particles inside the feasible region can be achieved as follows:

If one of the constraints of the link  $l$  is violated, the exploration term ( $\omega V^l$ ) for the corresponding dimensions are set to zero, which will restrict the next movement in these two dimensions to be only influenced by the personal and the global best positions as it is shown in figure 3. Of course these positions are located in the feasible region, therefore, the resulted velocity vector will bring the particle back to the feasible region.

The Assumption of having only one feasible region is a reasonable assumption, because it indicates that if a gain value violated one of the constraints after it was increased, then continuing to increase it (while keeping the other gains with the same values) will keep violating this constraint. The same applies for when the gain is decreased.

Based on the foregoing, it is now only required to locate the initial swarm inside the feasible region. A

simple strategy is to define only one suitable position  $\{K_{p,0}, K_{d,0}\}$  (e. g. by tuning the controller through trails and errors). After that, one can define an interval in the neighborhood of this position, let it be for example  $\{[K_{p,0} - \Delta K_p, K_{p,0} + \Delta K_p], [K_{d,0} - \Delta K_d, K_{d,0} + \Delta K_d]\}$ . Finally, random initial positions can be allocated with a unified distribution in this interval. It is possible that some of the initial positions may violate one or more of the constraints (if the chosen interval is too large), however, they will return in the following iterations to the feasible region thanks to the proposed method.

## 4 EXPERIMENTAL RESULTS

The proposed tuning method has been tested on a 7-DOF robot, which is built of specially designed modules called PowerCube from the company ‘‘Schunk’’. The experiment was carried on for the joints (3,4,6) of the robot as it is shown in figure 4. All the joints are rotational and actuated by brushless dc-motors.

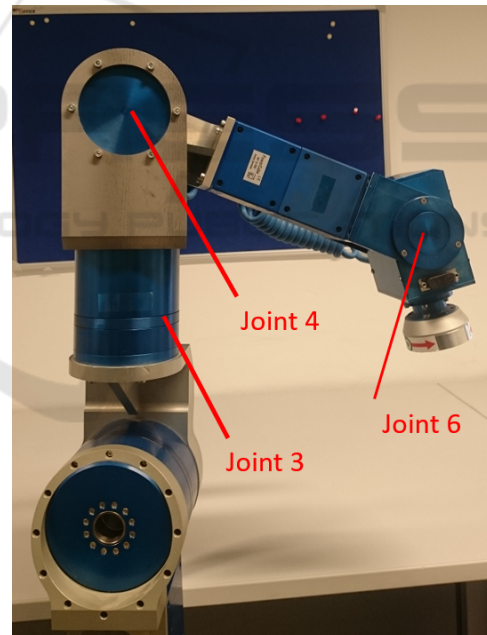


Figure 4: PowerCube Robot.

The trajectory tracking performance of the PD controllers of the robot is tested according to the desired trajectories shown in figure 5. These trajectories represent point-to-point movements with sinusoidal velocity profiles.

To define the constraints for the chosen joints, the value  $e_{max} = \frac{\pi}{90} \approx 0.035[rad]$  was chosen as the maximal position error. The PowerCube modules provide the user with current measurements for

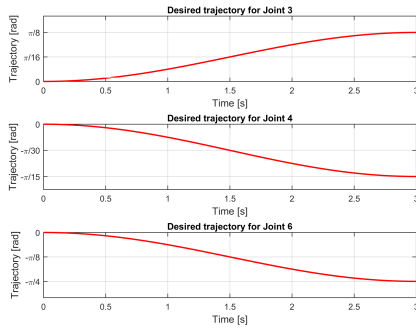


Figure 5: Desired Trajectories in Joint Space.

every motor instead of torque measurements, that is why a maximum current constraints is used in the experiment instead of a maximum torque. The maximum allowed current in all of the modules equals  $15[A]$ , but for the optimization procedure only the half of this value is defined as the current limit ( $I_{max} = 7.5[A]$ ). Oscillations detection is carried on after detecting at least  $N_{zc} = 20$  zero-crossings of the rescaled error signal  $e_{zc}(t)$ . if the condition  $h(N_{zc}) > 0.5$  is met, then oscillations are considered to be occurred and the movement of the robot must be stopped.

Regarding the PSO algorithm, a swarm size of 10 particles is chosen and the maximum number of generations is set to 20. In addition, a tolerance value of the cost function is set to  $0.03[rad.s]$ . To define initial positions for the particle swarm, an acceptable gain set of values was defined through trials and errors which are  $K_{p,0} = [90, 60, 120]$  and  $K_{d,0} = [5, 2, 8]$ . Based on these values, two intervals for the initial gains are defined as follows:  $[K_{p,0} - K_{p,0}/2, K_{p,0} + K_{p,0}/2]$  and  $[K_{d,0} - K_{d,0}/2, K_{d,0} + K_{d,0}/2]$ .

The initial positions were then determined randomly from inside this interval with a uniform distribution. The search space for all the  $K_p$  gains is defined to be  $[1 - 500]$  and for the  $K_d$  gains  $[0 - 50]$ , which make the maximum gain limits be relatively high compared to the initial gains. However, the maximum velocity value  $V_{max}$  was set to be equal 20% of these maximum limits in order to avoid big leaps in the particle movement.

After applying the PSO on the PD-controllers for the three joints of the robot, the following optimal gain values was found:

$K_p = \{182.20, 107.46, 205.8\}$  and  $K_d = \{10.29, 5.55, 17.83\}$ . The optimal objective value is:  $IAE = 0.0376[rad.s]$ .

The convergence of the objective value during the searching procedure is shown in figure 6, while figure 7 shows the position error diagram according to the optimal gain values.

One may notice that some low amplitude oscillations

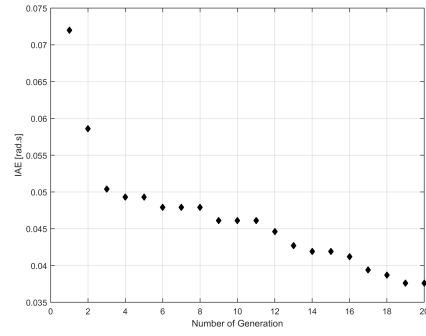


Figure 6: Convergence of the Objective Value After 20 Generations.

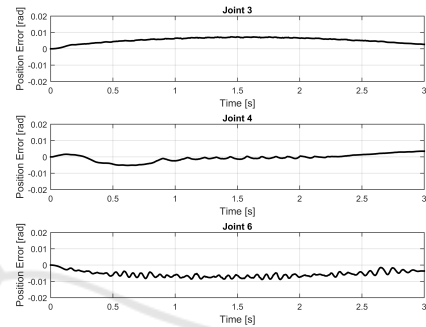


Figure 7: Position Error Signals According to the Optimal Gains.

appear on the error diagram of the joint 6. These oscillations, however, are not high enough to violate the oscillation constraint. The fact that the optimal gains are causing such oscillations indicates that there is a tradeoff between the tolerated oscillations and the accuracy of the trajectory tracking movement. This also means that the PSO technique was able to direct the particles toward the limits of the feasible region to the position where the optimal gain values are expected to be.

## 5 CONCLUSION

In this paper, an auto-tuning method of PD-controllers for robotic manipulators has been proposed. The suggested approach uses the particle swarm optimization in order to find the optimal control parameters. The main contribution of this work was to address the practical challenges that faces such a task and to propose suitable solutions for it. For this sake, the necessary constraints that guarantee a stable and safe movement for the robot while searching for the optimal gains have been defined. Additionally, a suitable way for the PSO to handle these constraints has been suggested. Finally, the proposed approach has been successfully experimented on a real robot.

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