Research on the Mission Critical Parameters Identification by using Kinematic Boundaries

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Abstract: Finalization of missile system requirement specifications in design phase is critical in order to achieve user's expectations and to avoid unexpected outcomes. This is guaranteed through well defined performance evaluation criteria. A methodology is devised to finalize and evaluate missile sub-systems characteristics with emphasis on its ability to meet mission specific goals. Missile system evaluation is achieved through computing kinematic boundaries against highly agile targets. Kinematic boundaries includes minimum and maximum launch points. These launch points are the sequel of 3-DoF missile and target engagement simulations. In order to reduce computation time, coarse and fine search has also been introduced. Mission critical sub-systems are identified through relative frequency analysis. Once requirements are finalized, technical challenges associated with respective sub-systems are eliminated by suggesting efficient missile launch strategies.

1 INTRODUCTION

Finalization of system requirement specification plays a vital role in terms of acceptable outcomes for the end user. Considerable amount of time and resources were often allocated in the preliminary design phases to avoid unexpected outcomes. The ample time dispense in the initial requirement finalization will not only expedite the development processes but also saves substantial amount of resources. In the recent years many tools and techniques have been introduced that ease in the analysis of proposed requirements. These tools not only reduce designers load but also mitigate the chances of human errors in safety critical applications. In complex aerospace programs, weaknesses and demerits associated with finalized system requirements may results due to insufficient resources or technical difficulties. Such imperfections can be annihilated by incorporating better deployment strategies.

The task of finalizing initial requirements starts with the development of prototypes. Prototypes are the abstract representation of the actual system. In case of complex and expensive systems prototypes are generally the computer based software modules. These modules includes the mathematical representations of vital sub-system characteristics. To get a quantitative assessment of overall system, the need to develop abstract level mathematical programs can not be neglected (Moore, 2015). For the proof-of-concept, the verification of system through prototypes with remarkable cost-benefit ratio has been achieved through automated tool like Model Analyzer/Checker (Storrle, 2015). As illustrated in our study, the utilization of abstract modules can play a vital role when evaluated in an actual dynamic environment.

Many complex simulations have been developed to evaluate system effectiveness in actual combat scenario. Such simulations are it self a big programs like TISES which provides system evaluations for THAAD, US anti-ballistic missile system (Dawn Horn, 1997). These complex simulations require detailed level of system modeling. In order to meet the needs of iterative design process, top level exemplar comprising salient features are more efficient in terms of time and cost.

Modern missile systems comprehend numerous subsystems that should be able to perform in an integrated environment. Thus the lofty or rigid attributes of one part will directly affect the performance of over all system. Like the aerodynamic of the missile is linked with the airframe structures, bulky structures have more strength but are less aerodynamically efficient. Similarly optimum functionality of guidance and control is associated with the stable output of seeker and actuator systems. So studies and simula-

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tions that ensure the balanced outcomes of each entity without compromising on the performance of overall system is pertinent in the realm of systems engineering (Kilian James C, 2016). In the preliminary design stage, simple interactive models are enough to see the influence of one element on the overall performance, but in the validation period the detailed hardware in loop simulations are often used to get the expected outcomes (Chang Xiaofei, 2012). However, our proposed scheme shall unmask the fact that even with these abstract models, one can get acceptable results when operational environment is also a part of our simulations.

Performance of the missile system is often compromised due to inadequate resources. To over come this challenge different tactics and weapon deployment strategies have been in use. To maximize engagement envelop (James M. Davies and Oxtoby., 2011) used probabilistic method to predict target location upon detection. (Vergez, 1998) proposed the implementation of extended Kalman filter (EKF) to estimate the target position from limited target acceleration data provided by onboard passive seekers. However such methodologies are difficult to meet the available time budget in short range missile concepts. In case of short range air-to-air combat, pilots also prefer their experiences instead of relying on onboard guidance optimization algorithms. Thus, our proposed evaluation algorithm intend to educate the training pilots to get maximum probability of kill by adopting those attack angle where efficiency of respective sub-system is maximum.

This paper recommends the usage of abstract level of air-to-air missile prototypes to evaluate system performance in early design phase. These evaluation studies are more efficient when incorporated with operational scenarios. The novelty associated with our research is to demonstrate the utility of missile kinematic model in predicting mission critical parameters. Also highlighted the application of proposed algorithm in exploiting trade-offs between other complex sub-systems. Technical challenges can be over come through adopting tactical deployment strategies which are also helpful to develop training simulators for the pilots.

2 MATHEMATICAL MODEL OF MISSILE AND TARGET

The three degree of freedom mathematical models have been used to develop missile and target engagement simulation. In this section these models shall be discussed in detail.

2.1 Missile and Target Model

A 3-DoF point mass simulation is used for the missile and target simulations. Two frame of references shall be used in this paper. One is north, east and down (NED) inertial frame of reference and other is the standard body frame, for reference see Fig. (1). Equation of motion governing the dynamics of missile under the influence of thrust, drag and gravity in inertial frame of reference are shown below;

$$\begin{bmatrix} \dot{V_m} \\ \dot{\Psi_m} \\ \dot{\gamma_m} \\ \dot{X_{i_m}} \\ \dot{Y_{i_m}} \\ \dot{Z_{i_m}} \end{bmatrix} = \begin{bmatrix} \frac{T_m - D_m}{m_m} - g\sin(\gamma_m) \\ \frac{a_{y_m}}{V_m \cos(\gamma_m)} \\ \frac{a_{z_m} - g\cos(\gamma_m)}{V_m} \\ V_m \cos(\gamma_m)\cos(\Psi_m) \\ V_m \cos(\gamma_m)\sin(\Psi_m) \\ V_m \sin(\gamma_m) \end{bmatrix} \tag{1}$$

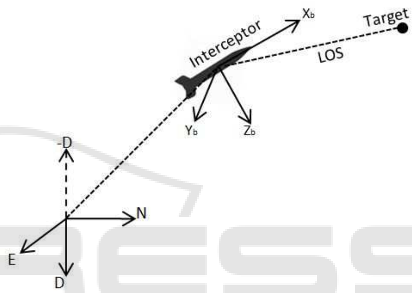

Figure 1: Inertial and body frame of reference.

Lift and drag coefficients in the form of look-up table, are the function of mach number and angle of attack. Thrust and mass are the function of time. Other parameter and variable used in this simulation are mentioned in the Tab. (1). Actual values of these parameters are omitted here because of regulatory restrictions.

Table 1: Missile parameters and variables.

Parameter	Description
S_{ref}	Surface Area(m^2)
d	Diameter(m)
m_m	Mass(Kg)
γ_m	Flight Path Angle(rad)
Ψ_m	Azimuth Angle(rad)
D_m	Drag(N)
$X_{i_m}, Y_{i_m}, Z_{i_m}$	Inertial Positions(m)
a_{y_m}, a_{z_m}	Measured Accelerations(m/sec^2)
T_m	Thrust(N)
V_m	Velocity(m/sec)

Missile guidance commands are generated form conventional PN guidance law using Eq. (2), in which the missile velocity vector rotate in a rate proportional to the rotation rate of line of sight, and in the same direction.

$$A_c = N\dot{\lambda}V_c \tag{2}$$

Here A_c is the commanded acceleration form PN law, N is the navigational constant, $\dot{\lambda}$ and V_c are the relative to the target line of sight rate and velocity vector in body frame of reference. Target is assumed to retain its velocity magnitude once the missile is launched from the aircraft. Target maneuvers in inertial frame are implemented by using Eq. (3).

$$\dot{V}_t = ng[\hat{v}_t \times \hat{n}] \tag{3}$$

Here n is the desired acceleration load factor, g is the gravitational acceleration. $\hat{v}_t$ is the target's velocity unit vector and $\hat{n}$ is the normal to plane unit vector, on which target maneuver is required. For example if target is required to perform a positive east side maneuver in NE frame, then normal to plane unit vector in NED inertial frame of reference is $[0,0,-1]$.

3 PROBLEM STATEMENT WITH FORMULATIONS

3.1 Problem Statement

Mission critical sub-system identification is the prima for complex aerospace programs. (Kascha Christian, 2015) proposed moving average method for model identification and performance evaluation through Monte Carlo simulation. In case of missiles, required dynamic performance against the agile targets make it difficult to finalize the sub-system specification through probabilistic models (Yi Ping, 2014). Hence research to identify and finalize key performance indicators in operational scenario is studied here. The lack of knowledge sharing between users and program designers also limit the utility of product to its full extent. Machine learning is suggested in (Roehm Tobias, 2013) to identify mismatch between user and developer. In this paper we devised knowledge based deployment strategies of missiles to overcome sub-system identification and limitations.

3.2 Proposed Formulations

The mathematical approach presented here is to first implement missile kinematic model against the agile target. Then missile sub-systems behaviour were analyzed with varying parameters. Mission specific target models with relative frequency distribution were studied to finalize critical sub-system characteristics. Finally, remedial tactics in terms of initial launch conditions shall be helpful in eliminating sub-systems requirement limitations.

3.2.1 Kinematic Boundaries Computation

Kinematic boundaries consists of missile maximum and minimum launch ranges. These boundaries are the result of multiple simulations between missile and target with monotonically varying initial conditions using coarse and fine search techniques. The initial geometry between missile and target is assumed to be unchanged at the time of launch so monotonic search is sufficient to provide required results 3.2.4. This algorithm is used as a basic tool to access the kinematic capability of the missile to hit the target, with pre-defined sub-system characteristics. The detail description to computed kinematic ranges is described below;

3.2.2 Compute Maximum Range$(Rmax)$

Rmax is computed first using coarse search method followed by fine search method. In coarse search, first fly-out starts from the farthest possible launch point w.r.t the target at specific aspect angle. The aspect angle is the angle between the body longitudinal axis of the target and the LOS vector (Ronghui Zhan, 2012). This point indicates maximum detection range of short range missile's Eq. (4). If missile hits the targets, fine search method will further tune the results. Upon miss, first fly-out comes closer to the target until hit. Target is assumed to perform evasive maneuver once missile is launched.

$$R_{max0} = \max(R_{detect}(\theta)) \tag{4}$$

Where R_{max0} is the starting point of first fly-out for Rmax, R_{detect} is the maximum detection range of missile seeker and θ is the target aspect angle. Binary search is used to compute the fine search between lower bound (LB) and upper bound (UB) provided by coarse search. In case of Rmax computation, lower bound is the last hit and upper bound in the last miss. The purpose of binary search is to reduce the coarse step error to fine step. The ratio of coarse to fine step is 20:1, which is good enough to reduce the computation time. Pseudocode to compute binary search is written below;

3.2.3 Compute Minimum Range $(Rmin)$

Rmin is computed first using coarse search method followed by fine search method. In coarse search, first fly-out starts from the nearest possible launch point w.r.t the target at specific aspect angle see Eq. (5).

Algorithm 1: Binary Search for Rmax Computation.

```
Data: Lower and upper values of coarse Rmax.
      ranges
Result: Fine Range for Rmax.
while difference of upper lower bound greater
 than fine step do
    Update missile and target geometry based
     on middle point;
    if miss then
        upper bound = middle point;
        middle point = upper
         bound-(difference)/2;
        go back to read middle value;
    else
        lower bound = middle point;
        middle point = lower
         bound+(difference)/2;
        go back to read middle value;
    end
end
```

Upon miss, first fly-out goes farther from the target until hit. Target is assumed to perform evasive maneuver once missile is launched.

$$R_{min0} = V_{m_0} t_{max_0} + \frac{1}{2} A_l t_{max_0}{}^2 \qquad (5)$$

V_{m_0} is the missile initial launch velocity, t_{max_0} is the time associated with launch button delay plus safe and arming sub-system activation delay. A_l is the initial longitudinal acceleration jerk from the launcher necessary for the safe release of the missile. Binary search is used to compute the fine search between lower bound (LB) and upper bound (UB) provided by coarse search. In case of Rmin computation, lower bound is the last miss and upper bound is the last hit. The purpose of binary search is to reduce the coarse step error to fine step. The ratio of coarse to fine step is in case of Rmin is also 20:1, to reduce the computation time. Pseudocode to compute Rmin binary search is mentioned below;

Algorithm 2: Binary Search for Rmin Computation.

```
Data: Lower and upper values of coarse Rmin.
      ranges
Result: Fine Range for Rmin.
while difference of upper lower bound greater
 than fine step do
    Update missile and target geometry based
     on middle point;
    if miss then
        lower bound = middle point;
        middle point = lower
         bound+(difference)/2;
        go back to read middle value;
    else
        upper bound = middle point;
        middle point = upper
         bound-(difference)/2;
        go back to read middle value;
    end
end
```

3.2.4 Sub-systems Dependence Formulations

Seeker system, structural characteristics and terminal energy are the key characteristics to be evaluated. Tab. (2) shows the maximum or minimum acceptable criteria for each sub-system qualification. The engagement simulation must stop upon expiry of any criteria listed below. New search point for Rmax and Rmin shall be adjusted to get the actual performance of the missile against the desired targets.

In dog fight scenarios, target aircraft perform evasive maneuvers or use after burner to out run the incoming missile. Thus the minimum energy required by the missile should be evaluated after rocket motor burns out.

Table 2: Sub-systems evaluation criteria.

Sub-System	Terminal Value	Description
Seeker	LA_{max}	Max. Look Angle
Seeker	TR_{max}	Max. Track Rate
Control Power	G_{max}	Max. Load
Propulsion	$-VC_{max}$	Min. Closure Rate

Feasible terminal values mentioned in Tab. (2) largely depends upon the available resources and users expectation. To get these estimates simulated analysis is proposed here. Utility of such analysis is to investigate the trade-offs that might exist between various section of the missile. Tab. (3) depicts the possible sub-systems design ranges that can be examined and formalized through kinematic simulation. The reference system details are also mentioned to compare the differences resulted form one specific sub-system variation.

Table 3: Sub-system design ranges.

Sub-System	Range	Ref. Value
Look Angle	[30 to 50]	40(deg)
Track Rate	[22 to 35]	30(deg/sec)
Closure Rates	[-400 to -600]	-500(m/sec)
Load or Control Power	[15 to 20]	15
Diameter	[5 to 7]	5(in)
Thrust	[T_m*1 to T_m*1.3]	T_m(t)(N)
Mass	[m_m*1 to m_m*1.3]	m_m(t)(Kg)

3.2.5 Mission Critical Parameter Identification

There is no generic formula to device requirement finalization. The ratified outcomes of a system highly depends on an organizational resources, mission objectives and user satisfactions. Relative frequencies of each sub-system in case of miss is devised to identify critical sub-systems (Lin Chin-Yew, 2016) and (Michael, 2015). Requirement finalization is achieved by forcing relative frequency of each sub-system at some moderate value without compromising on the missile performance. Kinematic envelop of missile is enhanced once the mission critical relative frequencies were reduced. Relative frequency of each section when missile miss the target is computed using Eq. (6). This formulation can also help to allocate major resource to those sub-systems that are mission imperative.

$$f_s = \frac{m_s}{\sum\limits_{\theta} m_s} \tag{6}$$

f_s is the relative frequency of each sub-system in case of miss, m_θ is the number of frequency of miss caused by this sub-system at particular aspect θ divided by the total number of missile miss at all aspect angles θ (0° to 360°) influenced by this sub-system.

3.2.6 Remedial Tactics

Shortcomings associated with our finalized requirements can be eliminated through incorporating efficient launch tactics. Frequency distribution in terms of comparative histograms are computed. These histograms show the number of miss caused by each sub-system at specific aspect angle θ. Such distributions can educate the pilots to avoid those aspect angles where frequency distribution of particular sub-system is maximum. Simulated results that can be helpful to understand proposed remedial strategies are discussed in section 4.

4 SIMULATION RESULTS

3-DoF simulated missile and target simulations shall be used to compute missile kinematic ranges. These ranges shall be the function of iterative fly-outs between missile and target engagements. RK4 numerical integration is used to update missile and target states (Z. Kalogiratou, 2010). Sub-system parametric identification and finalization are carried against the reference missile mentioned in Tab. (3) using initial conditions tabulated Tab. (4). Following assumptions are made to compute kinematic boundaries;

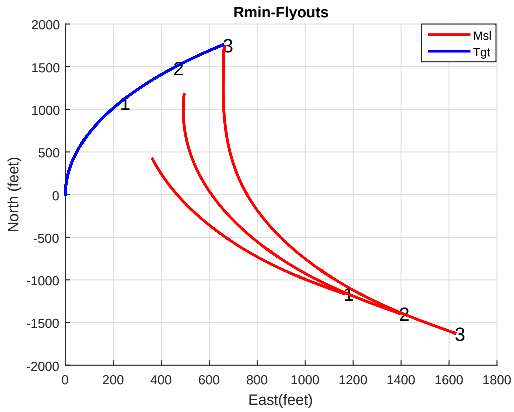

Figure 2: Rmin fly-outs against 5g's target turn.

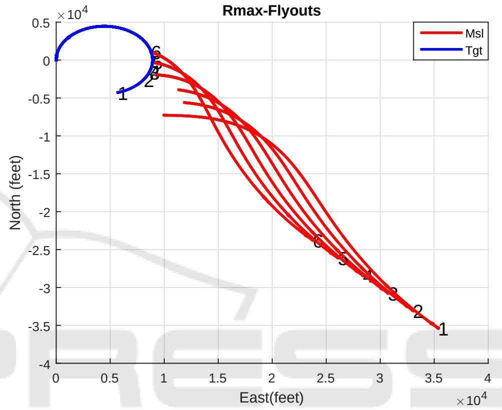

Figure 3: Rmax fly-outs against 3g's target turn.

1. Missile seeker always looked-on to the target.
2. Missile initial velocity vector aligned with the line of sight vector.
3. Target aircrafts starts evasive maneuvering once missile is fired from the aircraft.
4. Target evasive load factor to compute Rmax and Rmin is 3g's and 5g's respectively.

Table 4: Initial conditions for parametric studies.

Msl. Vel	Tgt. Vel	Msl. Alt	Tgt. Alt
$200m/sec$	$200m/sec$	$10,000ft$	$10,000ft$

Rmin and Rmax fly-outs at specific aspect angle are shown in Fig. (2) and Fig. (3) respectively. These fly-outs are iteratively updated at every miss till hit is achieved as mentioned in Sec. (3.2.2) and Sec. (3.2.3). These fly-outs models with varying evaluation parameters mentioned in Tab. (2) help us to identify parametric influence on over all performance. Design parameter of each subsystem is compared with the reference system mentioned in Tab. (3). This table also provides the case study values for each section to be

Table 5: Sub-system relative frequencies.

Case	$V_m(m/sec)$	$V_t(m/sec)$	$H_m(ft)$	$H_t(ft)$	n	$-VC_{max}$	LA_{max}	TR_{max}
1	200	200	10,000	10,000	0	0.761	0.238	0.00
2	300	200	10,000	10,000	0	0.860	0.139	0.00
3	400	200	10,000	10,000	0	0.899	0.100	0.00
4	200	200	10,000	10,000	3	0.850	0.122	0.03
5	200	200	15,000	5,000	0	0.813	0.186	0.00

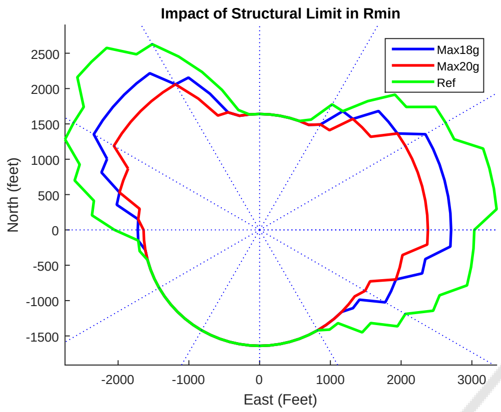

Figure 4: Rmin variation with structural g's.

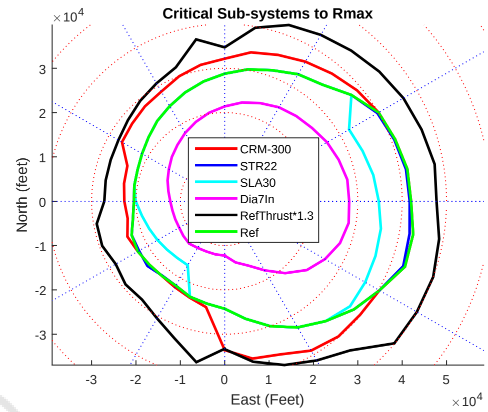

Figure 6: Sub-systems more sensitive to Rmax computation.

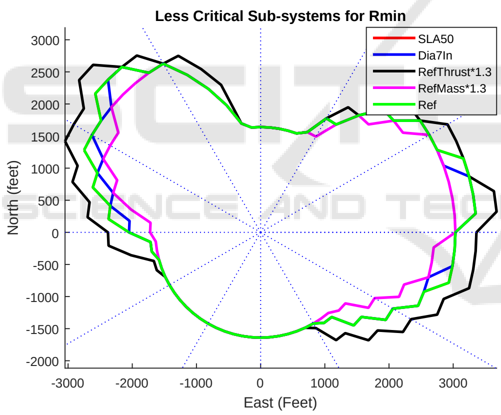

Figure 5: Sub-systems less sensitive to Rmin computation.

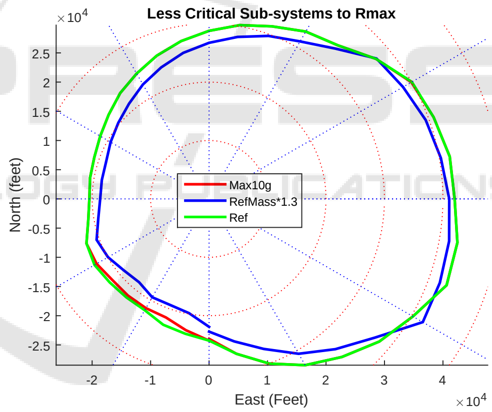

Figure 7: Sub-systems less sensitive to Rmax computation.

compared. Figures associated with Rmin computations can conclude that improved structural loading or control power in terms of allowable g's Fig. (4) and seeker track rates Fig. (9) gives close combat advantage to the pilots. On the other hand, designer can choose moderate values of missile mass, diameter, thrust and seeker look angles as elaborated through Fig. (5) for close combat missions.

Influence of critical sub-systems for Rmax computations are elaborated through Fig. (6). Figures associated with Rmax computations can conclude that improved aerodynamic, propulsion and seeker look angles can provide long range combat advantages to the pilots. Moderate values of missile mass and control power in terms of allowable g's are sufficient for long range attack scenarios Fig. (7).

The Rmin sub-system limitation in the form of frequency distribution against the stationary target and moving target is shown in Fig. (8) and Fig. (9) respectively. Green pattern which reflects the missile seeker track rate limitation, shows the major limitation in Rmin computations. Black pattern is the indication of missile hit once seeker track rate is enough to track the close range target. This distribution clearly indicates that in close combat, seeker track rate is one of the dominant element to consider.

Similarly, Rmax sub-system limitation in the form of frequency distribution against the stationary target and moving target is shown in Fig. (10) and Fig. (11) respectively. Red pattern reflects the missile energy

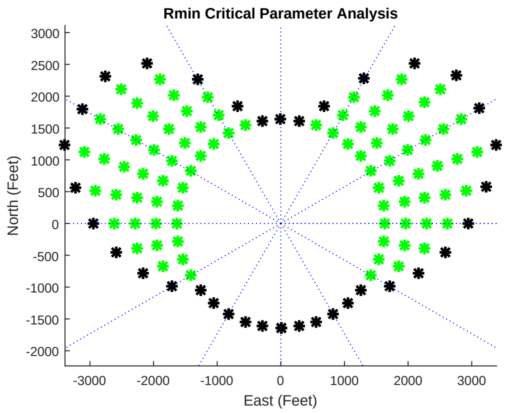

Figure 8: Critical sub-systems in Rmin against stationary targets.

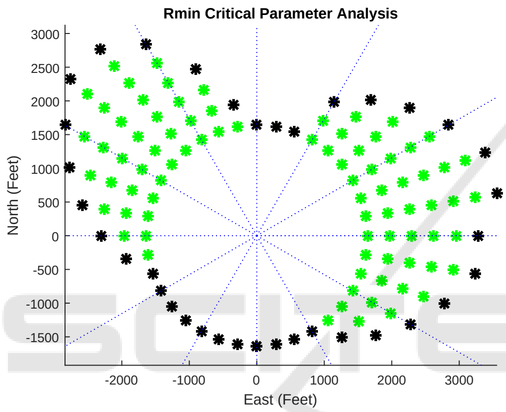

Figure 9: Critical sub-systems in Rmin against maneuvering targets.

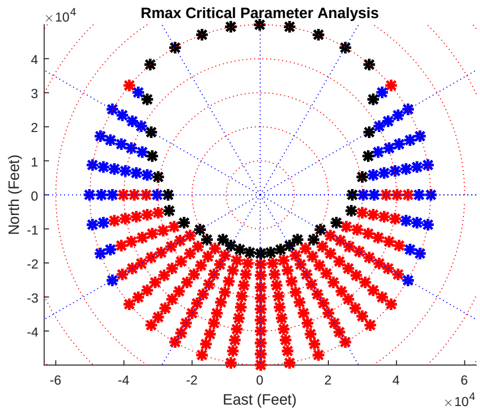

Figure 10: Critical sub-systems in Rmax against stationary targets.

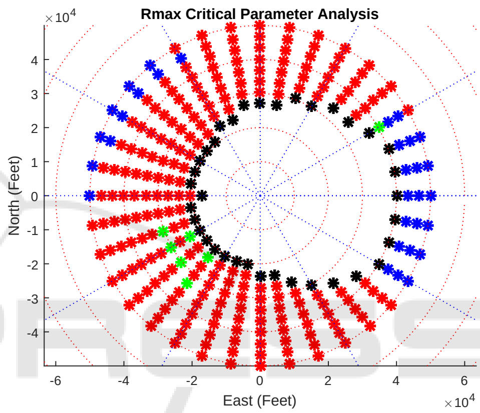

Figure 11: Critical sub-systems in Rmax against maneuvering targets.

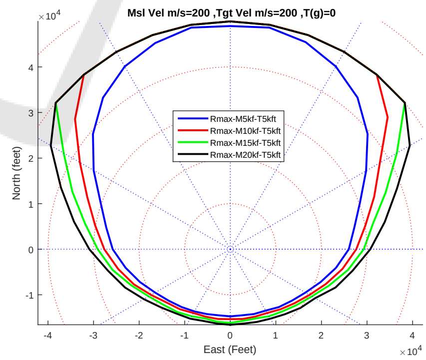

Figure 12: Rmax variation with altitude.

limitation required to perform the hard turns in terminal engagements. This energy requirement is in terms of minimum closure rate $V_t - V_m$ allowed once rocket motors expires. Another dominant section is the missile seeker look angle constraint. Black pattern is the indication of missile hit when is meets the required evaluation criteria see Tab. (2). Hence, it can be stated that rocket motor and seeker look angles are the primary parameters in long range missile designs.

Relative frequency of evaluation criteria in case of missile fly-out termination is listed in Tab. (5). This table shows the detailed operational evaluation of missile against maneuvering target at different speeds and altitude. Reference missile terminal energy and seeker look angle limits the longer kinematic ranges. Thus for long range engagements, missile propulsion and seeker look angles characteristics can not be undermined.

As mentioned before in Sec. (3.2.6) missile launch tactics plays significant role in mitigating the design challenges. Increase in the launch aircraft speed and altitude enhance missile long range kinematic capabilities. Increase in Rmax of reference missile Tab. (3) is possible through increasing relative speed and altitude w.r.t the target. Advantages associated with relative initial geometry are shown in Fig. (12) and velocity Fig. (13). Such type of tactical remedies may overcome the design limitations.

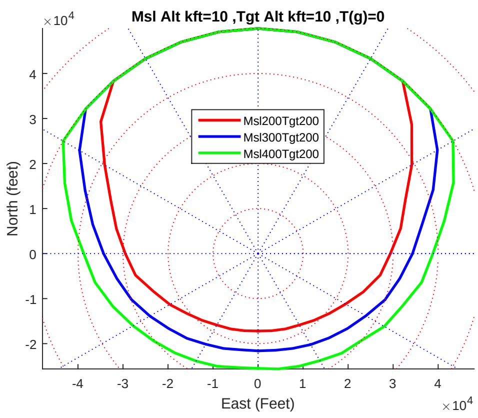

Figure 13: Rmax variation with speed.

5 CONCLUSIONS

Identification of mission critical parameters is very imperative to meet user requirements. The utility of automated tools based on the system mathematical behaviour is necessary to meet iterative design processes. By incorporating operational scenarios, these evaluation models becomes more efficient. Technical challenges associated with finalizing system specification can be overcome by utilizing remedial tactics. Missile system is being evaluated through developing kinematic model against highly agile targets. Missile seeker, propulsion and structural loading or control power are the key sub-systems to be evaluated. Individual impact of each sub-system is carefully examined in an operational environment. Identification of mission critical parameters are proposed through relative frequency distributions. Sub-system design limitations are overcome through highlighting missile launch strategies. It is concluded that missile seeker track track rate greatly influence close combat situations. Long range combat advantage is possible through improved propulsion and seeker look angle limit.

In future, adversary's missile dynamics can be included in friendly missile kinematic model, to high light those parameters that can provide situational advantage in one-to-one air combat scenarios.

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